Coherent & Incoherent Tune Shift Induced by the Nonlinear Flat-Chamber Resistive-Wall Wake Field with Application to the LHC Collimator Experiment in the SPS

Frank Zimmermann, RLC Meeting 26.08.2005

wake potential & nonlinear deflection

Potential for nonlinear resistive-wall impedance between two parallel plates was derived by Piwinski (DESY 94-068, Eq. (52)) and re-written by Bane, Irwin, and Raubenheimer (NLC ZDR p. 594).

$$V(x,y,x_0,y_0) = -\kappa f_R(\tau)\left[\frac{-x_-\sinh x_- + y_+\sin y_+}{\cosh x_- + \cos y_+} + \frac{x_-\sinh x_- + y_-\sin y_-}{\cosh x_- - \cos y_-}\right]$$

$$y_+ = \frac{\pi}{2b}(y+y_0),\ y_- = \frac{\pi}{2b}(y-y_0),\ x_- = \frac{\pi}{2b}(x-x_0)$$

$2b$: full gap

$$\kappa = \frac{1}{2}\frac{N r_p}{\gamma\sigma_z}\frac{L}{b}\sqrt{\lambda\sigma_z} \qquad f_R(\tau) = \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{d\tau'}{\sqrt{\tau'}} e^{-\frac{(\tau+\tau')^2}{2}} \qquad \langle f_R \rangle = 0.816$$

$$\lambda = \rho/(120\pi)$$

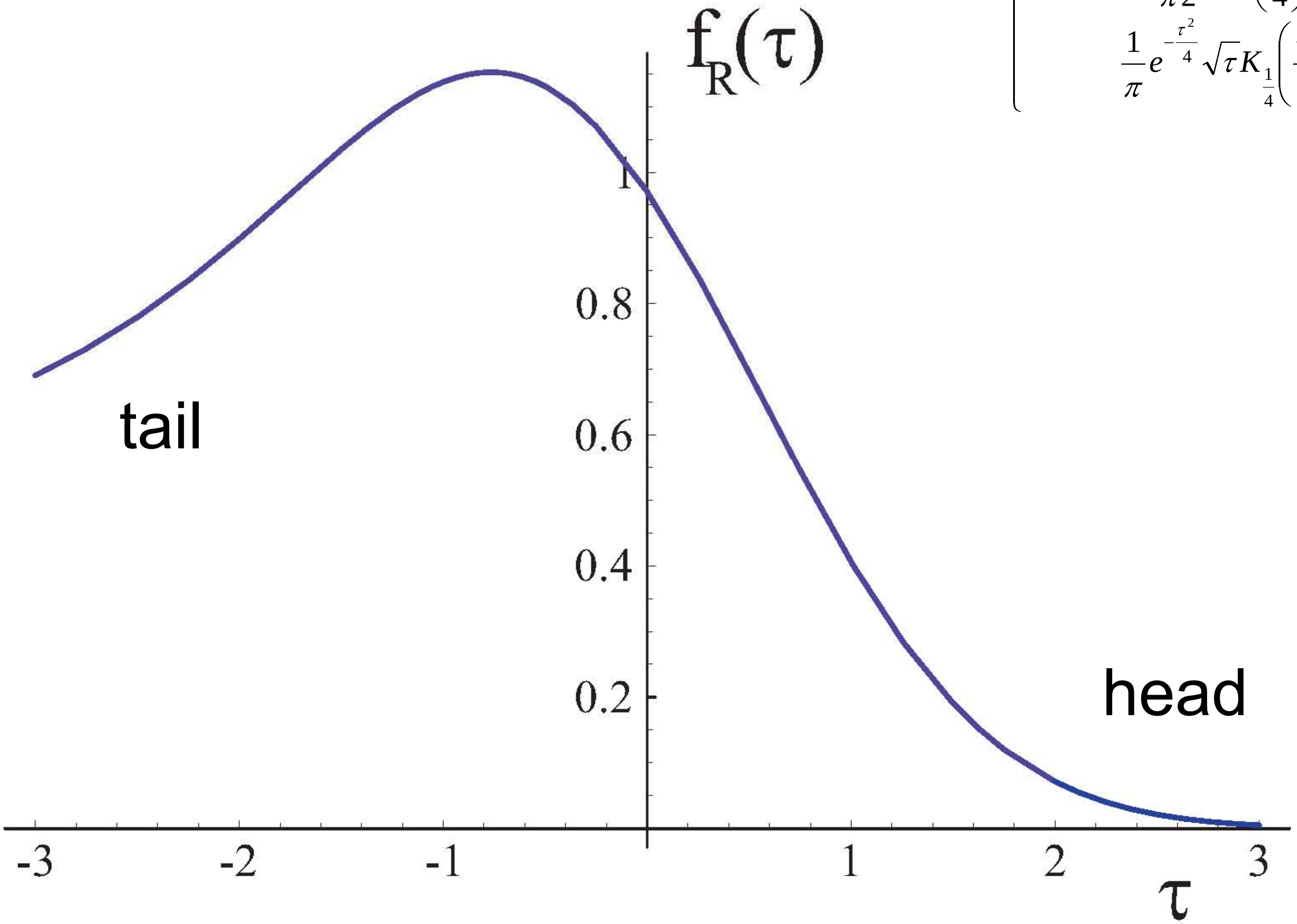
nonlinear kick to test particle:

$$\Delta y' = -\frac{\partial V}{\partial y}$$

Piwinski formula applies if $\delta_s << \mathrm{Min}\left(d_{wall}, d_{bw}, 1/(bk^2)\right)$

time dependence of kick along the bunch is described by f_R

$$f_R(\tau) = \begin{cases} \frac{1}{\sqrt{2}} e^{-\frac{\tau^2}{4}} \sqrt{-\tau} \left(I_{-\frac{1}{4}}\left(\frac{\tau^2}{4}\right) + I_{\frac{1}{4}}\left(\frac{\tau^2}{4}\right) \right) & \text{for} \quad \tau < 0 \\ \frac{1}{\pi 2^{1/4}} \Gamma\left(\frac{1}{4}\right) & \text{for} \quad \tau = 0 \\ \frac{1}{\pi} e^{-\frac{\tau^2}{4}} \sqrt{\tau} K_{\frac{1}{4}}\left(\frac{\tau^2}{4}\right) & \text{for} \quad \tau > 0 \end{cases}$$



coherent tune shift:

$$\Delta Q_y = -\frac{1}{\left[\mathrm{erf}\left(\frac{b-y_{co}}{\sqrt{2}\sigma_y}\right)\right]^2}\frac{\beta}{4\pi}\kappa\langle f_R\rangle_\tau$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-b+2y_{co}}^{b}\int_{-b+2y_{co}}^{b}\frac{1}{\kappa f_R(\tau)}\left.\frac{\partial(\Delta y'(x,y_c,x_0,y_c+y_0,\tau))}{\partial y_c}\right|_{y_c=y_{co}}\frac{e^{-\frac{(y-y_{co})^2}{2\sigma_y^2}-\frac{(y_0-y_{co})^2}{2\sigma_y^2}-\frac{x^2}{2\sigma_x^2}-\frac{x_0^2}{2\sigma_x^2}}}{(2\pi)^2\sigma_x^2\sigma_y^2}\,dy\,dy_0\,dx\,dx_0$$

