Multi-Layer Transverse Impedances

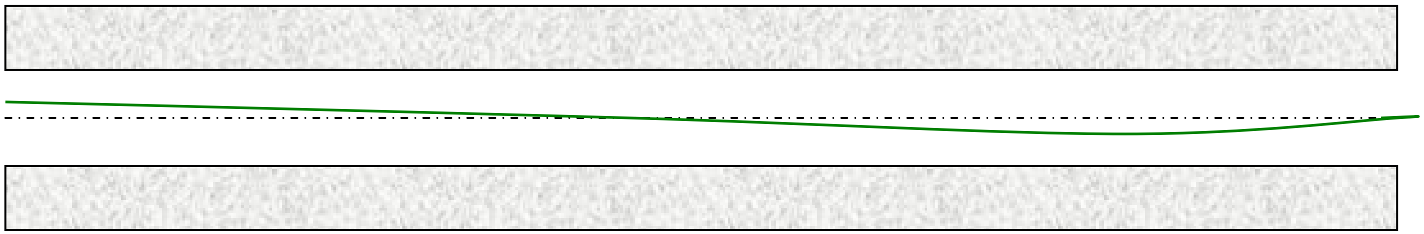
Alexey Burov

June 04 2004

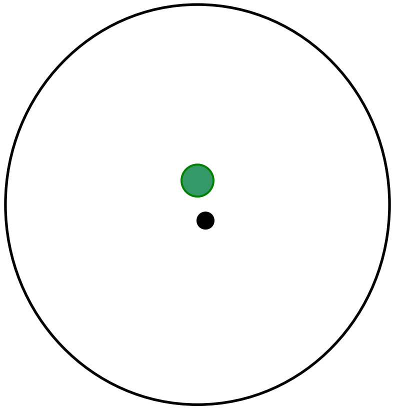
Long Wave Approach

- The case of interest is long wavelength one:

  wave length >> aperture.



- In this case, longitudinal gradients can be neglected, the chamber sees the beam as it were a parallel to its axis. In other words, the problem reduces to 2D one.



Electric and Magnetic Dipoles

- The beam as a source of the fields is presented as a superposition of its
  - charge = electric
  - and current = magnetic

  dipoles.
- Due to linearity of Maxwell equations, these two sources can be considered separately, and the superposition principle applied.
- In the long-wave case, beam local oscillations are much slower than the wave-guide frequencies. In other words, long-wave approximation is adiabatic or quasi-static approximation.

Electric Dipole

- Electric dipole excites quasi-static scalar potential

$$\Phi = \frac{A_0}{\beta}\left(\frac{a}{r} - \frac{r}{a}\right)\cos\theta\,\exp(-i\omega t)\,; \qquad A_0 = \frac{2Ix_0}{ca}$$

where the first term in the brackets is the direct field of the dipole, while the second is another independent solution of the Laplas equation in the vacuum. The amplitude of the second term is taken to satisfy the boundary condition on the metallic surface,

$$\Phi\big|_{r=a} = 0\,.$$

- This second term describes chamber response on the electric dipole, assuming some metal as the innermost layer.
- This response is trivial and does not depend neither on the metal properties, nor on the other layers.

Magnetic Dipole

- As the magnetic (or current) dipole, the beam excites longitudinal component of the vector potential

$$A_z = A(r)\cos\theta\,\exp(-i\omega t)\,.$$

- In vacuum, the vector potential satisfies the same Poisson equation as the scalar one,

$$\Delta A_z = -4\pi\rho_b\beta\,; \qquad \rho_b = \frac{I}{\beta c}\left(\delta(\vec{r}-\vec{r}_0)-\delta(\vec{r})\right).$$

- Thus, in vacuum it behaves as

$$A(r) = A_0\left(\frac{a}{r} - G\frac{r}{a}\right).$$

- The chamber response is in the constant G, which has to be found from the field solutions in the outer layers and the whole set of the boundary conditions.
- The transverse impedance is expressed then as

$$Z_x = -i\frac{E_x - \beta H_y}{I x_0} = i\frac{Z_0\beta}{2\pi a^2}\left(G - \frac{1}{\beta^2}\right)$$

Medium

- In a medium, the vector potential satisfies Bessel equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dA}{dr}\right)-\frac{A}{r^2}-\kappa^2 A=0\ ,$$

where

$$\kappa^2=-\frac{\omega^2\mu\varepsilon}{c^2}\ ,\quad \mathrm{Re}\,\kappa>0\ ,\quad \varepsilon=\varepsilon'+\frac{4\pi i\sigma}{\omega}\ .\quad \delta\equiv 1/\mathrm{Re}\,\kappa\geq 0.$$

- The boundary conditions require continuity of both the potential A itself and the tangential magnetic field $\mu^{-1}dA/dr$. At infinity, the potential goes to 0.

