Emittance, Intensity and Luminosity Evolution in LHC3

John Jowett, Amy Nicholson

Model of time evolution including beam-gas interactions (nuclear fragmentation and inelastic scattering, electromagnetic dissociation), intra-beam scattering (Bjorken-Mtingwa from MAD at all points in emittance space), radiation damping, optional RF noise to control longitudinal emittance. Allow for different emittances and intensity between beams, proton or ion beams. Detailed gas compositions taken into account.

This notebook is not meant to be evaluated - it is just some excerpts displayed in the meeting.

Load initialisation main LHC Ions notebook to get database of beams, physical effects.

Notation

Beam-gas Conditions

In[321]:=

$$\text{Notation}\left[\left(\sum_g n_g\,\sigma_{bg}\right) \Longleftrightarrow \text{sumngSigmaBeamGas}\right];$$

$$\text{IntroduceSymbol}\left[\text{sumngSigmaBeamGas, "is the sum over all residual gases of the product of density and interaction cross-section.", } \frac{\text{Barn}}{\text{Meter}^3}\right];$$

In[325]:=

$$\text{Notation}\left[\frac{4\,\pi\,r_p{}^2\,c\,Z_{ion}{}^2}{2\,\beta^2\,\gamma\,A_{ion}{}^2}\,\beta_{ave}\sum_g n_g\,Z_g\,(Z_g+1)\,\ln\left[183\,Z_g{}^{-\frac{1}{3}}\right] \Longleftrightarrow \text{sumMCS}\right];$$

$$\text{IntroduceSymbol}\left[\text{sumMCS, "is the sum over all pipe regions of the rms angle of MCS per unit time times the average beta function.", } \frac{\text{Meter}}{\text{Second}}\right];$$

Pessimistic MS where the emittance drops to 1/e of its initial value in 10 hours.

In[327]:=

$$\text{mcsPessimistic} = \text{sumMCS} \to \frac{1}{10\,\text{Hour}}\,(\epsilon\ /.\ \text{geometricEmittanceDef}\ //.\ \text{LHCIonBeam}[\text{Pb, Collision}])\ //\ \text{NumerDat};$$

Calculates the MS factor involving gas atomic numbers.

In[328]:=
```
msFac[el_?ElementQ] := Module[{z = AtomicNumber[el]},
  z (z + 1) Log[183./z^(1/3)]]
```

Calculates the MS factor for each residual gas element. This gives us a general expression in terms of gas compositions.

In[329]:=
```
gasWeightFactor =
 Plus @@ ((((({ng[#], MolecularStructure[#]}) & /@ residualGases) //.
       {ng[g_], {{el_?ElementQ, n_}, rest___}} :>
        {n ng[g] msFac[el], {ng[g], {rest}}}) /. {_, {}} → {} // Flatten)
```

Out[329]=
```
235.39 ng[CH4] + 518.89 ng[CO] + 844.067 ng[CO2] +
 20.8379 ng[H2] + 346.014 ng[H2 O] + 29.8706 ng[He]
```

vacuumTest, vacuumLHCPR674pessimistic are lists of residual gas densities in the accelerator structure. Both are pessimistic cases where the gas densities are higher than will probably be seen. vacuumLHCPR674conditioned is less pessimistic.

In[331]:=
```
vacuumTest =
  {ng[H2] → 9. 10^12 Meter^-3, ng[He] → 1. 10^13 Meter^-3, ng[CH4] → 1. 10^12 Meter^-3,
   ng[H2O] → 5. 10^12 Meter^-3, ng[CO] → 4. 10^12 Meter^-3, ng[CO2] → 2. 10^12 Meter^-3};
```

In[332]:=
```
vacuumLHCPR674pessimistic = {ng[H2] → 7.7/4 10^12 Meter^-3,
   ng[He] → 1./4 10^12 Meter^-3, ng[CH4] → 1.8/4 10^13 Meter^-3, ng[H2O] → 5./4 10^12 Meter^-3,
   ng[CO] → 2.6/4 10^12 Meter^-3, ng[CO2] → 4.2/4 10^12 Meter^-3};
```

In[333]:=
```
vacuumLHCPR674conditioned = {ng[H2] → 1.3`*^12/Meter^3, ng[He] → 2.5`*^11/Meter^3,
   ng[CH4] → 1.9 10^11/Meter^3, ng[H2 O] → 1.25`*^11/Meter^3, ng[CO] → 1.0 10^11/Meter^3, ng[CO2] → 2.8 10^11/Meter^3};
```

This is the (we hope very pessimistic) set of gas densities quoted in the LHC Design Report. In that report they are chosen to get 100 h lifetime for each gas separately.

In[632]:=
```
vacuum100hpergas = {ng[H2] → 1.2`*^15/Meter^3, ng[He] → 6.9`*^14/Meter^3, ng[CH4] → 1.8 10^14/Meter^3,
   ng[H2 O] → 1.8`*^14/Meter^3, ng[CO] → 1.2 10^14/Meter^3, ng[CO2] → 7.9 10^13/Meter^3};
```

Returns the cross sections of the various residual gases

In[334]:=

```
NuclearCrossSection[Pb, C, pion/Aion, NuclearCrossSectionModel → AbrasionAblation]
```

Out[334]=

```
3.61053 Barn
```

Some sets of data for nuclear cross sections. For lead ions we have data from the abrasion-ablation model and the RELDIS electromagnetic disocciation:

In[335]:=

```
beamGasAbrasionAblationRELDIS =
  sumngSigmaBeamGas → Plus @@ (ng[#] (NuclearCrossSection[Pb, #, pion/Aion,
            NuclearCrossSectionModel → AbrasionAblation] + NuclearCrossSection[
            Pb, #, pion/Aion, NuclearCrossSectionModel → RELDIS]) & /@ residualGases)
```

Out[335]=

$$\left(\sum_g n_g\, \sigma_{bg}\right) \to$$
$$13.8691\,\text{Barn}\, n_g[CH_4] + 12.5414\,\text{Barn}\, n_g[CO] + 19.6271\,\text{Barn}\, n_g[CO_2] + 4.20667\,\text{Barn}\, n_g[H_2] + 11.2924\,\text{Barn}\, n_g[H_2 O] + 3.07884\,\text{Barn}\, n_g[He]$$

For protons, we can use the Barashenkov formula which is quite accurate.

