Resistive Wall Impedance

New results by BZ for LHC collimators

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What we had so far ...

$$Z_{m=1}^{\perp,\,\text{thick}}(\omega) = (\operatorname{sgn}\omega + j)\;\frac{Z_0\,L\,\delta_0\,\mu_r}{2\,\pi\,b^3}\cdot\sqrt{\frac{\omega_0}{|\omega|}}$$

$$Z_{m=1,\,\text{ibp}}^{\perp,\,\text{thick}}(\omega) = (1 + j\;\operatorname{sgn}\omega)\;\frac{Z_0\,L}{2\,\pi\,b^2}\;\frac{1}{-j + \operatorname{sgn}\omega\left(1 + b\sqrt{\frac{\sigma_c\,\mu_0}{2\,\mu_r}}\;\sqrt{|\omega|}\right)}$$

$$Z_{m=1}^{\perp,\,\text{thin}}(\omega) = \frac{c\,L}{\pi\,b^3\,\sigma_c\,t_w\cdot\omega}$$

$$Z_{m=1,\,\text{ibp}}^{\perp,\,\text{thin}}(\omega) = \frac{Z_0\,L}{2\,\pi\,b^2}\cdot\frac{1}{\frac{1}{2}\,b\,t_w\,\sigma\,\mu_0\cdot\omega - j}$$

$$Z_{m=1,}^{\perp,\,\text{LV, n=2}}(\omega) = \frac{Z_0\,L}{2\,\pi\,b^2}\cdot\left[\frac{b\,\mu_0\,\omega\left(\sqrt{\frac{c^2\,\mu_0^2\,\omega}{\omega - j\;c^2\,\mu_0\,\sigma}} + \sqrt{c^2\,\mu_0^2}\tanh\left(t_w\sqrt{j\;\mu_0\,\sigma\,\omega}\right)\right)}{2\sqrt{c^2\,\mu_0^2}\sqrt{\frac{c^2\,\mu_0^2\,\omega}{\omega - j\;c^2\,\mu_0\,\sigma}} + \frac{2\,c^2\,\mu_0^2\,\omega\;\tanh\left(t_w\sqrt{j\;\mu_0\,\sigma\,\omega}\right)}{\omega - j\;c^2\,\mu_0\,\sigma}} - j\right]^{-1}$$