- Note that the skin effect (skin depth δ) is driven not only by the resistivity, but by the imaginary part of the magnetic permeability as well.

- The set of equations is now complete. The problem is reduced to finding coefficients for the Bessel functions from boundary conditions. This is just linear algebra.

Algebraic Details

- Convenient basis functions for a given layer are

$$\mathrm{shb}(\kappa r) = \kappa a\left[\mathrm{I}_1(\kappa r)\,\mathrm{K}_1(\kappa a) - \mathrm{I}_1(\kappa a)\,\mathrm{K}_1(\kappa r)\right]$$
$$\mathrm{chb}(\kappa r) = \kappa a\left[\mathrm{I}_1'(\kappa a)\,\mathrm{K}_1(\kappa r) - \mathrm{I}_1(\kappa r)\,\mathrm{K}_1'(\kappa a)\right]$$

where a is the inner boundary of this layer.

- For the outermost layer, the potential goes as $\mathrm{K}_1(\kappa r)$
- An advantage of these functions is that

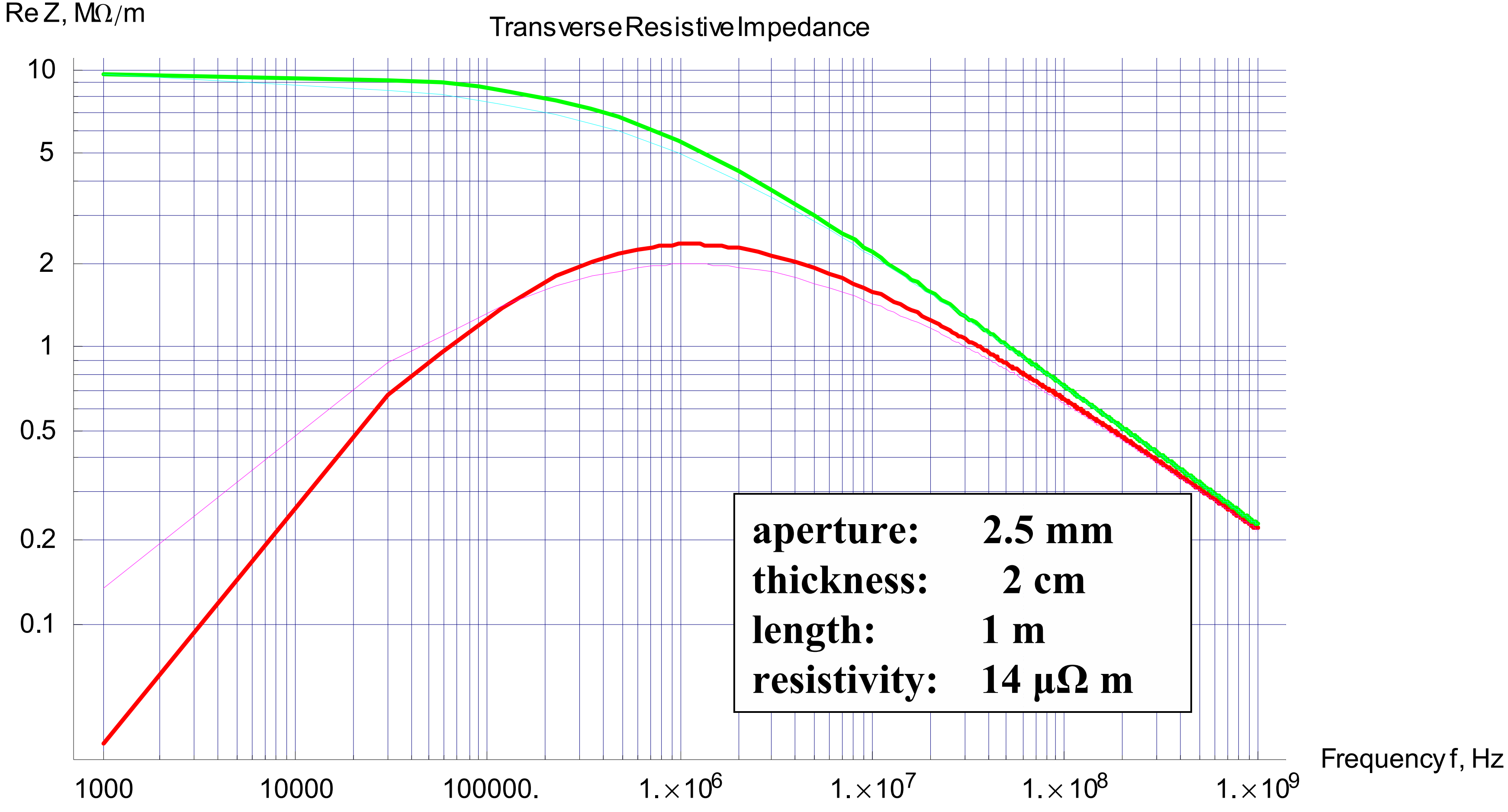
$$\mathrm{shb}(\kappa a) = 0\,, \quad \mathrm{sh\,b}'(\kappa a) = 1$$
$$\mathrm{chb}(\kappa a) = 1\,, \quad \mathrm{ch\,b}'(\kappa a) = 0$$

