TRANSVERSE RESISTIVE-WALL IMPEDANCE : COMPARISON ZOTTER - BUROV/LEBEDEV - CLASSICAL THICK-WALL

E. Métral

- **I re-derived most of Zotter's equations and we are currently checking our (slightly) different results…**
- **However, both formulae lead to the same numerical result as Burov-Lebedev for an LHC collimator ! (no factor 100 obtained…)**

FROM ZOTTER'S THEORY : FIELD MATCHING (1/2)

$$z_{\perp,\mathrm{round}}^{\mathrm{RW}} = j\,\frac{L\,Z_0}{\pi\,b^2} \times \frac{\beta - \dfrac{(1-j)\,r\,q\,Q_\eta}{2\,\gamma^2}}{1+2\,j\,p-\beta\left[\,(1+j)\dfrac{q}{r}Q_\alpha-(1-j)\dfrac{r\,q}{2}Q_\eta\,\right]+\dfrac{q^2\,Q_\eta\,Q_\alpha-p^2}{\gamma^2}}$$

$$r = \mu'\,\beta\,k\,\delta$$

$$\delta = \sqrt{\frac{1}{\pi\,f\,\mu\,\sigma}}$$

$$Q_\alpha = \frac{Q_2-\alpha_2\,P_2}{1-\alpha_2}$$

$$Q_\eta = \frac{Q_2-\eta_2\,P_2}{1-\eta_2}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$q = k\,b$$

$$p = \frac{k^2\,\delta^2}{2}$$

$$P_2 = \frac{I_1'(x)}{I_1(x)}$$

$$b = \text{Radius of the object}$$

$$L = \text{Length of the object}$$

$$k = \frac{2\pi\,f}{\beta\,c}$$

$$Q_2 = \frac{K_1'(x)}{K_1(x)}$$

$$\mu = \mu_0\,\mu' = \mu_0\,\mu_r\,(1+j\tan\vartheta_M)$$

$$x = \nu\,b$$

$$Z_0 = 120\,\pi$$

$$\varepsilon_c = \varepsilon_0\,\varepsilon' = \varepsilon_0\,\varepsilon_r + \frac{\sigma}{j\,2\pi\,f}$$

α_2* and *η_2* are determined by the boundary conditions at the outer chamber wall *r = d

$$\nu = \frac{1+j}{\delta}$$

⟹ Approximation here for a metal

⟹ In the general case

$$\nu = k\sqrt{1-\beta^2\,\varepsilon'\,\mu'}$$

FROM ZOTTER'S THEORY : FIELD MATCHING (2/2)