In[659]:=

```
beamGasBarashenkov = sumngSigmaBeamGas →
    Plus @@ (ng[#] (NuclearCrossSection[H, #, NuclearCrossSectionModel →
              BarashenkovInelastic]) & /@ residualGases) // N
```

Out[659]=

$$\left(\sum_g n_g\, \sigma_{bg}\right) \to$$
$$0.432952\,\text{Barn}\, n_g[CH_4] + 0.560335\,\text{Barn}\, n_g[CO] + 0.867718\,\text{Barn}\, n_g[CO_2] + 0.09\,\text{Barn}\, n_g[H_2] + 0.397383\,\text{Barn}\, n_g[H_2 O] + 0.113003\,\text{Barn}\, n_g[He]$$

Machine Conditions

Rules for Simplifying Special Cases

Time-Dependent Quantities

Initial Conditions

Differential Equations

"S" notations are used as switches to turn physics processes on and off

In[372]:=
```
Notation[S_lumi ⟺ switchLumi];
IntroduceSymbol[switchLumi, "is the switch for luminosity.", 1];
```

In[374]:=
```
Notation[S_rad ⟺ switchRadiationDamping];
IntroduceSymbol[switchRadiationDamping,
 "is the switch for radiation damping.", 1];
```

In[376]:=
```
Notation[S_bg ⟺ switchBeamGas];
IntroduceSymbol[switchBeamGas, "is the switch for beam-gas losses.", 1];
```

In[378]:=
```
Notation[S_ibs ⟺ switchIBS];
IntroduceSymbol[switchIBS, "is the switch for intra-beam scattering.", 1];
```

In[380]:=
```
Notation[T_IBSx ⟺ TIBSx];
IntroduceSymbol[TIBSx,
 "is the growth time of the horizontal emittance from IBS.", Second];
```

In[382]:=
```
Notation[T_IBSl ⟺ TIBSl];
IntroduceSymbol[TIBSl,
 "is the growth time of the longitudinal emittance from IBS.", Second];
```

The switchConst..... can be changed to zero to for the longitudinal emittance to be constant.

In[384]:=
```
Notation[S_lemitt ⟺ switchConstantLongitudinalEmittance];
IntroduceSymbol[switchConstantLongitudinalEmittance,
 "is the switch that forces the longitudinal emittance to be constant.", 1];
```

The diffusion term can be added to artificially blow up the longitudinal emittance in order to shrink the transverse emittance.

In[386]:=
```
Notation[S_diff ⟺ switchDiffusion];
IntroduceSymbol[switchDiffusion,
 "is the switch for the extra diffusion term for the longitudinal emittance.",
 1];
```

In[388]:=
```
Notation[S_ms ⟺ switchMultipleScattering];
IntroduceSymbol[switchMultipleScattering,
 "is the switch for multiple scattering.", 1];
```

The most general equations, assuming all asymmetries and all physics processes apply. They can be simplified using the rules defined above. The TIBS functions return a damping time associated with intra-beam scattering.

In[390]:=

$$
\begin{aligned}
&\mathtt{ODEs} = \mathtt{Flatten}\Big[\Big\{\mathtt{Nb1}'[\mathtt{t}] == -\frac{\sigma_{\mathtt{tot}}}{\mathtt{k}_{\mathtt{b}}}\,(\mathtt{lumi}\ /.\ \mathtt{luminosityDef})\ \mathtt{n}_{\mathtt{exp}}\ \mathtt{switchLumi} - \\
&\qquad \mathtt{switchBeamGas}\ \mathtt{vion}\ \mathtt{sumngSigmaBeamGas}\ \mathtt{Nb1}[\mathtt{t}], \\
&\quad \mathtt{Nb2}'[\mathtt{t}] == -\frac{\sigma_{\mathtt{tot}}}{\mathtt{k}_{\mathtt{b}}}\,(\mathtt{lumi}\ /.\ \mathtt{luminosityDef})\ \mathtt{n}_{\mathtt{exp}}\ \mathtt{switchLumi} - \\
&\qquad \mathtt{switchBeamGas}\ \mathtt{sumngSigmaBeamGas}\ \mathtt{vion}\ \mathtt{Nb2}[\mathtt{t}], \\
&\quad \mathtt{emittx1}'[\mathtt{t}] == -\frac{2}{\tau_{\mathtt{x}}}\,\mathtt{emittx1}[\mathtt{t}]\ \mathtt{switchRadiationDamping} + \\
&\qquad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emittx1}[\mathtt{t}]}{\mathtt{TIBSx}[\mathtt{Nb1}[\mathtt{t}],\ \mathtt{emittx1}[\mathtt{t}],\ \mathtt{emittl1}[\mathtt{t}]]} + \mathtt{sumMCS}\ \mathtt{switchMultipleScattering}, \\
&\quad \mathtt{emittx2}'[\mathtt{t}] == -\frac{2}{\tau_{\mathtt{x}}}\,\mathtt{emittx2}[\mathtt{t}]\ \mathtt{switchRadiationDamping} + \\
&\qquad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emittx2}[\mathtt{t}]}{\mathtt{TIBSx}[\mathtt{Nb1}[\mathtt{t}],\ \mathtt{emittx1}[\mathtt{t}],\ \mathtt{emittl2}[\mathtt{t}]]} + \mathtt{sumMCS}\ \mathtt{switchMultipleScattering}, \\
&\quad \mathtt{emitty1}'[\mathtt{t}] == -\frac{2}{\tau_{\mathtt{x}}}\,\mathtt{emitty1}[\mathtt{t}]\ \mathtt{switchRadiationDamping} + \\
&\qquad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emitty1}[\mathtt{t}]}{\mathtt{TIBSx}[\mathtt{Nb1}[\mathtt{t}],\ \mathtt{emittx1}[\mathtt{t}],\ \mathtt{emittl1}[\mathtt{t}]]} + \mathtt{sumMCS}\ \mathtt{switchMultipleScattering}, \\
&\quad \mathtt{emitty2}'[\mathtt{t}] == -\frac{2}{\tau_{\mathtt{x}}}\,\mathtt{emitty2}[\mathtt{t}]\ \mathtt{switchRadiationDamping} + \\
&\qquad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emitty2}[\mathtt{t}]}{\mathtt{TIBSx}[\mathtt{Nb1}[\mathtt{t}],\ \mathtt{emittx1}[\mathtt{t}],\ \mathtt{emittl2}[\mathtt{t}]]} + \mathtt{sumMCS}\ \mathtt{switchMultipleScattering}, \\
&\quad \mathtt{emittl1}'[\mathtt{t}] == \mathtt{switchConstantLongitudinalEmittance} \\
&\qquad \Bigg(\frac{-4}{\mathtt{taux}}\,(\mathtt{emittl1}[\mathtt{t}] - \mathtt{emittl10}\ \mathtt{switchDiffusion})\ \mathtt{switchRadiationDamping} + \\
&\qquad\quad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emittl1}[\mathtt{t}]}{\mathtt{TIBSl}[\mathtt{Nb1}[\mathtt{t}],\ \mathtt{emittx1}[\mathtt{t}],\ \mathtt{emittl1}[\mathtt{t}]]} - \\
&\qquad\quad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emittl1}[\mathtt{t}]}{\mathtt{TIBSl}[\mathtt{Nb10},\ \mathtt{emittx10},\ \mathtt{emittl10}]}\ \mathtt{switchDiffusion}\Bigg), \\
&\quad \mathtt{emittl2}'[\mathtt{t}] == \mathtt{switchConstantLongitudinalEmittance} \\
&\qquad \Bigg(\frac{-4}{\mathtt{taux}}\,(\mathtt{emittl2}[\mathtt{t}] - \mathtt{emittl20}\ \mathtt{switchDiffusion})\ \mathtt{switchRadiationDamping} + \\
&\qquad\quad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emittl2}[\mathtt{t}]}{\mathtt{TIBSl}[\mathtt{Nb1}[\mathtt{t}],\ \mathtt{emittx1}[\mathtt{t}],\ \mathtt{emittl2}[\mathtt{t}]]} - \\
&\qquad\quad \frac{\mathtt{S}_{\mathtt{ibs}}\ \mathtt{emittl2}[\mathtt{t}]}{\mathtt{TIBSl}[\mathtt{Nb10},\ \mathtt{emittx10},\ \mathtt{emittl20}]}\ \mathtt{switchDiffusion}\Bigg),\ \mathtt{InitialConditions}\Big\}\Big]
\end{aligned}
$$

