Activity Status: **Fast Transverse Beam Diagnostics**

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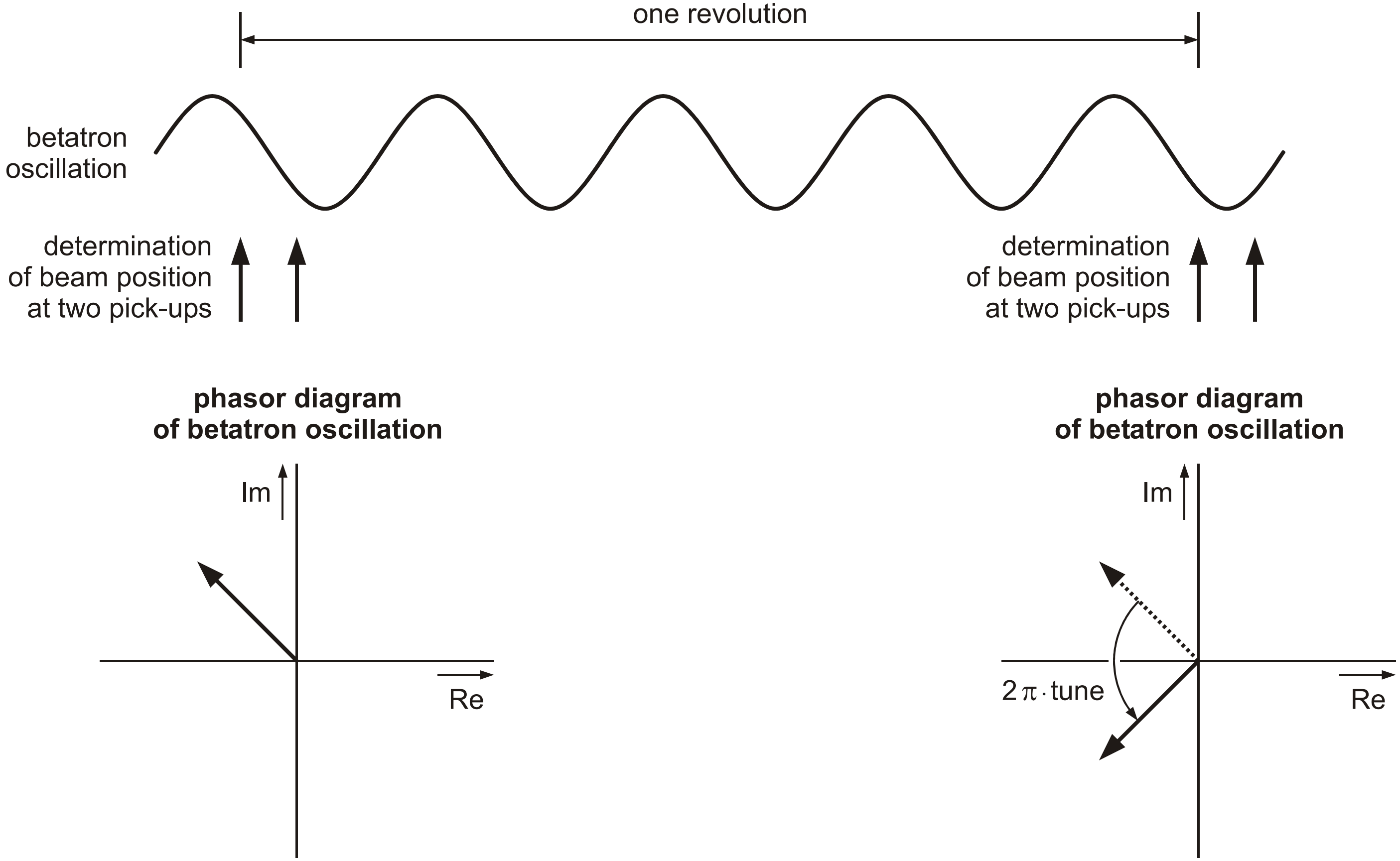
LCE-Meeting

Actual achieved Points

- tune measurement of single bunch within one turn
- fast coupled (rigid) bunch model usable in Simulink
- principle test of fast modal analysis
- first simulations of ‘conventional’ coupled bunch feedbacks