introduce new coordinates
and perform 2 integrations

$$X \equiv x - x_0,\quad Y \equiv y + y_0 + 2y_{co}$$

$$U \equiv x + x_0,\quad W \equiv y - y_0$$

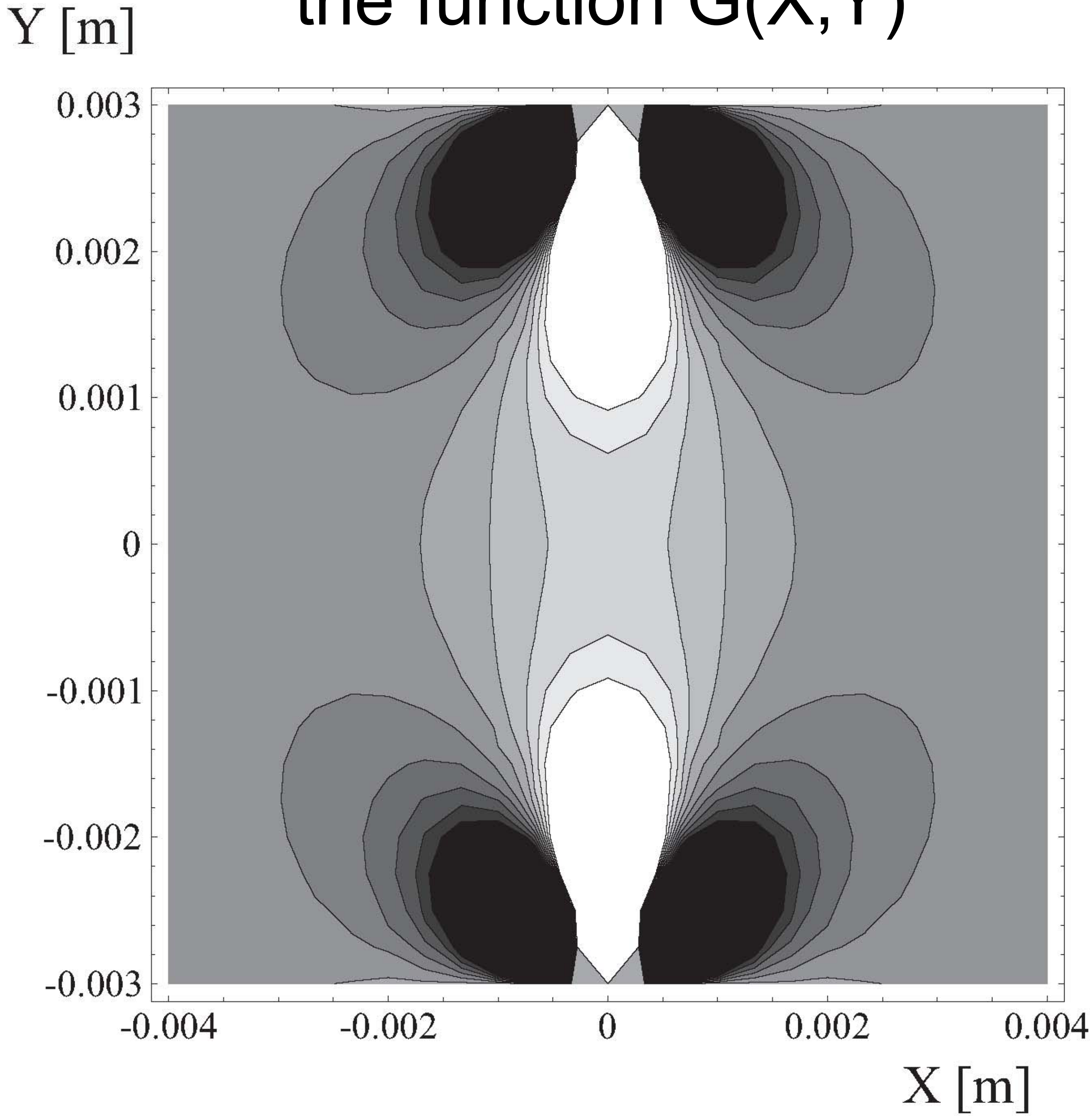
$$\Delta Q_y = -\frac{\mathrm{erf}\left(\frac{b-y_{co}}{\sigma_y}\right)}{\left[\mathrm{erf}\left(\frac{b-y_{co}}{\sqrt{2}\sigma_y}\right)\right]^2}\frac{\beta\kappa\langle f_R\rangle_\tau}{4\pi}\int_{-\infty}^{\infty}\int_{-2b+4y_{co}}^{2b}G(X,Y))\frac{e^{-\frac{Y^2}{4\sigma_y^2}-\frac{X^2}{4\sigma_x^2}}}{(4\pi)\,\sigma_x\sigma_y}\,dX\,dY$$

$$G(X,Y) \equiv -\frac{\pi^2 8}{b^3}\left(\cos\left(\frac{\pi Y}{2b}\right)+\cosh\left(\frac{\pi X}{2b}\right)\right)^{-3}\left[\begin{array}{l}\left\{2\pi Y\sin\left(\frac{\pi Y}{2b}\right)-8b\cos\left(\frac{\pi X}{2b}\right)\right\}\cosh^2\left(\frac{\pi X}{2b}\right)\\ -\left\{4b\cos\left(\frac{\pi Y}{b}\right)+12b+\pi Y\sin\left(\frac{\pi Y}{b}\right))\right\}\cosh\left(\frac{\pi X}{2b}\right)-4\pi Y\sin\left(\frac{\pi Y}{2b}\right)\\ -8b\cos\left(\frac{\pi Y}{2b}\right)-\pi X\left(\cos\left(\frac{\pi Y}{b}\right)-3\right)\sinh\left(\frac{\pi X}{2b}\right)+\pi X\cos\left(\frac{\pi Y}{2b}\right)\sinh\left(\frac{\pi X}{b}\right)\end{array}\right]$$

