GENERAL FORMULA FOR THE RESISTIVE WALL IMPEDANCE DERIVED FROM L. VOS' FORMALISM

E. Metral

THEORY (1/3)

- **General formula for 2 layers**

$$z_T^{ul} = \frac{Z_0}{2\pi b^2} \times \frac{1}{-j + \dfrac{\omega b \mu_0}{2 Z_2} \times \dfrac{1 + \dfrac{Z_2}{Z_1}\,\mathrm{Tanh}\left(\gamma_1 s_1\right)}{1 + \dfrac{Z_1}{Z_2}\,\mathrm{Tanh}\left(\gamma_1 s_1\right)}}$$

ul ⇔ length of 1 m

$$\gamma_1 = \sqrt{\left(\frac{\omega}{\beta c}\right)^2 + j\,\omega\,\mu_1\left(\sigma_1 + j\,\omega\,\varepsilon_1\right)}$$

$$Z_i = \sqrt{\frac{j\,\omega\,\mu_i}{j\,\omega\,\varepsilon_i + \sigma_i}}$$

$$s_1 = \text{Thickness of layer 1}$$

$$b = \text{Beam pipe radius}$$

$$Z_0 = 120\,\pi$$

$$\mu_0 = 4\,\pi \times 10^{-7}$$

THEORY (2/3)

- Consider the case where the layer 2 is vacuum

  - $\left| \gamma_1 \, s_1 \right| >> 1$

$$\Rightarrow \quad z_T^{ul} = (1 + j)\,\frac{Z_0}{2\pi b^2} \times \frac{1}{-j + 1 + b\sqrt{\dfrac{\mu_0\,\sigma_1\,\omega}{2}}}$$

$\Rightarrow$ The thick-wall formula with inductive bypass is recovered

THEORY (3/3)

- $\left| \gamma_1 \, s_1 \right| << 1$

$\Rightarrow$

$$z_T^{ul} = \frac{Z_0}{2 \pi b^2} \times \frac{1}{- j + \dfrac{b \, \mu_0 \, s_1 \, \sigma_1 \, \omega}{2}}$$