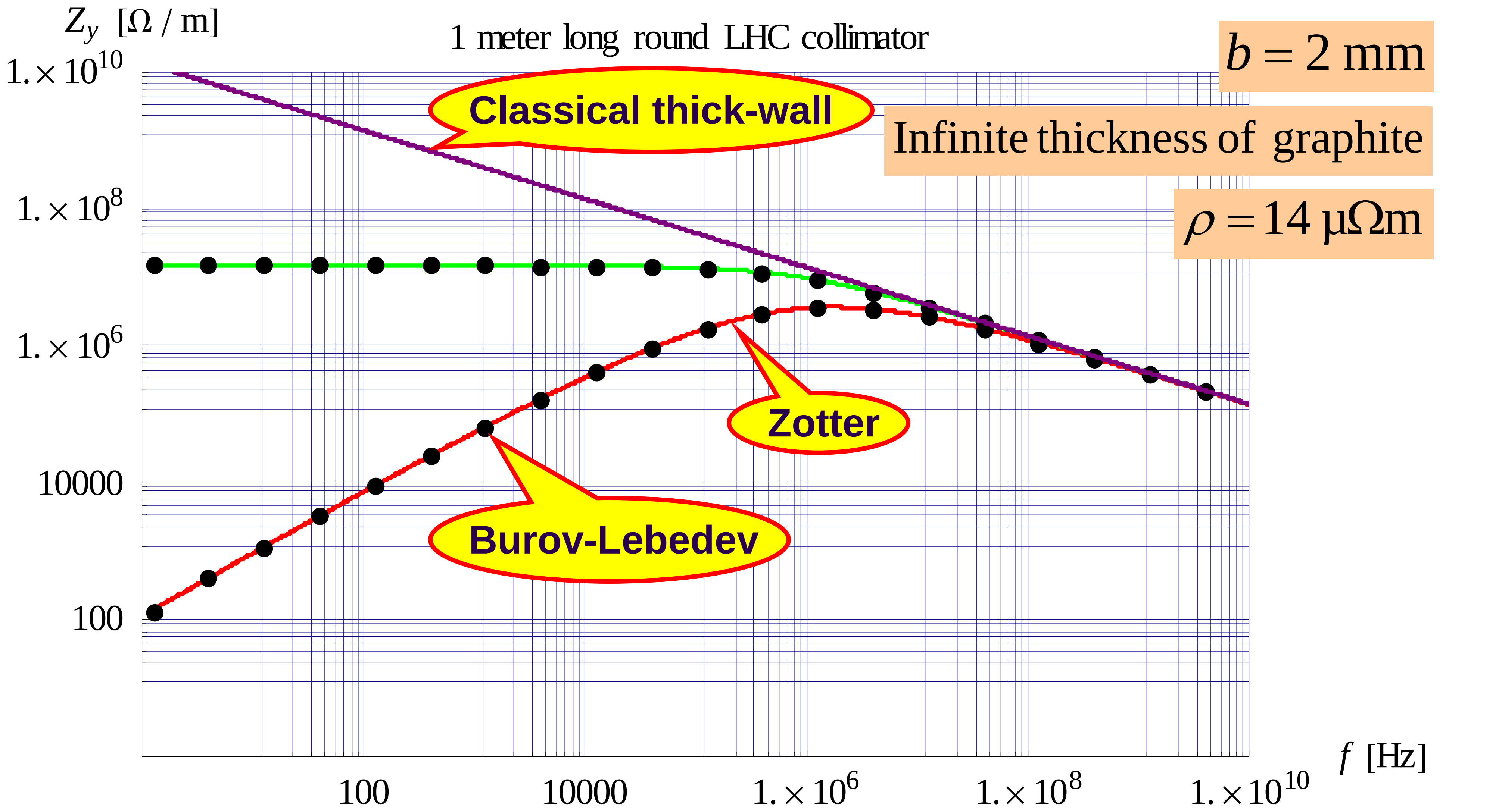
- **Infinitely thick wall (INF)** $\alpha_2^{\text{INF}} = \eta_2^{\text{INF}} = 0$