Show the general form of our differential equations:

In[391]:=

```
TableForm[ODEs // Union]
```

Out[391]//TableForm=

$$\epsilon_{l1}[0] == \epsilon_{l10}$$

$$\epsilon_{l2}[0] == \epsilon_{l20}$$

$$\epsilon_{x1}[0] == \epsilon_{x10}$$

$$\epsilon_{x2}[0] == \epsilon_{x20}$$

$$\epsilon_{y1}[0] == \epsilon_{y10}$$

$$\epsilon_{y2}[0] == \epsilon_{y20}$$

$$N_{b1}[0] == N_{b10}$$

$$N_{b2}[0] == N_{b20}$$

$$\epsilon_{l1}'[t] == S_{\text{lemitt}} \big(-\frac{4\, S_{\text{rad}}\, (-\epsilon_{l10}\, S_{\text{diff}} + \epsilon_{l1}[t])}{\tau_x} - \frac{S_{\text{diff}}\, S_{\text{ibs}}\, \epsilon_{l1}[t]}{T_{\text{IBSl}}[N_{b10}, \epsilon_{x10}, \epsilon_{l10}]} + \frac{S_{\text{ibs}}\, \epsilon_{l1}[t]}{T_{\text{IBSl}}[N_{b1}[t], \epsilon_{x1}[t], \epsilon_{l1}}$$

$$\epsilon_{l2}'[t] == S_{\text{lemitt}} \big(-\frac{4\, S_{\text{rad}}\, (-\epsilon_{l20}\, S_{\text{diff}} + \epsilon_{l2}[t])}{\tau_x} - \frac{S_{\text{diff}}\, S_{\text{ibs}}\, \epsilon_{l2}[t]}{T_{\text{IBSl}}[N_{b10}, \epsilon_{x10}, \epsilon_{l20}]} + \frac{S_{\text{ibs}}\, \epsilon_{l2}[t]}{T_{\text{IBSl}}[N_{b1}[t], \epsilon_{x1}[t], \epsilon_{l2}}$$

$$\epsilon_{x1}'[t] == \left(\frac{4\, \pi\, r_p^2\, c\, Z_{\text{ion}}^2}{2\, \beta^2\, \gamma\, A_{\text{ion}}^2}\, \beta_{\text{ave}} \sum_g n_g\, Z_g\, (Z_g + 1)\, \ln\left[183\, Z_g^{-\frac{1}{3}}\right] \right) S_{\text{ms}} - \frac{2\, S_{\text{rad}}\, \epsilon_{x1}[t]}{\tau_x} + \frac{}{T_{\text{IBSx}}[}$$

$$\epsilon_{x2}'[t] == \left(\frac{4\, \pi\, r_p^2\, c\, Z_{\text{ion}}^2}{2\, \beta^2\, \gamma\, A_{\text{ion}}^2}\, \beta_{\text{ave}} \sum_g n_g\, Z_g\, (Z_g + 1)\, \ln\left[183\, Z_g^{-\frac{1}{3}}\right] \right) S_{\text{ms}} - \frac{2\, S_{\text{rad}}\, \epsilon_{x2}[t]}{\tau_x} + \frac{}{T_{\text{IBSx}}[}$$

$$\epsilon_{y1}'[t] == \left(\frac{4\, \pi\, r_p^2\, c\, Z_{\text{ion}}^2}{2\, \beta^2\, \gamma\, A_{\text{ion}}^2}\, \beta_{\text{ave}} \sum_g n_g\, Z_g\, (Z_g + 1)\, \ln\left[183\, Z_g^{-\frac{1}{3}}\right] \right) S_{\text{ms}} - \frac{2\, S_{\text{rad}}\, \epsilon_{y1}[t]}{\tau_x} + \frac{}{T_{\text{IBSx}}[}$$

$$\epsilon_{y2}'[t] == \left(\frac{4\, \pi\, r_p^2\, c\, Z_{\text{ion}}^2}{2\, \beta^2\, \gamma\, A_{\text{ion}}^2}\, \beta_{\text{ave}} \sum_g n_g\, Z_g\, (Z_g + 1)\, \ln\left[183\, Z_g^{-\frac{1}{3}}\right] \right) S_{\text{ms}} - \frac{2\, S_{\text{rad}}\, \epsilon_{y2}[t]}{\tau_x} + \frac{}{T_{\text{IBSx}}[}$$