$\Rightarrow$ **The thin-wall formula with inductive bypass is recovered**

NUMERICAL APPLICATION

- The layer 2 is vacuum

- The impedance parameters are

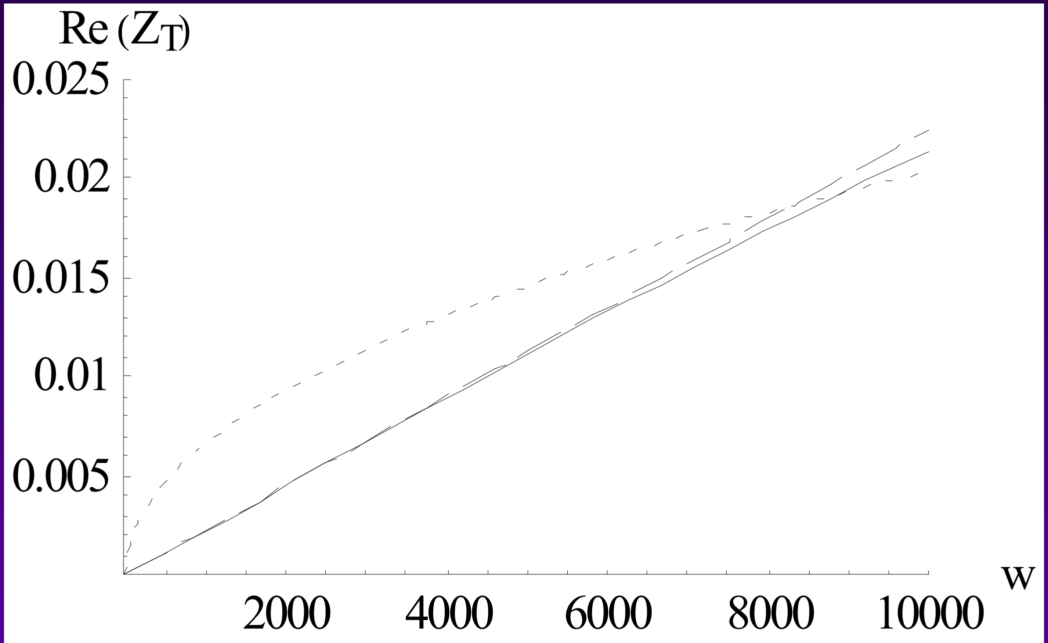
$$b = 2 \text{ mm}$$