- **Perfect conductor (PC)**

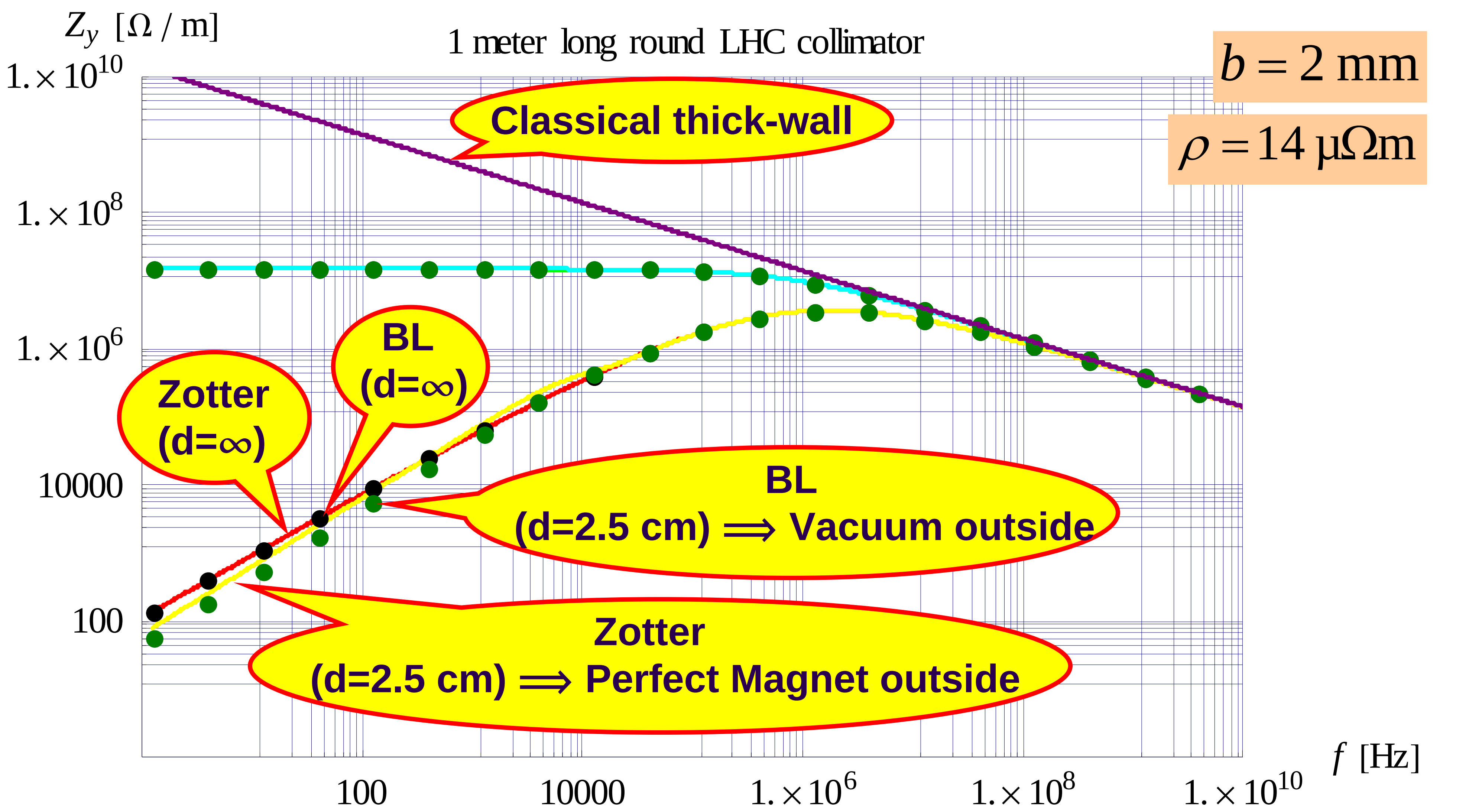
$$\alpha_2^{\text{PC}} = \frac{K_1(y)\, I_1(x)}{I_1(y)\, K_1(x)} \qquad \eta_2^{\text{PC}} = \frac{K_1'(y)\, I_1(x)}{I_1'(y)\, K_1(x)}$$

$$y = \nu\, d$$

- **Perfect magnet (PM)** $\alpha_2^{\text{PM}} = \eta_2^{\text{PC}}$ $\eta_2^{\text{PM}} = \alpha_2^{\text{PC}}$