$$N_{b1}'[t] == -\left(\sum_g n_g\, \sigma_{\text{bg}}\right) S_{\text{bg}}\, v_{\text{ion}}\, N_{b1}[t] - \frac{f_0\, n_{\text{exp}}\, \sigma_{\text{tot}}\, S_{\text{lumi}}\, N_{b1}[t]\, N_{b2}[t]}{\pi \left(\sqrt{\beta_{x1}\, \epsilon_{x1}[t]} + \sqrt{\beta_{x2}\, \epsilon_{x2}[t]}\right) \left(\sqrt{\beta_{y1}\, \epsilon_{y1}[t]} + \sqrt{\beta_{y2}\, \epsilon_{y2}[t]}\right)}$$

$$N_{b2}'[t] == -\left(\sum_g n_g\, \sigma_{\text{bg}}\right) S_{\text{bg}}\, v_{\text{ion}}\, N_{b2}[t] - \frac{f_0\, n_{\text{exp}}\, \sigma_{\text{tot}}\, S_{\text{lumi}}\, N_{b1}[t]\, N_{b2}[t]}{\pi \left(\sqrt{\beta_{x1}\, \epsilon_{x1}[t]} + \sqrt{\beta_{x2}\, \epsilon_{x2}[t]}\right) \left(\sqrt{\beta_{y1}\, \epsilon_{y1}[t]} + \sqrt{\beta_{y2}\, \epsilon_{y2}[t]}\right)}$$

Much other material skipped here.Paste some results below.

Protons, Injection, no diff. Multiple Scattering only

In[678]:=

```
modelSimplifications = Flatten[{Slumi → 0, Sbg → 0, Srad → 0, Sibs → 0,
   Sdiff → 0, Slemitt → 0, Sms → 1, ipBeta, roundBeams, equalBeams}]
```

Out[678]=

$$\{S_{\text{lumi}} \to 0,\ S_{\text{bg}} \to 0,\ S_{\text{rad}} \to 0,\ S_{\text{ibs}} \to 0,\ S_{\text{diff}} \to 0,\ S_{\text{lemitt}} \to 0,\ S_{\text{ms}} \to 1,\ \beta_{x1} \to \beta^*,\ \beta_{x2} \to \beta^*,\ \beta_{y1} \to \beta^*,\ \beta_{y2} \to \beta^*,\ \epsilon_{y1} \to \epsilon_{x1},\ \epsilon_{y2} \to \epsilon_{x2},\ \epsilon_{y10} \to \epsilon_{x10},\ \epsilon_{y20} \to \epsilon_{x20},\ \beta_{y1} \to \beta_{x1},\ \beta_{y2} \to \beta_{x2},\ N_{b2} \to N_{b1},\ N_{b20} \to N_{b10},\ \epsilon_{x2} \to \epsilon_{x1},\ \epsilon_{x20} \to \epsilon_{x10},\ \epsilon_{y2} \to \epsilon_{y1},\ \epsilon_{y20} \to \epsilon_{y10},\ \epsilon_{l2} \to \epsilon_{l1},\ \epsilon_{l20} \to \epsilon_{l10}\}$$

In[679]:=

```
modelInitialConditions =
 {Nb10 → Nb, emittx10 → ε /. geometricEmittanceDef, emittl10 → 2.5 eVs}
```

Out[679]=

$$\left\{N_{b10}\to N_b,\ \epsilon_{x10}\to \frac{\epsilon_n}{\sqrt{-1+\gamma_{\text{ion}}^2}},\ \epsilon_{l10}\to 2.5\ \text{eVs}\right\}$$

In[680]:=

```
modelODEs
```

Out[680]=

$$\Big\{\epsilon_{l1}[0]==\epsilon_{l10},\ \epsilon_{x1}[0]==\epsilon_{x10},\ N_{b1}[0]==N_{b10},\ \epsilon_{l1}'[t]==0,$$
$$\epsilon_{x1}'[t]==\frac{4\,\pi\,r_p{}^2\,c\,Z_{\text{ion}}{}^2}{2\,\beta^2\,\gamma\,A_{\text{ion}}{}^2}\,\beta_{\text{ave}}\sum_g n_g\,Z_g\,(Z_g+1)\,\ln\left[183\,Z_g{}^{-\frac{1}{3}}\right],\ N_{b1}'[t]==0\Big\}$$

In[681]:=

```
modelODEsN // Simplify // TableForm
```

Out[681]//TableForm=

$$\epsilon_{l1}[0.]==2.5$$
$$\epsilon_{x1}[0.]==7.81895\times 10^{-9}$$
$$N_{b1}[0.]==1.15\times 10^{11}$$
$$\epsilon_{l1}'[t]==0$$
$$\epsilon_{x1}'[t]==1.93877\times 10^{-9}$$
$$N_{b1}'[t]==0$$

In[682]:=

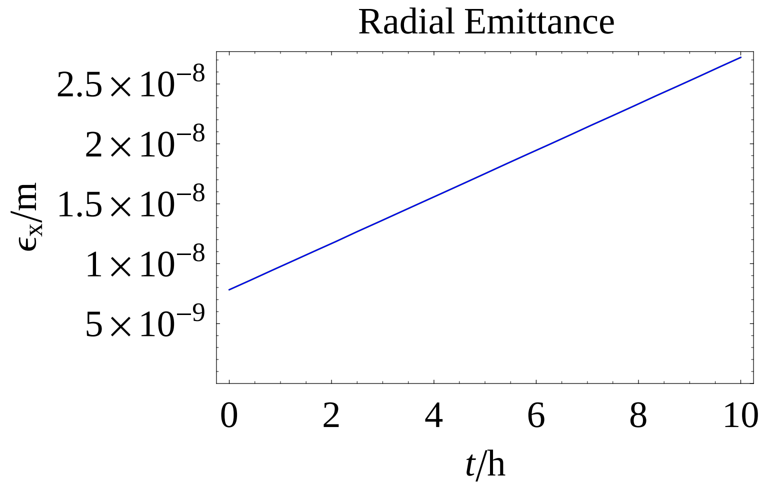
```
IBSfitmodelHCollision = {TIBSx[nb_, ex_, el_] :> 10072.814975574618`
     ibsTlLHCCollision[nb, ex/Meter, el/eVs] Second, TIBSl[nb_, ex_, el_] :>
    12987.866208439438` ibsTxLHCCollision[nb, ex/Meter, el/eVs] Second};
```