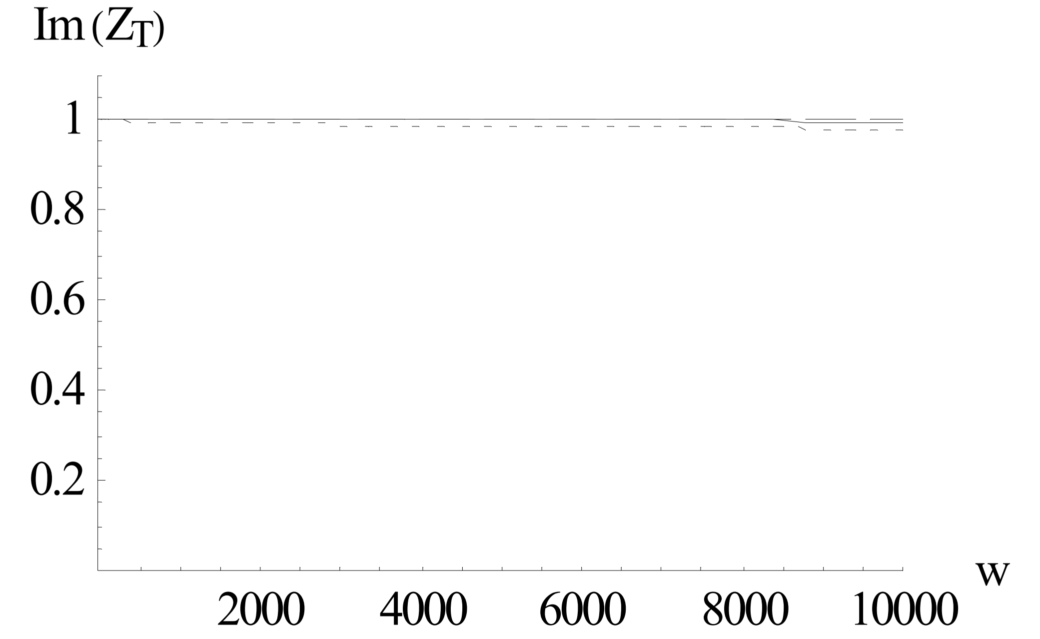
$$\rho_1 = 14 \ \mu\Omega\text{m}$$

$$s_1 = 2.5 \text{ cm}$$

$$\mu_1 = \mu_0$$

$$\varepsilon_1 = \varepsilon_0 = \frac{1}{Z_0 \, c}$$





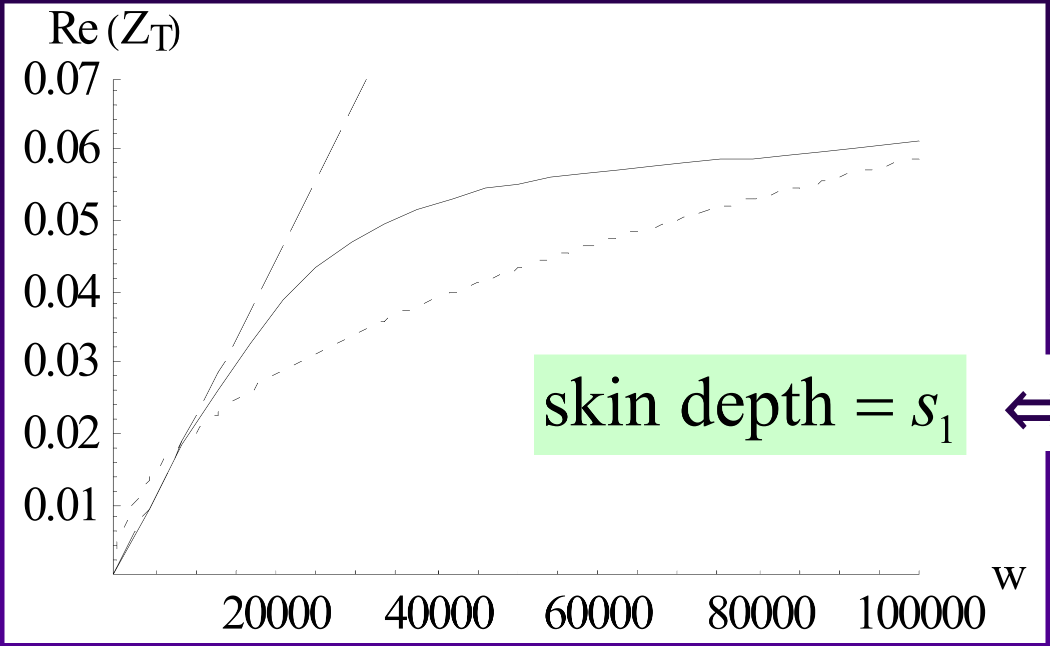