- With these functions, the linear algebra reduces for n layers to a sequence of n linear equations with a 2-diagonal matrix.

Impedance of LHC collimator

- The transverse impedance for LHC collimators:

red =Re Z, green = -Im Z, magenta=Re Z by L. Vas, cyan= -Im Z by L. Vas.



SPS Kicker Parameters

- SPS MKE kicker is an open to beam peace of 4A4 ferrite with half-gap 16 mm, thickness 60 mm, and length 1.66 m. Currently, 5 of them are installed and 4 more are planned to be in the ring in 2006. Material properties are quoted by L. Vas in CERN-S L-2000-010 AP:

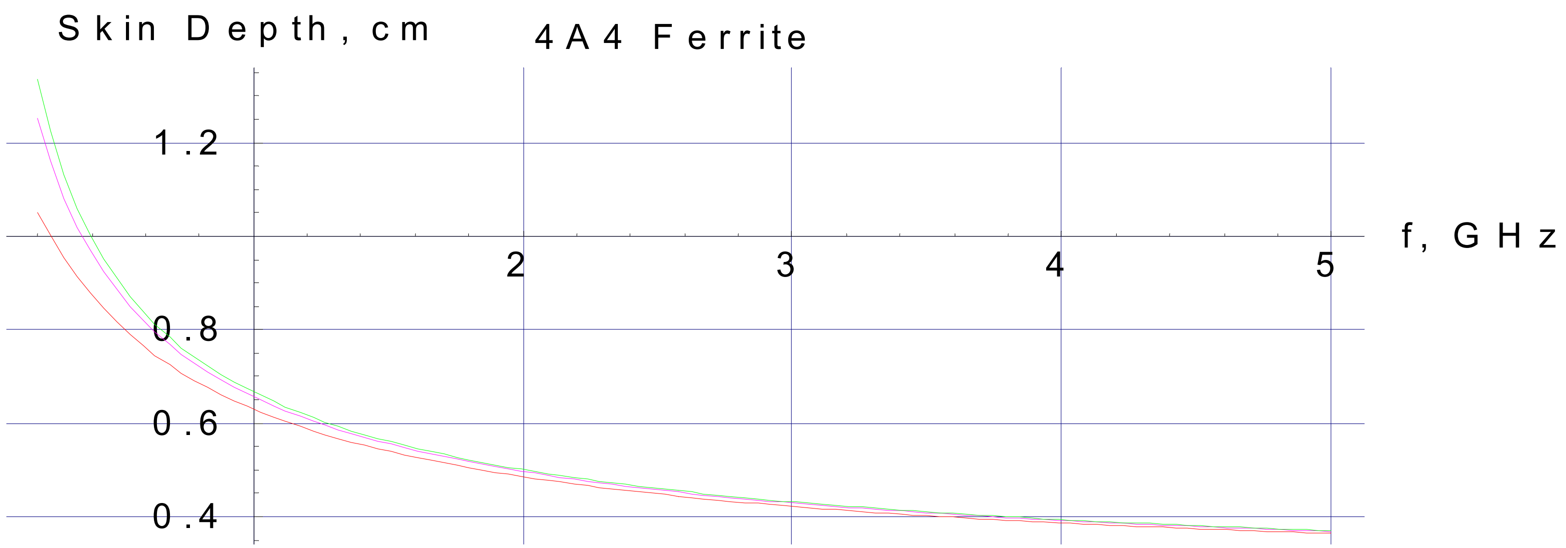
$$\varepsilon' = 12$$

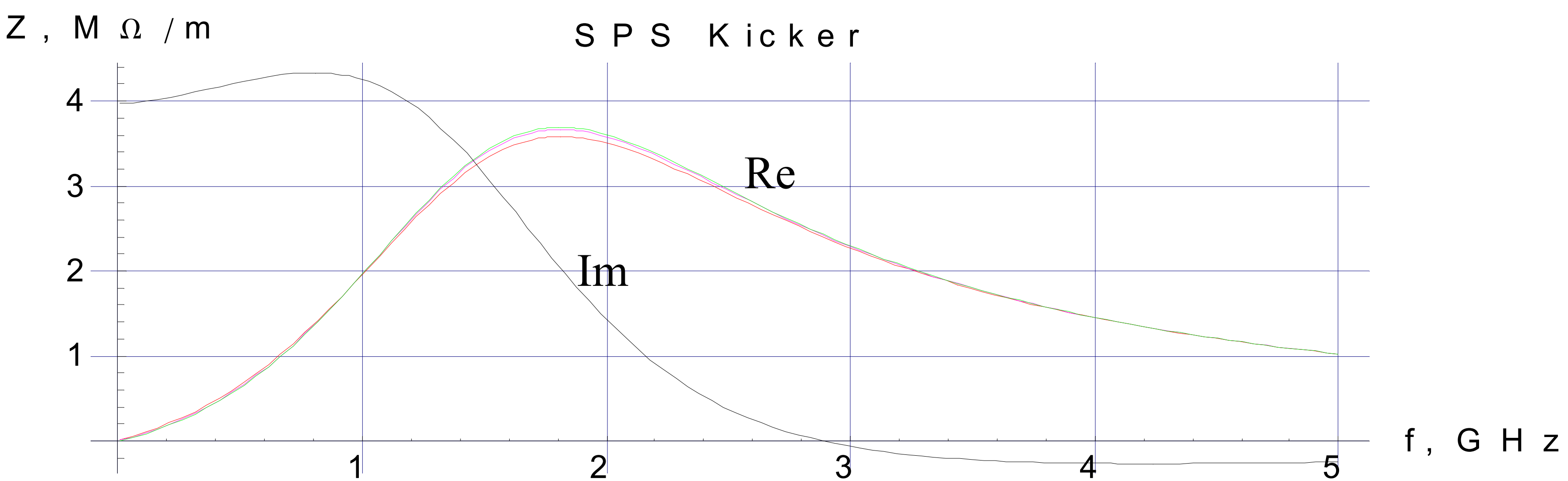
$$\mu(\omega) = \frac{460}{1 - i\omega\tau} + 1\,; \qquad \frac{1}{2\pi\tau} = 20\ \mathrm{MHz}$$

$$\rho = 10 - 30\ \Omega\,\mathrm{m}$$

- The skin depth of the 4A4 ferrite and the kicker impedance are calculated with the described method and presented in the Figs. below. Red, magenta and green lines correspond to resistivity (Re Z) for 10, 20 and 30 Ohm m. Black line is Im Z.

Kicker Impedance





Alexey Burov

Remarks on Kicker Impedance

- The skin depth in the interesting frequency range is almost independent of the resistivity when that varies between 10 and 30 Ohm m. The same is true for the impedance.
- The skin depth is always small compared with the thickness; thus, the thickness is not important.
- From here, it is also follows that all possible waveguide modes are 10 GHz and higher; thus, for the bunch length of $\approx$1m they do not contribute to the impedance.
- H. Tsutsui and L. Vos suggested an approach for the kicker impedance calculations (LHC Project Note 234, Sep. 2000) based on the contribution of the excited waveguide modes. They did not assume any dissipative terms in the quasi-static contribution. The analysis above shows that both assumptions are not correct; so is their result, which is about 5 times smaller than that presented in the Fig. above.

More Remarks

- Note that the kicker is flat; thus, the vertical driving (dipole, coherent) impedance is about 0.8 of the numbers above found for the round geometry. The detuning (quadrupole, incoherent) impedance is equal to the driving one and has the same sign.
- That is why, for zero longitudinal mode they add together giving 1.6 factor for the effective vertical impedance at small chromaticity.

Conclusions

- Resistive-type impedances may be of interest at wavelengths much longer than aperture only. Otherwise, they are negligible compared with geometrical impedances.
- The method is suggested for effective analytical calculations for arbitrary multi-layer structures.
- The method is applied to designed LHC collimator and SPS MKE kicker. Numbers found for the last one are in good agreement with the beam based measurements at SPS (E. Shaposhnikova, Jan. 2004, Chamonix).