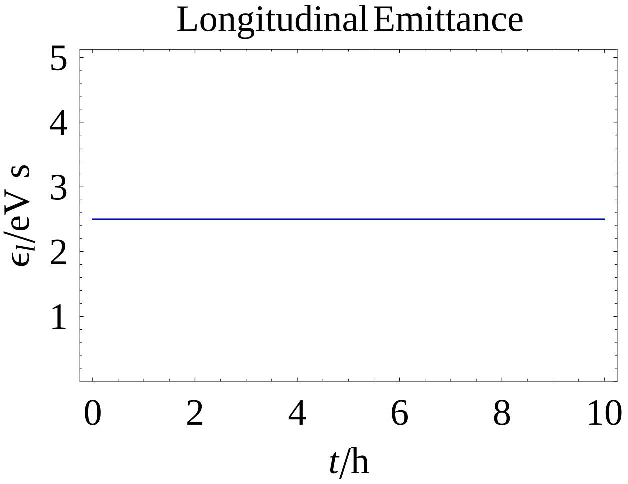
In[683]:=

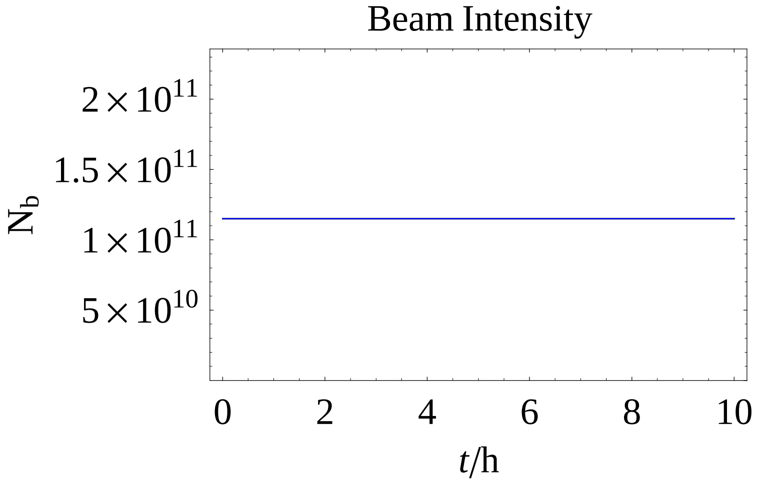
```
modelNumerical[f_] :=
 NumerDat[f //. Flatten[{IBSfitmodelHCollision, modelInitialConditions, mcsRule,
     LHCIonBeam[H, Injection], beamGasBarashenkov, vacuum100hpergas,
     ionVelocity, radiationDampingDef // N, betaAverage → 100 Meter}]]
```

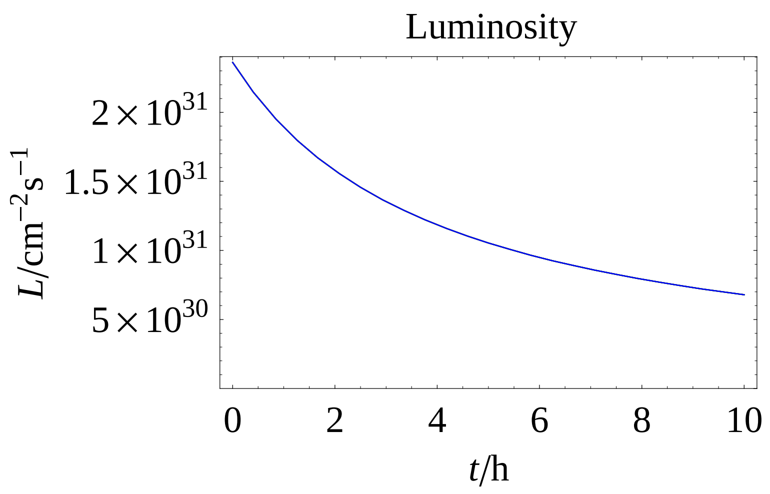
In[684]:=

```
plotFunctions[]
```


Radial Emittance
ϵ_x/m
2.5×10^-8
2×10^-8
1.5×10^-8
1×10^-8
5×10^-9
0
2
4
6
8
10
t/h


Longitudinal Emittance
ϵ_l/eV s
5
4
3
2
1
0
2
4
6
8
10
t/h


Beam Intensity
N_b
2×10^11
1.5×10^11
1×10^11
5×10^10
0
2
4
6
8
10
t/h



Out[684]=

```
{- Graphics -, - Graphics -, - Graphics -, - Graphics -}
```

In[586]:=

```
Table[tauInstantaneous[numericalLuminosity[nexp]], {nexp, 0, 3}] /. t → 0
```

Out[586]=

```
{407.567, 407.567, 407.567, 407.567}
```

In[587]:=

```
Table[tauLife[numericalLuminosity[nexp]], {nexp, 0, 4}]
```

In[588]:=

```
DisplayTogether[Table[plotEmittx[nexp], {nexp, 0, 4}]]
```

In[589]:=

```
DisplayTogether[Table[plotEmittl[nexp], {nexp, 0, 3}]]
```

```
updateTables
```

Protons, Collision no diff., Multiple Scattering only

In[685]:=

$$\text{modelSimplifications} = \text{Flatten}[\{S_{\text{lumi}} \to 0,\ S_{\text{bg}} \to 0,\ S_{\text{rad}} \to 0,\ S_{\text{ibs}} \to 0,\ S_{\text{diff}} \to 0,\ S_{\text{lemitt}} \to 0,\ S_{\text{ms}} \to 1,\ \text{ipBeta},\ \text{roundBeams},\ \text{equalBeams}\}]$$

Out[685]=

$$\{S_{\text{lumi}} \to 0,\ S_{\text{bg}} \to 0,\ S_{\text{rad}} \to 0,\ S_{\text{ibs}} \to 0,\ S_{\text{diff}} \to 0,\ S_{\text{lemitt}} \to 0,\ S_{\text{ms}} \to 1,\ \beta_{x1} \to \beta^*,\ \beta_{x2} \to \beta^*,\ \beta_{y1} \to \beta^*,\ \beta_{y2} \to \beta^*,\ \epsilon_{y1} \to \epsilon_{x1},\ \epsilon_{y2} \to \epsilon_{x2},\ \epsilon_{y10} \to \epsilon_{x10},\ \epsilon_{y20} \to \epsilon_{x20},\ \beta_{y1} \to \beta_{x1},\ \beta_{y2} \to \beta_{x2},\ N_{b2} \to N_{b1},\ N_{b20} \to N_{b10},\ \epsilon_{x2} \to \epsilon_{x1},\ \epsilon_{x20} \to \epsilon_{x10},\ \epsilon_{y2} \to \epsilon_{y1},\ \epsilon_{y20} \to \epsilon_{y10},\ \epsilon_{l2} \to \epsilon_{l1},\ \epsilon_{l20} \to \epsilon_{l10}\}$$