Full = General formula

Dashed = Thin-wall formula

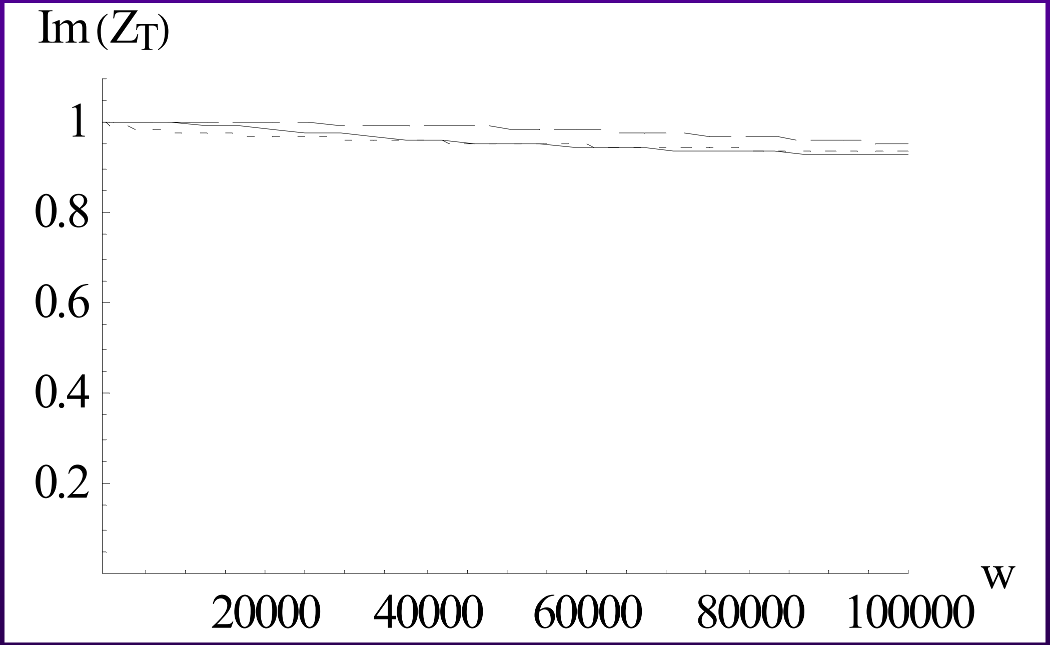
Dotted = Thick-wall formula

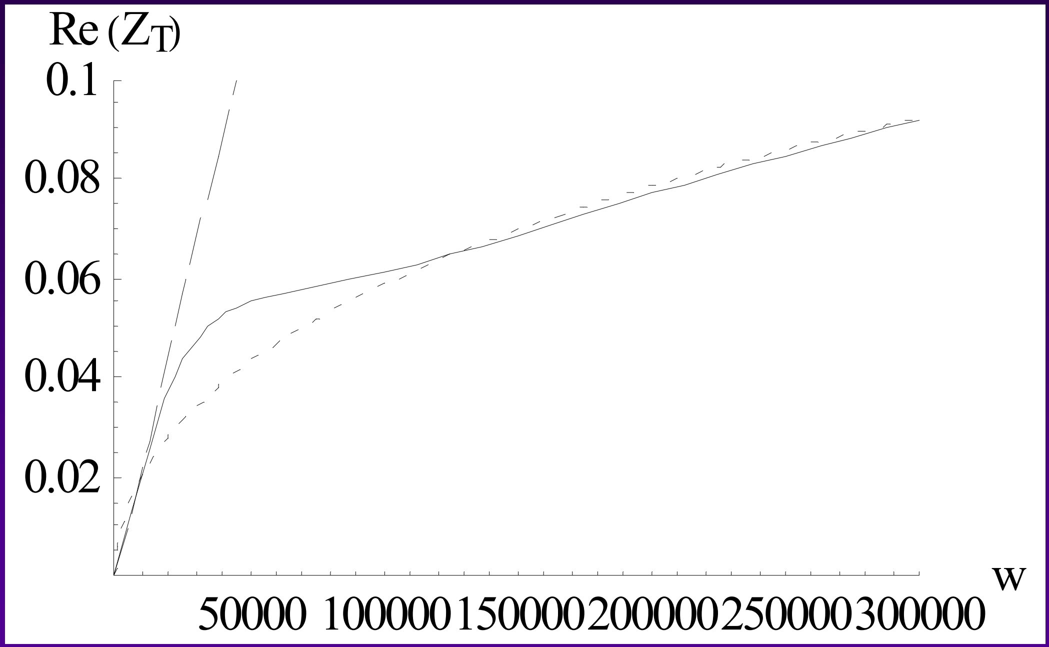
Remark : The plots are normalized by