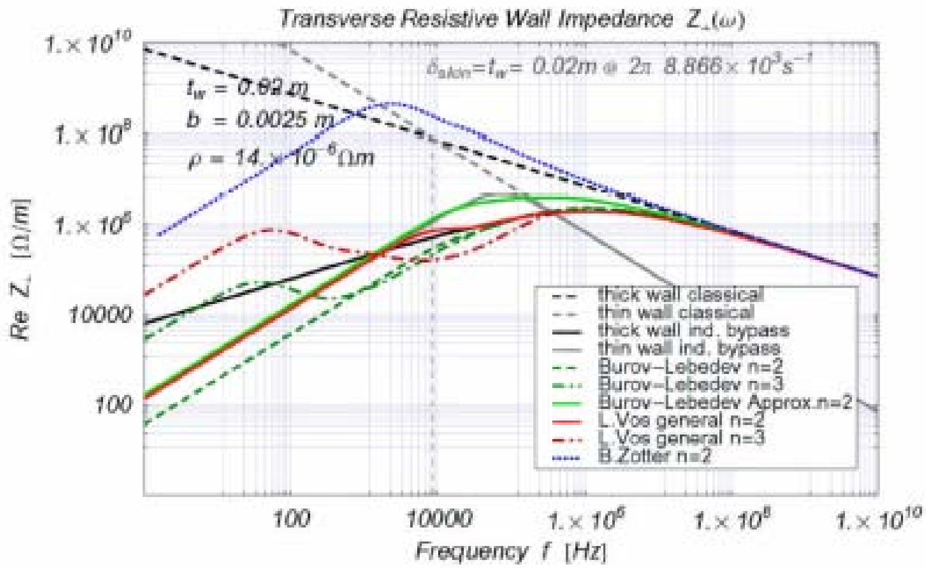


Z_y [Ω / m]
1 meter long round LHC collimator
$b = 2$ mm
Infinite thickness of graphite
$\rho = 14$ μΩm
Classical thick-wall
Zotter
Burov-Lebedev
$1.\times 10^{10}$
$1.\times 10^{8}$
$1.\times 10^{6}$
10000
100
100
10000
$1.\times 10^{6}$
$1.\times 10^{8}$
$1.\times 10^{10}$
f [Hz]



Z_y [Ω / m]
1 meter long round LHC collimator
$b = 2$ mm
$\rho = 14$ μΩm
Classical thick-wall
BL (d=∞)
Zotter (d=∞)
BL (d=2.5 cm) ⟹ Vacuum outside
Zotter (d=2.5 cm) ⟹ Perfect Magnet outside
$1. \times 10^{10}$
$1. \times 10^{8}$
$1. \times 10^{6}$
10000
100
100
10000
$1. \times 10^{6}$
$1. \times 10^{8}$
$1. \times 10^{10}$
f [Hz]

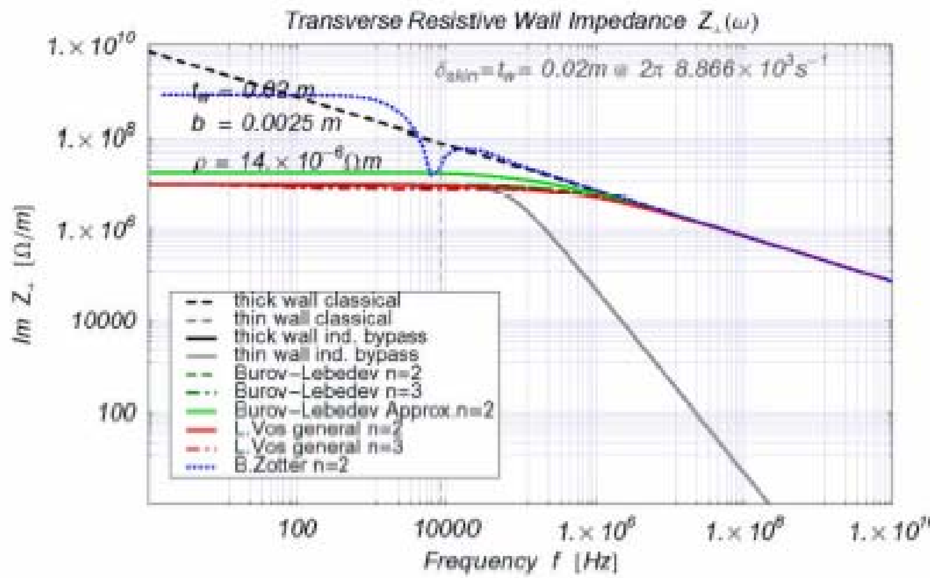
To be compared with…



Zotter's impedance model

Predictions for the collimator impedance from calculations and formulae by various authors (compiled & plotted by A. Koschik).

Blue curve is B. Zotter's result.



F. Zimmermann, LCE 22.10.04 10

CONCLUSION AND FUTURE WORK

- **Burov-Lebedev and Zotter (and also Vos and Tsutsui) seem to give the same result for the LHC collimator ! $\Longrightarrow$ Good news for LHC…**

- **More discussions on February 11, 2005, when B. Zotter will give his talk…**

- **The resistive-wall impedance can be derived without making the approximation $x = k\,b/\gamma << 1$ and without assuming a metal for the 1st layer $\Longrightarrow$ I derived it analytically and I will discuss with B. Zotter**

- **I am looking also at the draft paper by B. Zotter and R.L. Gluckstern on the "Transverse Impedance of a resistive tube of finite length"**