In[686]:=

```
modelInitialConditions =
 {Nb10 → Nb, emittx10 → ϵ /. geometricEmittanceDef, emittl10 → 2.5 eVs}
```

Out[686]=

$$\left\{N_{b10}\to N_b,\ \epsilon_{x10}\to \frac{\epsilon_n}{\sqrt{-1+\gamma_{ion}^2}},\ \epsilon_{l10}\to 2.5\ \mathrm{eVs}\right\}$$

In[687]:=

```
modelODEs
```

Out[687]=

$$\Big\{\epsilon_{l1}[0] == \epsilon_{l10},\ \epsilon_{x1}[0] == \epsilon_{x10},\ N_{b1}[0] == N_{b10},\ \epsilon_{l1}'[t] == 0,$$

$$\epsilon_{x1}'[t] == \frac{4\,\pi\, r_p^2\, c\, Z_{ion}{}^2}{2\,\beta^2\,\gamma\, A_{ion}{}^2}\ \beta_{ave}\sum_g n_g\, Z_g\,(Z_g+1)\ \ln\left[183\, Z_g{}^{-\frac{1}{3}}\right],\ N_{b1}'[t] == 0\Big\}$$

In[688]:=

```
IBSfitmodelHCollision = {TIBSx[nb_, ex_, el_] :→ 10072.814975574618`
     ibsTlLHCCollision[nb, ex/Meter, el/eVs] Second, TIBSl[nb_, ex_, el_] :→
    12987.866208439438` ibsTxLHCCollision[nb, ex/Meter, el/eVs] Second};
```

In[693]:=

```
modelNumerical[f_] :=
 NumerDat[f //. Flatten[{IBSfitmodelHCollision, modelInitialConditions, mcsRule,
     LHCIonBeam[H, Collision], beamGasBarashenkov, vacuum100hpergas,
     ionVelocity, radiationDampingDef // N, betaAverage → 100 Meter}]]
```

In[694]:=

```
modelODEsN // Simplify // TableForm
```

Out[694]//TableForm=

$\epsilon_{l1}[0.] == 2.5$

$\epsilon_{x1}[0.] == 5.02645\times 10^{-10}$

$N_{b1}[0.] == 1.15\times 10^{11}$

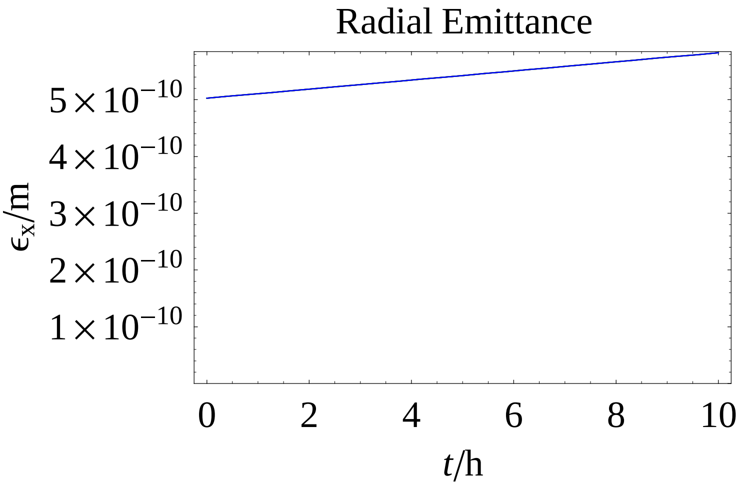
$\epsilon_{l1}'[t] == 0$

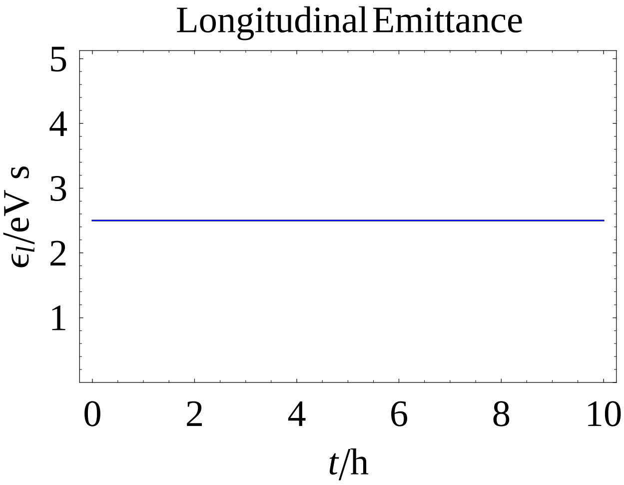
$\epsilon_{x1}'[t] == 8.0122\times 10^{-12}$