$Z_{m=1,}^{\perp,\,\text{BL}}(\omega) =$ algorithm

Field Matching by B.Zotter

Field calculations staring from Maxwell’s equations in frequency domain,

$$curl\,\vec{H} = \vec{J} + j\omega\vec{D}\,,$$

$$curl\,\vec{E} = -j\omega\vec{B}\,,$$

$$div\,\vec{B} = 0\,,$$

$$div\,\vec{D} = \rho\,.$$

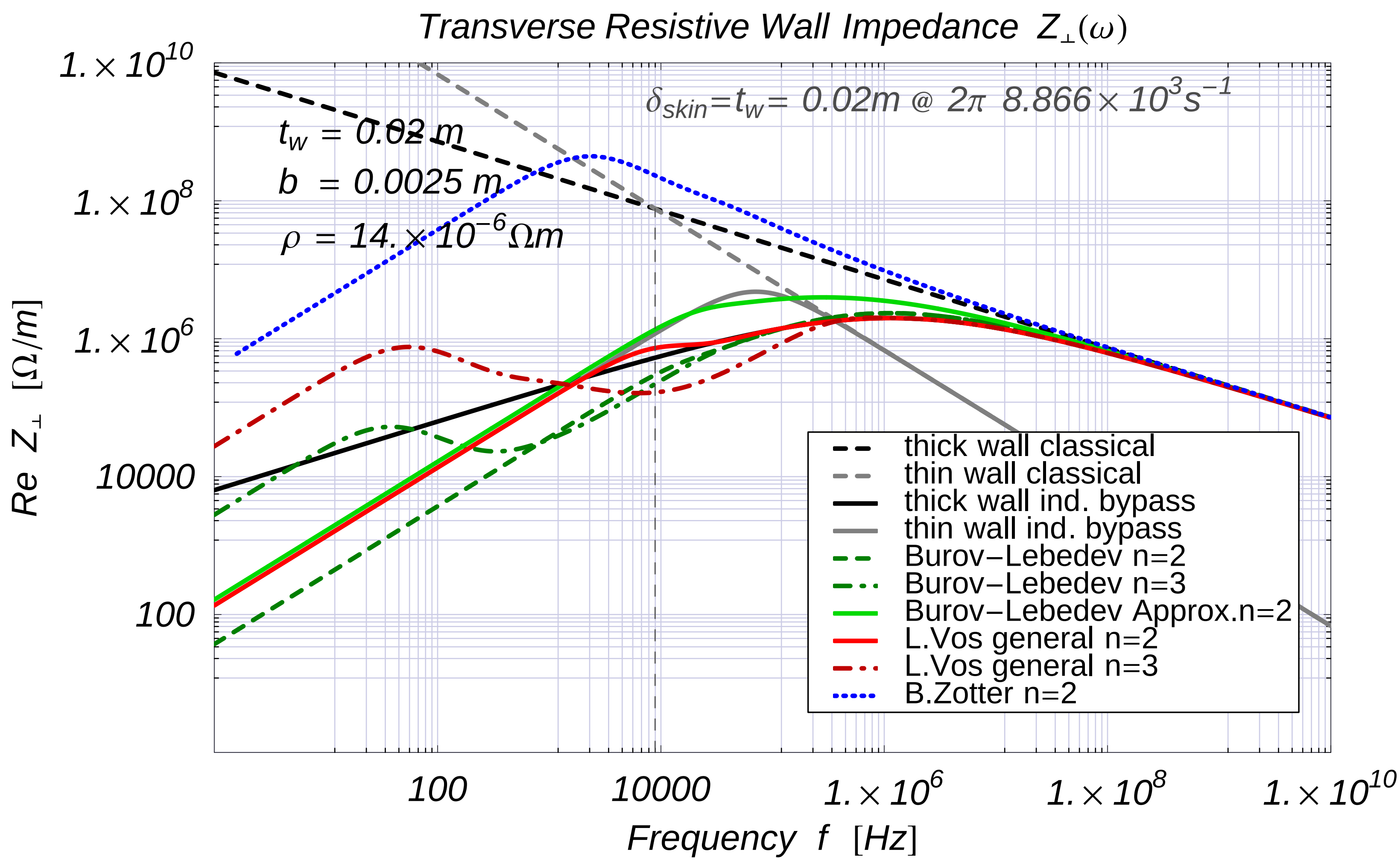
Source term model (Beam) is an infinitesimally thin ring of charge with an azimuthal density modulation [Gluckstern].

Field matching at boundaries:

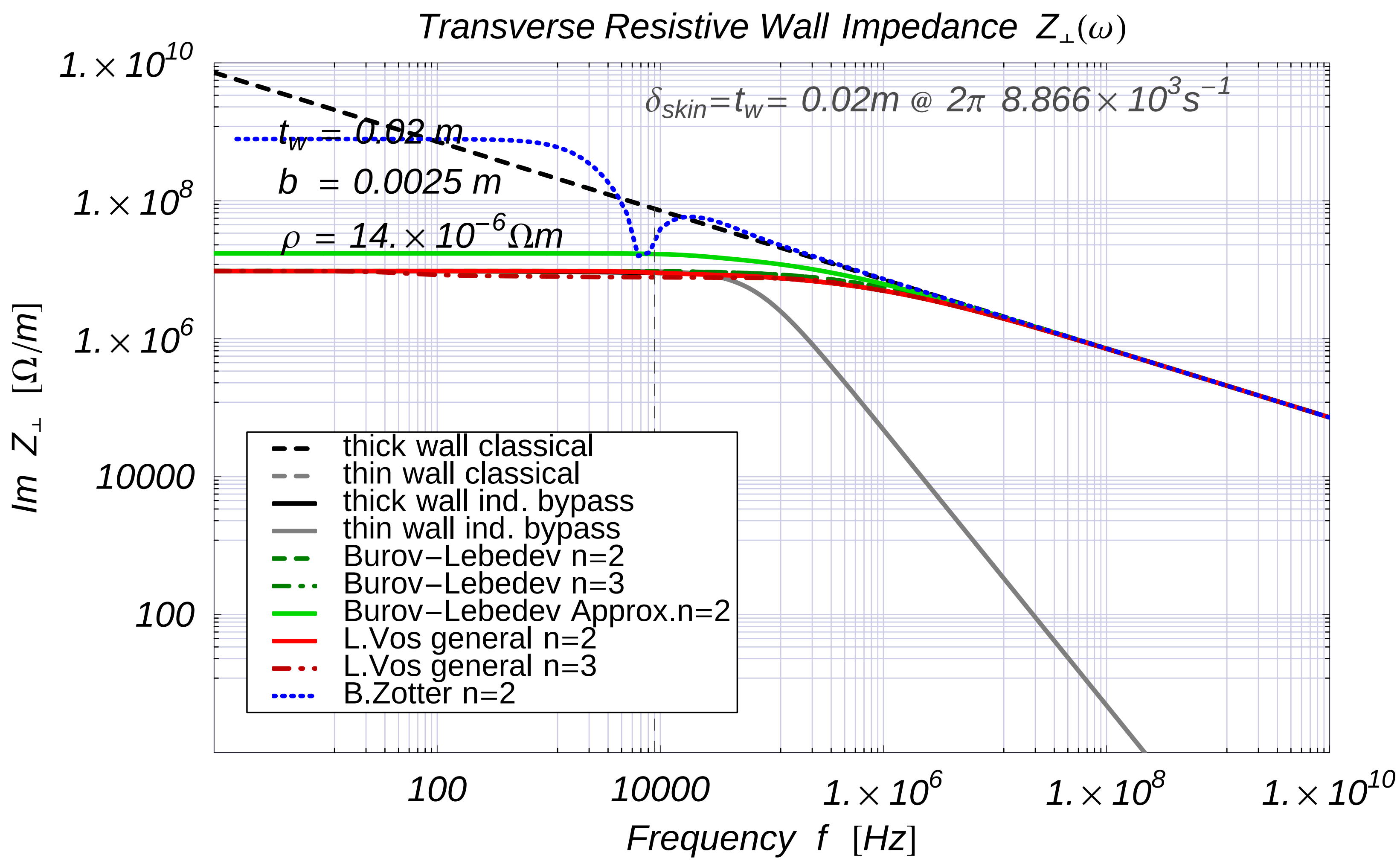
$$E_z^{(p+1)} = E_z^{(p)}, \quad E_\theta^{(p+1)} = E_\theta^{(p)}, \quad G_z^{(p+1)} = G_z^{(p)}, \quad G_\theta^{(p+1)} = G_\theta^{(p)}\,,$$

where $\vec{G} = Z_0\vec{H}$.

LHC Collimators



LHC Collimators



Zero Crossing of $\mathrm{Im} Z$

Transverse impedances properties [Chao, p.76], since wake functions must be real:

$$\begin{aligned} \mathrm{Re}\ Z_m^{\perp}(\omega) &= -\mathrm{Re}\ Z_m^{\perp}(-\omega) \quad \text{odd function} \\ \mathrm{Im}\ Z_m^{\perp}(\omega) &= \quad \mathrm{Im}\ Z_m^{\perp}(-\omega) \quad \text{even function} \end{aligned} \tag{1}$$