One Turn Tune Measurement



tune information is necessary for fast modal analysis

Fast Coupled (Rigid) Bunch Model

- we would like to use the discrete state space formalism (discrete models are faster!)
- one simulation step is about ¼ betatron oscillation long
- we have to transform the state space description to the diagonal from to get it discrete

what does this mean …

State Space Formalism: one Oscillator

equation of motion (driven oscillator):

$$\frac{d^2 x(t)}{d\,t^2} + \omega^2\, x(t) = \omega^2\, u(t)$$

continuous state space representation:

$$\frac{d}{d\,t}\begin{pmatrix}\dot{x}\\ x\end{pmatrix} = \overbrace{\begin{pmatrix}0 & -\omega^2\\ 1 & 0\end{pmatrix}}^{=\mathbf{A}}\begin{pmatrix}\dot{x}\\ x\end{pmatrix} + \overbrace{\begin{pmatrix}\omega^2\\ 0\end{pmatrix}}^{=\mathbf{B}} u$$

$$y = \underbrace{\begin{pmatrix}0 & 1\end{pmatrix}}_{=\mathbf{C}}\begin{pmatrix}\dot{x}\\ x\end{pmatrix} + \underbrace{0}_{=\mathbf{D}}\, u$$

discrete state space representation:
(by using the solution of the homogenous equation of motion!)

$$\begin{pmatrix}\dot{x}/\omega\\ x\end{pmatrix}_{n+1} = \overbrace{\begin{pmatrix}\cos\omega T & -\sin\omega T\\ \sin\omega T & \cos\omega T\end{pmatrix}}^{=\mathbf{A}}\begin{pmatrix}\dot{x}/\omega\\ x\end{pmatrix}_{n} + \overbrace{\begin{pmatrix}\sin\omega T\\ 0\end{pmatrix}}^{=\mathbf{B}} u_n$$

$$y_n = \underbrace{\begin{pmatrix}0 & 1\end{pmatrix}}_{=\mathbf{C}}\begin{pmatrix}\dot{x}/\omega\\ x\end{pmatrix}_{n} + \underbrace{0}_{=\mathbf{D}}\, u_n$$

State Space Formalism: two Oscillators

equation of motion :

$$\frac{d^2}{dt^2}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \overbrace{\begin{pmatrix} \omega^2 + \alpha\,\omega^2 & -\alpha\,\omega^2 \\ -\alpha\,\omega^2 & \omega^2 + \alpha\,\omega^2 \end{pmatrix}}^{=\mathbf{U}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \overbrace{\begin{pmatrix} \omega^2 + \alpha\,\omega^2 & 0 \\ 0 & \omega^2 + \alpha\,\omega^2 \end{pmatrix}}^{=\mathrm{K}_p} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

continuous state space representation:

$$\frac{d}{dt}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_1 \\ x_2 \end{pmatrix} = \overbrace{\begin{pmatrix} 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$-\mathbf{U}$}} \\ 0 & 0 & & \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}^{=\mathbf{A}} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_1 \\ x_2 \end{pmatrix} + \overbrace{\begin{pmatrix} \mathrm{K}_p \\ 0 \quad 0 \\ 0 \quad 0 \end{pmatrix}}^{=\mathbf{B}} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{=\mathbf{C}} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_1 \\ x_2 \end{pmatrix}$$

Discrete State Space Description: two Oscillators

we diagonalise the matrix U in the homogenous equation of motion with:

$$\mathbf{T} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\frac{d^2}{dt^2}\overbrace{\mathrm{T}^{-1}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}^{=\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix}} + \overbrace{\mathrm{T}^{-1}\begin{pmatrix} \omega^2 + \alpha\,\omega^2 & -\alpha\,\omega^2 \\ -\alpha\,\omega^2 & \omega^2 + \alpha\,\omega^2 \end{pmatrix}\mathrm{T}}^{=\begin{pmatrix} \Omega_1 & 0 \\ 0 & \Omega_2 \end{pmatrix}}\ \mathrm{T}^{-1}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

then we get two independend ‘oscillators’ and can write directly:

$$\begin{pmatrix} \dot{\tilde{x}}_1/\Omega_1 \\ \tilde{x}_1 \\ \dot{\tilde{x}}_2/\Omega_2 \\ \tilde{x}_2 \end{pmatrix}_{n+1} = \overbrace{\begin{pmatrix} \cos\Omega_1 T & -\sin\Omega_1 T & 0 & 0 \\ \sin\Omega_1 T & \cos\Omega_1 T & 0 & 0 \\ 0 & 0 & \cos\Omega_2 T & -\sin\Omega_2 T \\ 0 & 0 & \sin\Omega_2 T & \cos\Omega_2 T \end{pmatrix}}^{=\mathbf{A}}\begin{pmatrix} \dot{\tilde{x}}_1/\Omega_1 \\ \tilde{x}_1 \\ \dot{\tilde{x}}_2/\Omega_2 \\ \tilde{x}_2 \end{pmatrix}_n + \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}}^{=\mathbf{B}}\begin{pmatrix} \sin(\Omega_1\,T)\,\tilde{u}_1 \\ \sin(\Omega_2\,T)\,\tilde{u}_2 \end{pmatrix}_n$$

Input- and Output-Variables

the input and output-variables should be in the not transformed representation:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \sin(\Omega_1 T)\tilde{u}_1 \\ \sin(\Omega_2 T)\tilde{u}_2 \end{pmatrix}_n = \begin{pmatrix} \sin(\Omega_1 T) & 0 \\ 0 & 0 \\ 0 & \sin(\Omega_2 T) \\ 0 & 0 \end{pmatrix} \mathbf{T}^{-1} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_n = \frac{1}{\sqrt{2}} \begin{pmatrix} \sin(\Omega_1 T) & \sin(\Omega_1 T) \\ 0 & 0 \\ \sin(\Omega_2 T) & -\sin(\Omega_2 T) \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_n$$

respectively:

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_n = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{\tilde{x}}_1/\Omega_1 \\ \tilde{x}_1 \\ \dot{\tilde{x}}_2/\Omega_2 \\ \tilde{x}_2 \end{pmatrix}_n \quad \Rightarrow \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_n = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{\tilde{x}}_1/\Omega_1 \\ \tilde{x}_1 \\ \dot{\tilde{x}}_2/\Omega_2 \\ \tilde{x}_2 \end{pmatrix}_n$$

we get …

Discrete State Space Description: two Oscillators

the fast model:

$$\begin{pmatrix} \dot{\tilde{x}}_1/\Omega_1 \\ \tilde{x}_1 \\ \dot{\tilde{x}}_2/\Omega_2 \\ \tilde{x}_2 \end{pmatrix}_{n+1} = \overbrace{\begin{pmatrix} \cos\Omega_1 T & -\sin\Omega_1 T & 0 & 0 \\ \sin\Omega_1 T & \cos\Omega_1 T & 0 & 0 \\ 0 & 0 & \cos\Omega_2 T & -\sin\Omega_2 T \\ 0 & 0 & \sin\Omega_2 T & \cos\Omega_2 T \end{pmatrix}}^{=\mathbf{A}} \begin{pmatrix} \dot{\tilde{x}}_1/\Omega_1 \\ \tilde{x}_1 \\ \dot{\tilde{x}}_2/\Omega_2 \\ \tilde{x}_2 \end{pmatrix}_n + \overbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} \sin(\Omega_1 T) & \sin(\Omega_1 T) \\ 0 & 0 \\ \sin(\Omega_2 T) & -\sin(\Omega_2 T) \\ 0 & 0 \end{pmatrix}}^{=\mathbf{B}} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_n$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_n = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{\tilde{x}}_1/\Omega_1 \\ \tilde{x}_1 \\ \dot{\tilde{x}}_2/\Omega_2 \\ \tilde{x}_2 \end{pmatrix}_n$$

This procedure is also applicable to a larger number of coupled oscillators!

To Practice …

Simulations to show…

With Simulink …

- SimpleTransverseDamper_discrete.mdl
- DiscreteStateSpaceModel1HO.mdl
- DiscreteStateSpaceModel4HO.mdl
- CBF16BiSPSwithNarrowBandImpedance.mdl