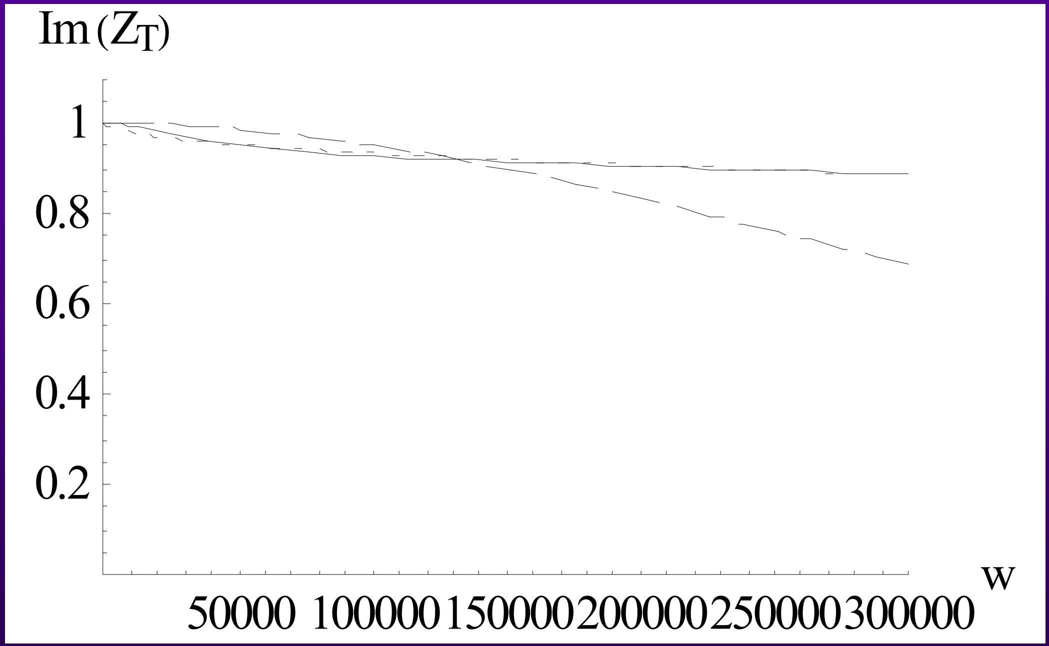
$$\frac{Z_0}{2\pi b^2}$$

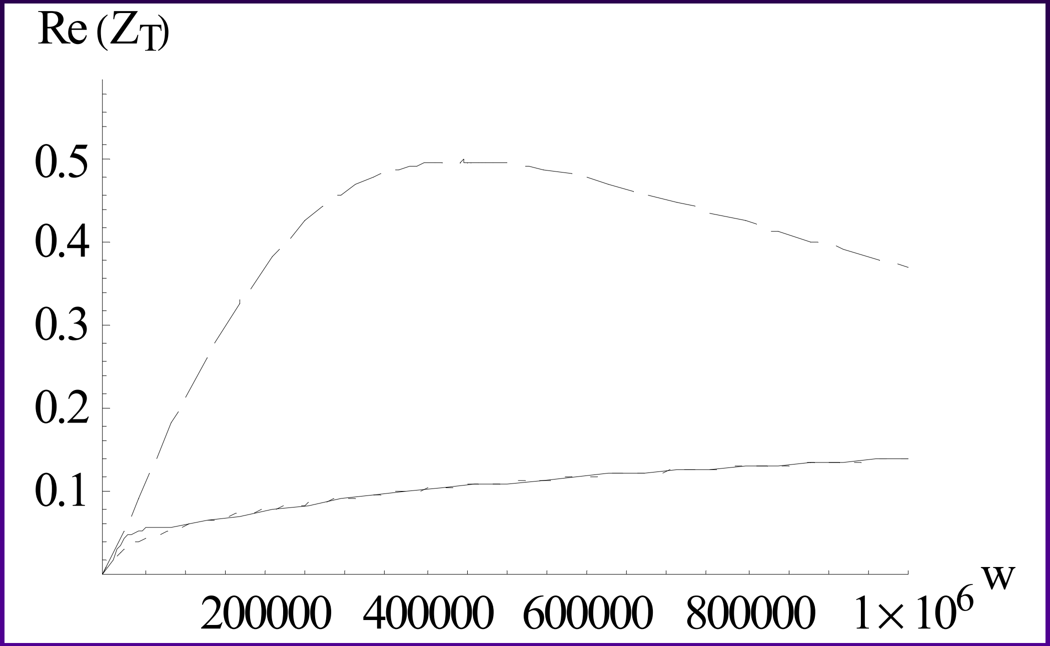


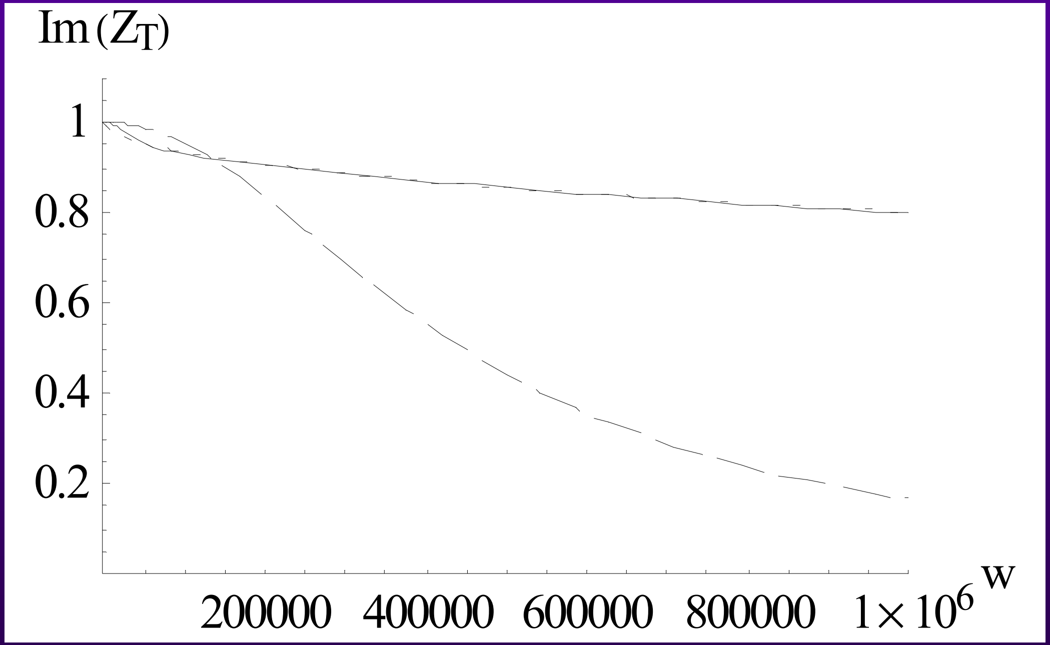
skin depth = s_1 $\Leftrightarrow$ $w \approx 35651$ rad/s