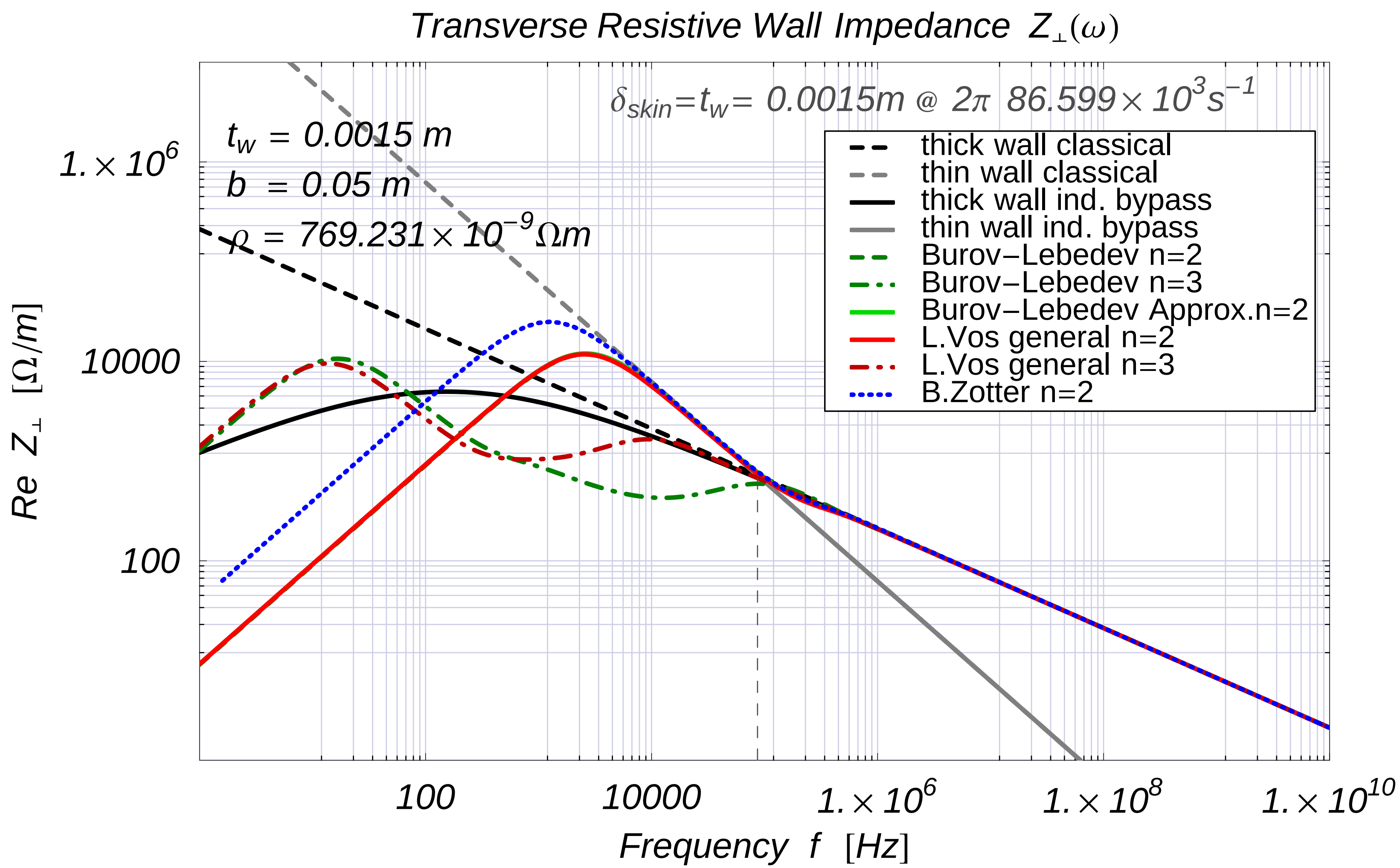
Wake function is Inverse Fourier transform of the impedance:

$$W_m^{\perp}(t) = -j \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\ Z_m^{\perp}(\omega)\ \exp\left(j\, \omega t\right) \tag{2}$$