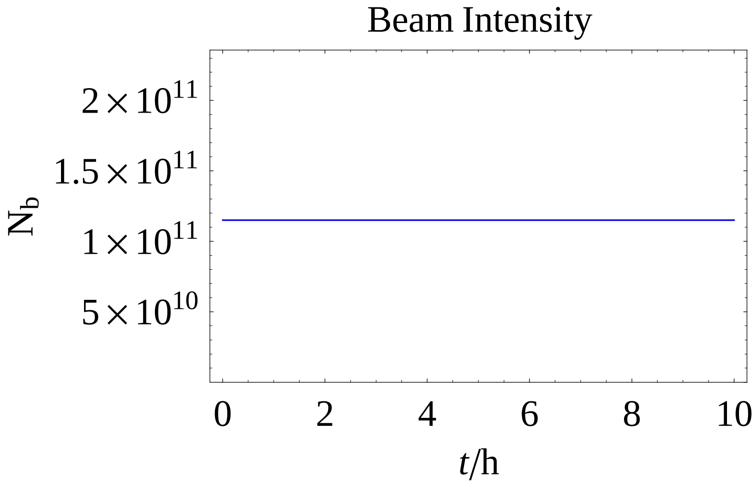
$N_{b1}'[t] == 0$

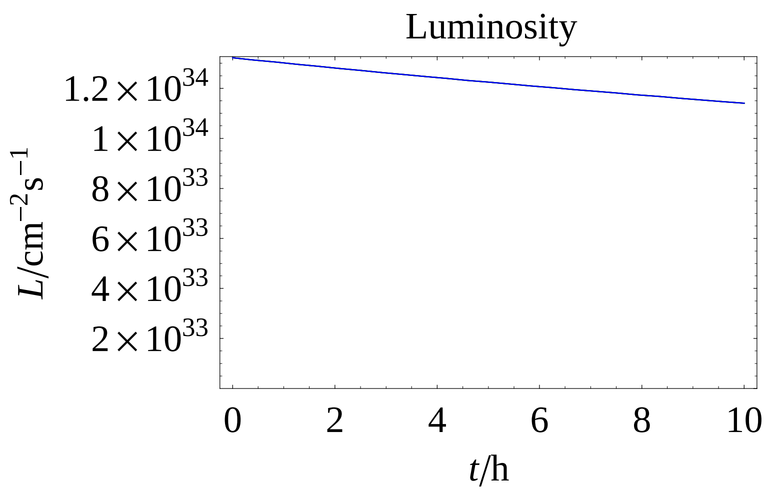
In[695]:=

```
plotFunctions[]
```


Radial Emittance
ϵ_x/m
5×10^{-10}
4×10^{-10}
3×10^{-10}
2×10^{-10}
1×10^{-10}
0
2
4
6
8
10
t/h


Longitudinal Emittance
ϵ_l/eV s
5
4
3
2
1
0
2
4
6
8
10
t/h


Beam Intensity
N_b
2×10^{11}
1.5×10^{11}
1×10^{11}
5×10^{10}
0
2
4
6
8
10
t/h



Out[695]=

```
{- Graphics -, - Graphics -, - Graphics -, - Graphics -}
```

In[696]:=

```
Table[tauInstantaneous[numericalLuminosity[nexp]], {nexp, 0, 3}] /. t → 0
```

Out[696]=

```
{62.735, 62.735, 62.735, 62.735}
```

```
DisplayTogether[Table[plotEmittx[nexp], {nexp, 0, 4}]]
```

```
DisplayTogether[Table[plotEmittl[nexp], {nexp, 0, 3}]]
```

```
updateTables
```

Protons, Collision no diff., all effects

In[698]:=

$\texttt{modelSimplifications} = \texttt{Flatten}[\{S_{\mathrm{lumi}} \to 1,\ S_{\mathrm{bg}} \to 1,\ S_{\mathrm{rad}} \to 0,\ S_{\mathrm{ibs}} \to 0,$
$\quad S_{\mathrm{diff}} \to 1,\ S_{\mathrm{lemitt}} \to 0,\ S_{\mathrm{ms}} \to 1,\ \texttt{ipBeta},\ \texttt{roundBeams},\ \texttt{equalBeams}\}]$

Out[698]=

$\{S_{\mathrm{lumi}} \to 1,\ S_{\mathrm{bg}} \to 1,\ S_{\mathrm{rad}} \to 0,\ S_{\mathrm{ibs}} \to 0,\ S_{\mathrm{diff}} \to 1,\ S_{\mathrm{lemitt}} \to 0,$
$S_{\mathrm{ms}} \to 1,\ \beta_{x1} \to \beta^*,\ \beta_{x2} \to \beta^*,\ \beta_{y1} \to \beta^*,\ \beta_{y2} \to \beta^*,\ \epsilon_{y1} \to \epsilon_{x1},\ \epsilon_{y2} \to \epsilon_{x2},$
$\epsilon_{y10} \to \epsilon_{x10},\ \epsilon_{y20} \to \epsilon_{x20},\ \beta_{y1} \to \beta_{x1},\ \beta_{y2} \to \beta_{x2},\ N_{b2} \to N_{b1},\ N_{b20} \to N_{b10},$
$\epsilon_{x2} \to \epsilon_{x1},\ \epsilon_{x20} \to \epsilon_{x10},\ \epsilon_{y2} \to \epsilon_{y1},\ \epsilon_{y20} \to \epsilon_{y10},\ \epsilon_{l2} \to \epsilon_{l1},\ \epsilon_{l20} \to \epsilon_{l10}\}$

In[699]:=

```
modelInitialConditions =
 {Nb10 → N_b, emittx10 → ϵ /. geometricEmittanceDef, emittl10 → 2.5 eVs}
```

Out[699]=

$$\left\{N_{b10} \to N_b,\ \epsilon_{x10} \to \frac{\epsilon_n}{\sqrt{-1+\gamma_{ion}^2}},\ \epsilon_{l10} \to 2.5\,\text{eVs}\right\}$$

In[700]:=

```
modelODEs
```

Out[700]=

$$\left\{\epsilon_{l1}[0] == \epsilon_{l10},\ \epsilon_{x1}[0] == \epsilon_{x10},\ N_{b1}[0] == N_{b10},\ \epsilon_{l1}'[t] == 0,\right.$$

$$\epsilon_{x1}'[t] == \frac{4\,\pi\,r_p{}^2\,c\,Z_{ion}{}^2}{2\,\beta^2\,\gamma\,A_{ion}{}^2}\,\beta_{ave}\sum_g n_g\,Z_g\,(Z_g+1)\,\ln\left[183\,Z_g{}^{-\frac{1}{3}}\right],$$

$$\left. N_{b1}'[t] == -\left(\sum_g n_g\,\sigma_{bg}\right) v_{ion}\,N_{b1}[t] - \frac{f_0\,n_{exp}\,\sigma_{tot}\,N_{b1}[t]^2}{4\,\beta^*\,\pi\,\epsilon_{x1}[t]}\right\}$$

In[701]:=

```
IBSfitmodelHCollision = {TIBSx[nb_, ex_, el_] :→ 10072.814975574618`
    ibsTlLHCCollision[nb, ex/Meter, el/eVs] Second, TIBSl[nb_, ex_, el_] :→
   12987.866208439438` ibsTxLHCCollision[nb, ex/Meter, el/eVs] Second};
```

In[704]:=

```
modelNumerical[f_] :=
 NumerDat[f //. Flatten[{IBSfitmodelHCollision, modelInitialConditions, mcsRule,
     LHCIonBeam[H, Collision], beamGasAbrasionAblationRELDIS, vacuum100hpergas,
     ionVelocity, radiationDampingDef // N, betaAverage → 100 Meter}]]
```