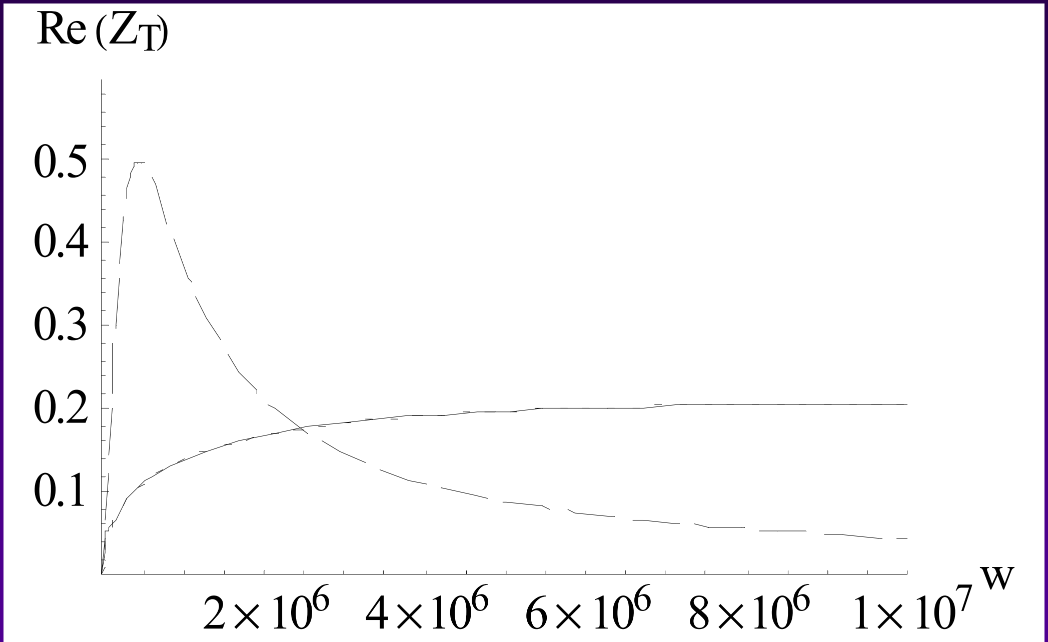


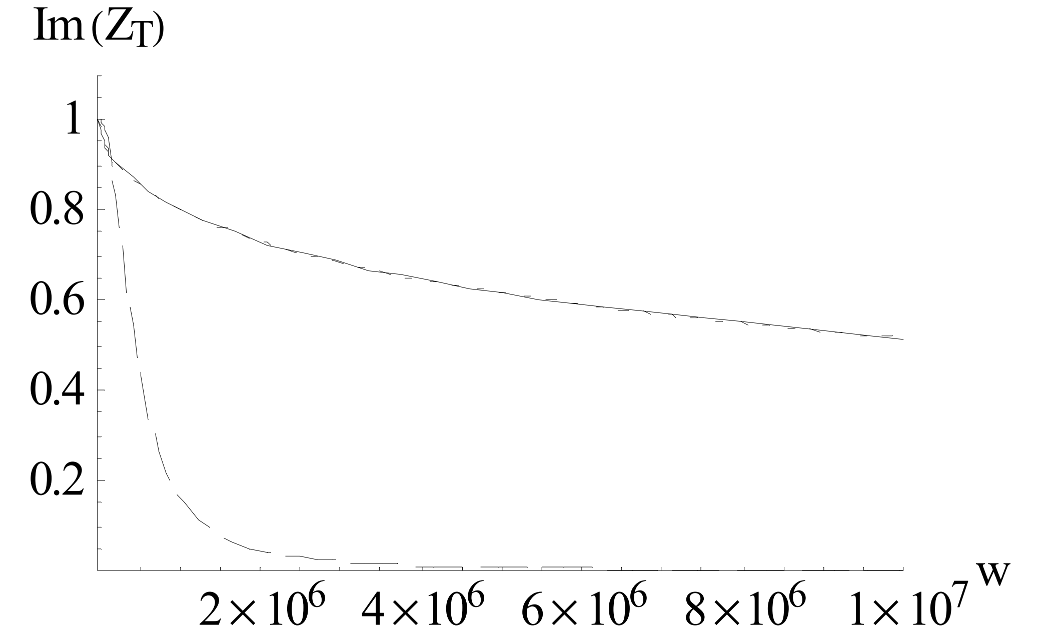

Re (Z$_T$)
0.1
0.08
0.06
0.04
0.02
w
50000 100000 150000 200000 250000 300000


Im (Z$_T$)
1
0.8
0.6
0.4
0.2
w
50000 100000 150000 200000 250000 300000

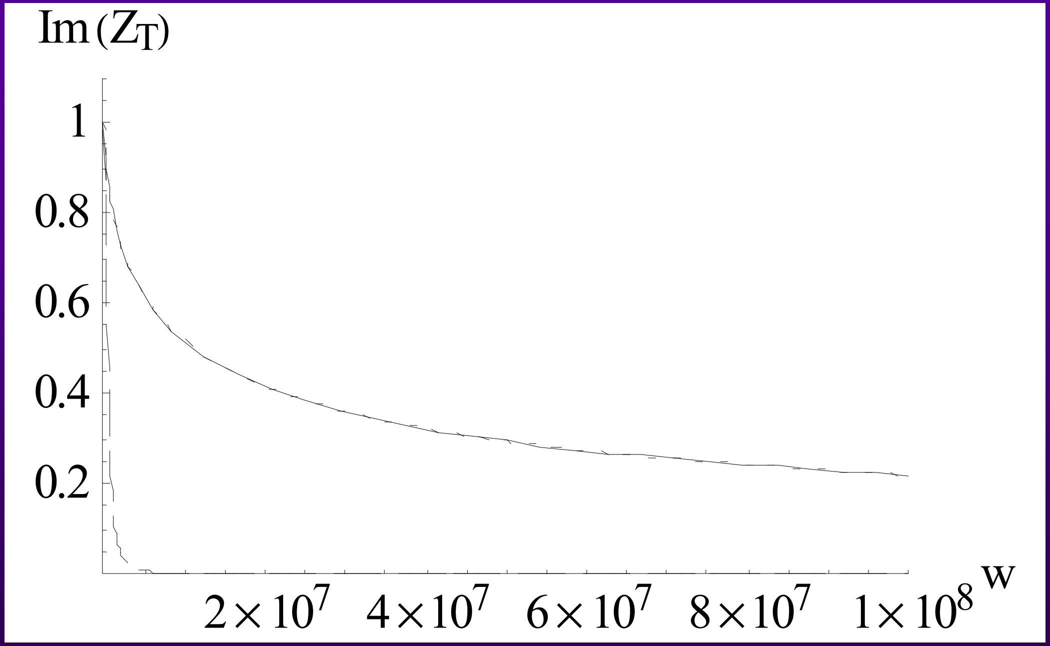

Re (Z_T)
0.5
0.4
0.3
0.2
0.1
200000
400000
600000
800000
1×10^6
w


Im (Z_T)
1
0.8
0.6
0.4
0.2
200000
400000
600000
800000
1×10^6
w


Re (Z_T)
0.5
0.4
0.3
0.2
0.1
w
2×10^6
4×10^6
6×10^6
8×10^6
1×10^7


Im (Z_T)
1
0.8
0.6
0.4
0.2
w
2×10^6
4×10^6
6×10^6
8×10^6
1×10^7


Re (Z_T)
0.5
0.4
0.3
0.2
0.1
2×10^7
4×10^7
6×10^7
8×10^7
1×10^8
w


Im (Z_T)
1
0.8
0.6
0.4
0.2
2×10^7
4×10^7
6×10^7
8×10^7
1×10^8
w

THEORETICAL PREDICTIONS COMPARED TO MEASUREMENTS (Caspers et al. in CERN-AB-2003-051 (RF)) (1/2)