Especially the transverse wake function satisfies $W_m(0) = 0$, which gives

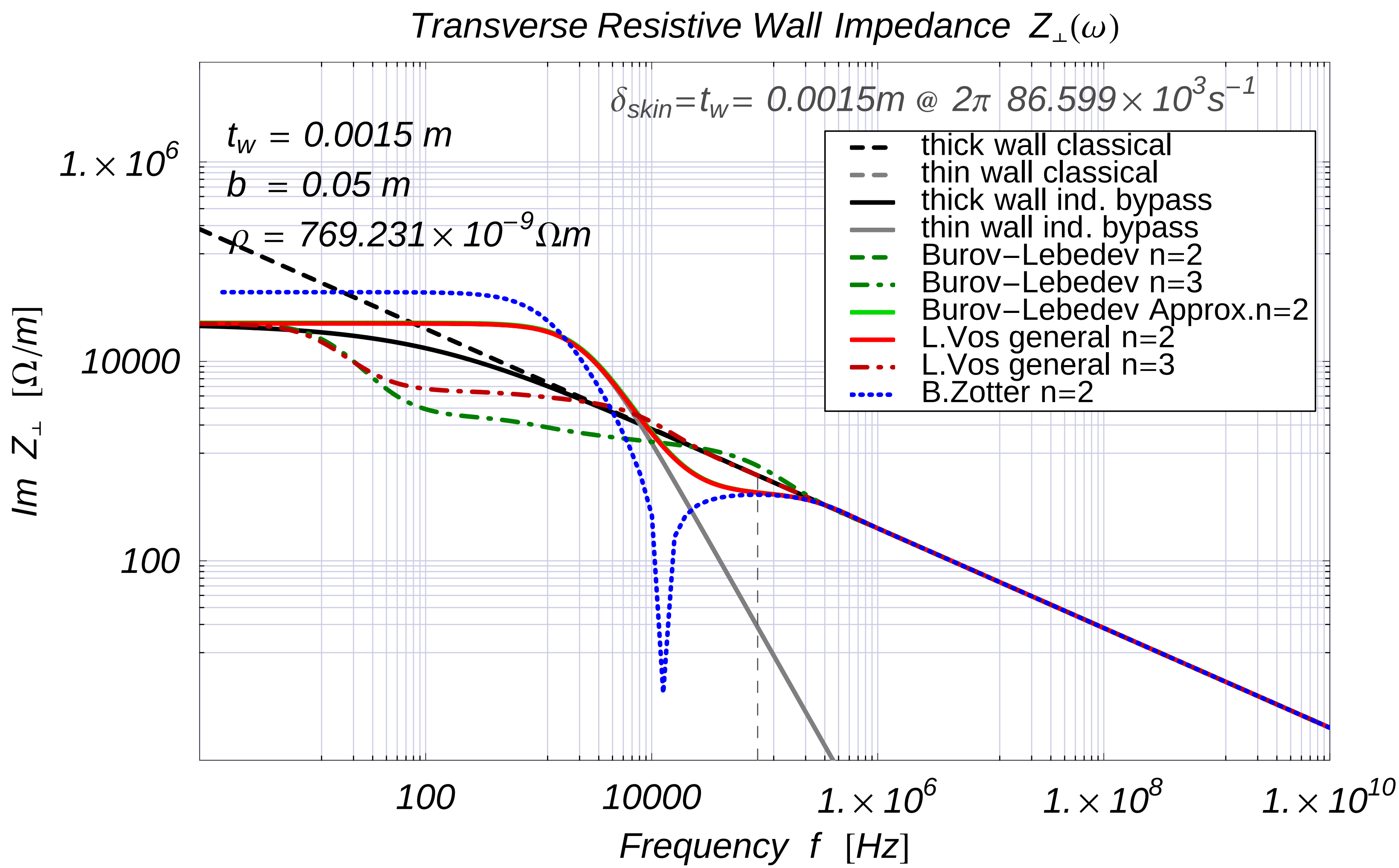
$$\begin{aligned} W_m^{\perp}(0) &= -j \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\ Z_m^{\perp}(\omega) = -j \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left[\underbrace{\mathrm{Re}\ Z_m^{\perp}(\omega)}_{\text{odd}} + \underbrace{\mathrm{Im}\ Z_m^{\perp}(\omega)}_{\text{even}} \right] \\ &= 2 \int_{0}^{+\infty} \frac{d\omega}{2\pi}\ \mathrm{Im}\ Z_m^{\perp}(\omega) \stackrel{!}{=} 0 \quad \Longrightarrow \mathrm{Im} Z(\omega) \text{ must change sign in } [0, \infty] \end{aligned} \tag{3}$$

Bench measurements revisited



"Bench Measurements of Low Frequency Transverse Impedance", CERN-AB-2003-051 (RF)

Bench measurements revisited



"Bench Measurements of Low Frequency Transverse Impedance", CERN-AB-2003-051 (RF)