In[705]:=

```
modelODEsN // Simplify // TableForm
```

Out[705]//TableForm=

$$\epsilon_{l1}[0.] == 2.5$$

$$\epsilon_{x1}[0.] == 5.02645\times 10^{-10}$$

$$N_{b1}[0.] == 1.15\times 10^{11}$$

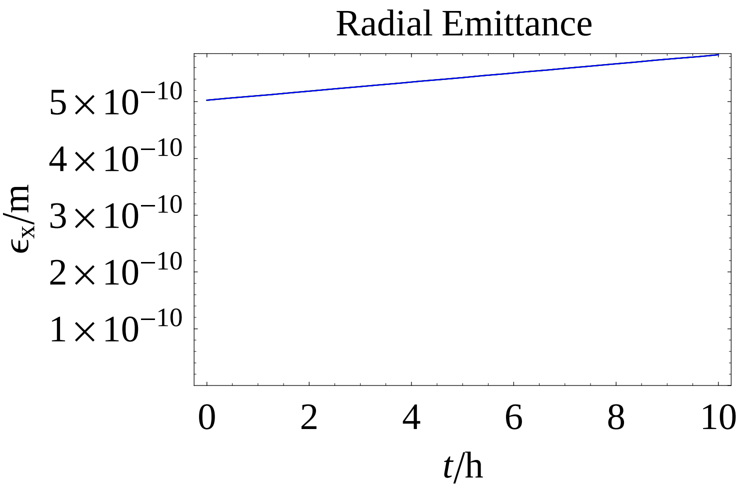
$$\epsilon_{l1}'[t] == 0$$

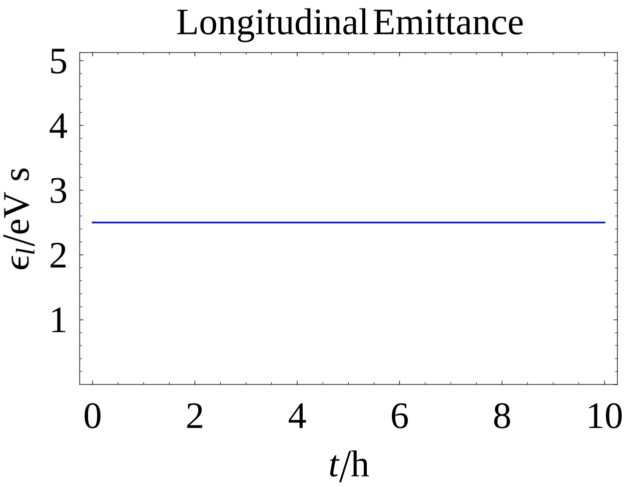
$$\epsilon_{x1}'[t] == 8.0122\times 10^{-12}$$

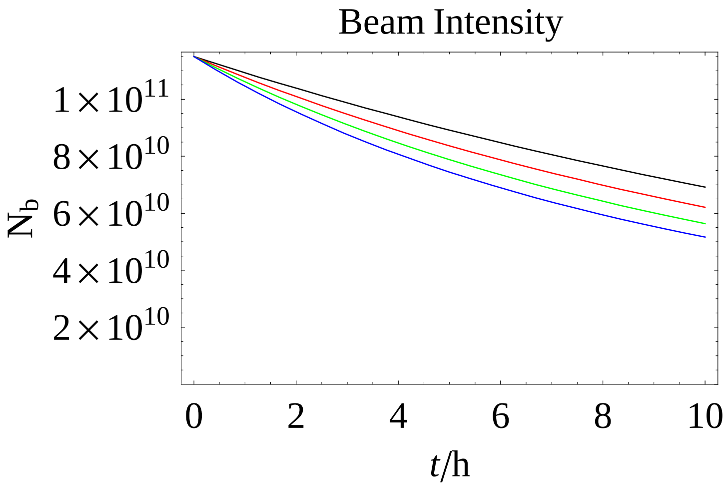
$$N_{b1}'[t] == N_{b1}[t]\left(-0.0508568 - \frac{6.76536\times 10^{-23}\,n_{exp}\,N_{b1}[t]}{\epsilon_{x1}[t]}\right)$$

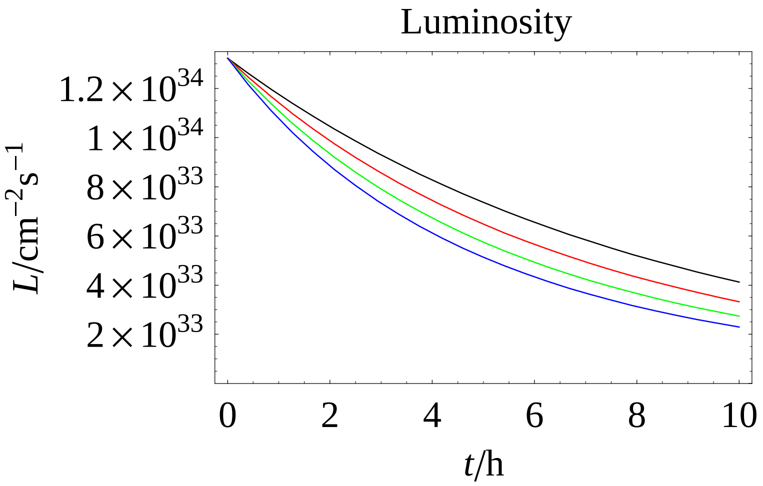
In[706]:=

```
plotFunctions[]
```


Radial Emittance
ϵ_x/m
5×10^{-10}
4×10^{-10}
3×10^{-10}
2×10^{-10}
1×10^{-10}
0
2
4
6
8
10
t/h


Longitudinal Emittance
ϵ_l/eV s
5
4
3
2
1
0
2
4
6
8
10
t/h


Beam Intensity
N_b
1×10^{11}
8×10^{10}
6×10^{10}
4×10^{10}
2×10^{10}
0
2
4
6
8
10
t/h



Out[706]=

```
{- Graphics -, - Graphics -, - Graphics -, - Graphics -}
```

In[707]:=

```
Table[tauInstantaneous[numericalLuminosity[nexp]], {nexp, 0, 3}] /. t → 0
```

Out[707]=

```
{8.49952, 6.729, 5.56894, 4.75005}
```

Conclusions

We have implemented a rather complete model for the evolution of beam intensity, emittances and luminosity. Any combination of effects can be included.

Although based on a different formulation of the multiple-Coulomb scattering on residual gas and using other sources for the nuclear cross sections, thes calculations are in reasonable agreement for beam lifetime with those presented previously, e.g., in the Design Report.

If the residual gas densities are really as high as stated then emittance growth from multiple Coulomb scattering will be a significant effect, particularly at injection.