- **Stainless steel pipe**

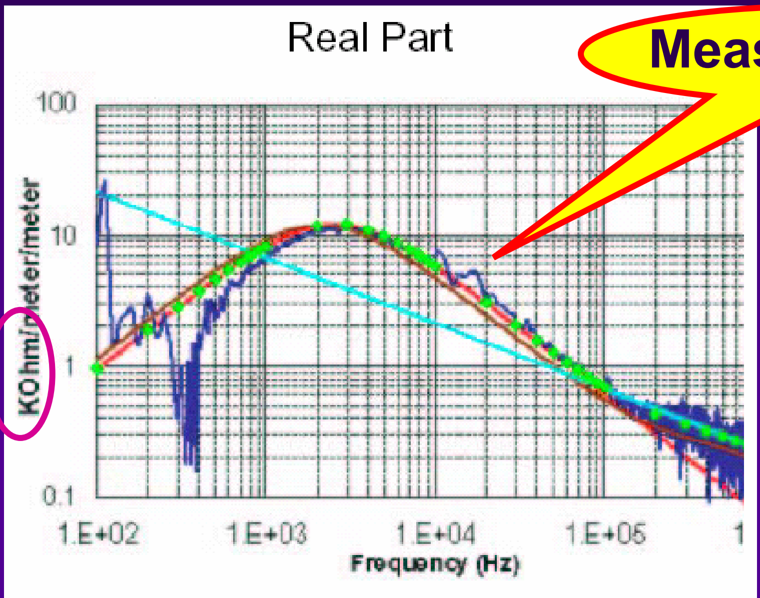
$$b = 50 \text{ mm}$$

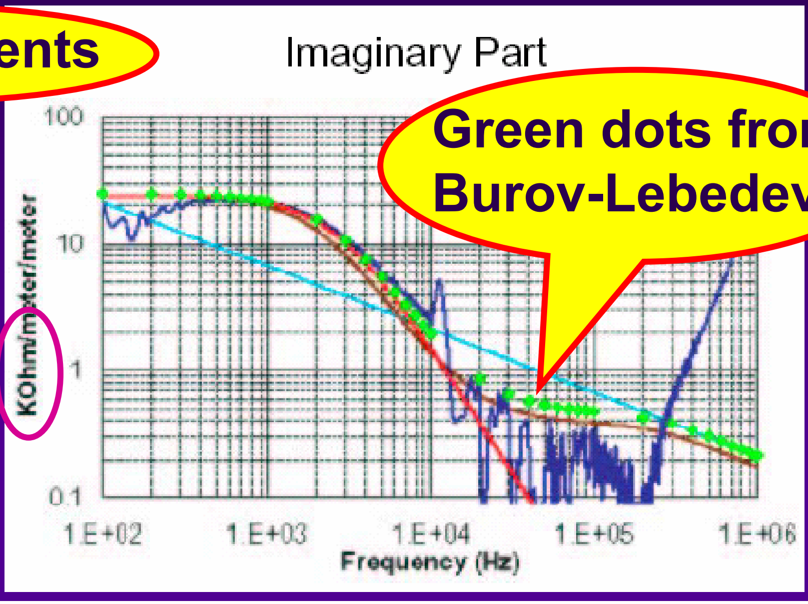
$$s_1 = 1.5 \text{ mm}$$

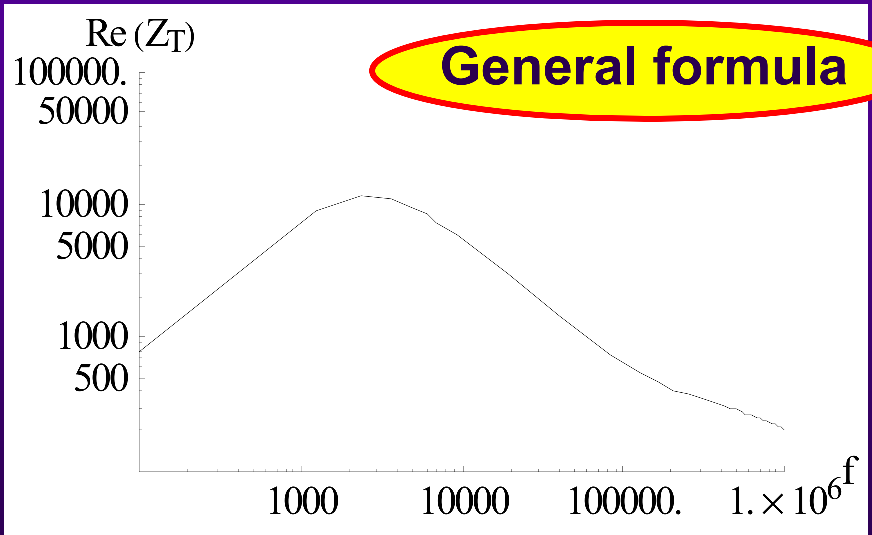
$$\sigma_1 = 1.3 \times 10^6 \ \Omega^{-1}\text{m}^{-1}$$

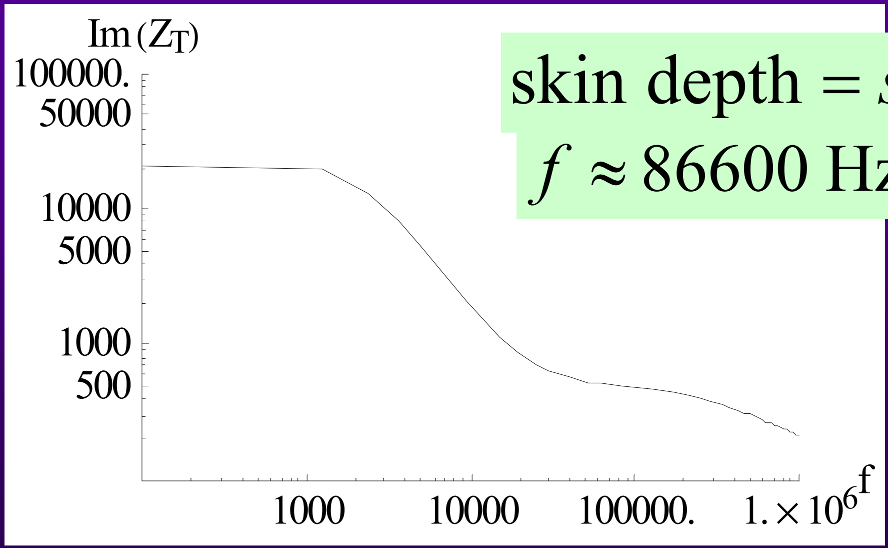
- **We will compare the transverse impedances (not normalized) per unit length in a LogLog plot**

THEORETICAL PREDICTIONS COMPARED TO MEASUREMENTS (Caspers et al. in CERN-AB-2003-051 (RF)) (2/2)









$\text{skin depth} = s_1$

$f \approx 86600 \text{ Hz}$

CONCLUSION (1/2)

- A general formula has been found by L. Vos for any multi-layer vacuum chamber
- For the case of 2 layers, with the vacuum for the second layer :
  - The general formula reduces to the "thin-wall formula with inductive bypass" for low frequencies. This formula can be derived from L. Vos' formalism and is the same as the one derived by B. Zotter. It is the same as the one used by Caspers et al., which is a corrected version of the one derived by Nassibian and Sacherer. Burov and Lebedev say that they also obtain the same formula in this approximation
  - The general formula reduces to the "thick-wall formula with inductive bypass" (which can be derived from L. Vos' formalism) for high frequencies
  - The general formula has a smooth transition between the "thin-wall formula with inductive bypass" and the "thick-wall formula with inductive bypass" for intermediate frequencies

CONCLUSION (2/2)

- **Low and high frequencies correspond to frequencies much smaller than or bigger than the frequency where the skin depth is equal to the thickness of the layer 1**

- **Intermediate frequencies stand for frequencies close to the frequency where the skin depth is equal to the thickness of the layer 1**

- **As concerns the comparison between the theoretical predictions of the new general formula and the measurements by Caspers et al. (in CERN-AB-2003-051 (RF))**
  - **The agreement is very good over the whole range of frequencies**
  - **The new general formula seems to be very close the one from Burov-Lebedev**