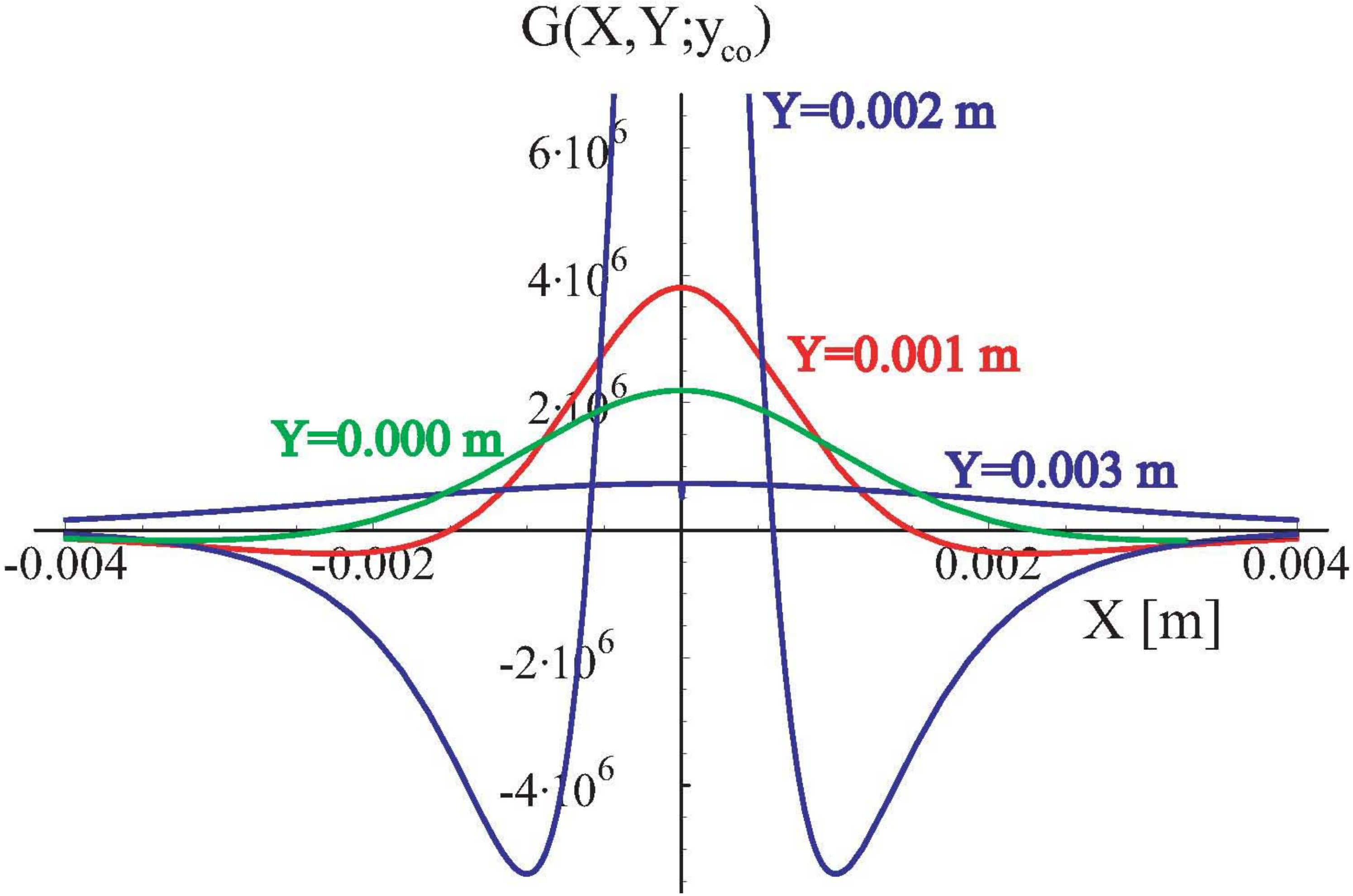
example parameters

bunch population	N_b	10^{11}
horizontal beta function	β_x	93 m
vertical beta function	β_y	25 m
normalized horizontal emittance	$\gamma\epsilon_x$	1.5 μm
normalized vertical emittance	$\gamma\epsilon_y$	1.5 μm
rms horizontal beam size	σ_x	0.72 mm
rms vertical beam size	σ_y	0.37 mm
rms bunch length	σ_z	0.21 m
vertical tune	Q_y	26.135
proton beam momentum	p	270 GeV/c
collimator half gap	b	1.5 mm
collimator thickness	d	30 mm
collimator resistivity	ρ	10 $\mu\Omega$m

the function G(X,Y)

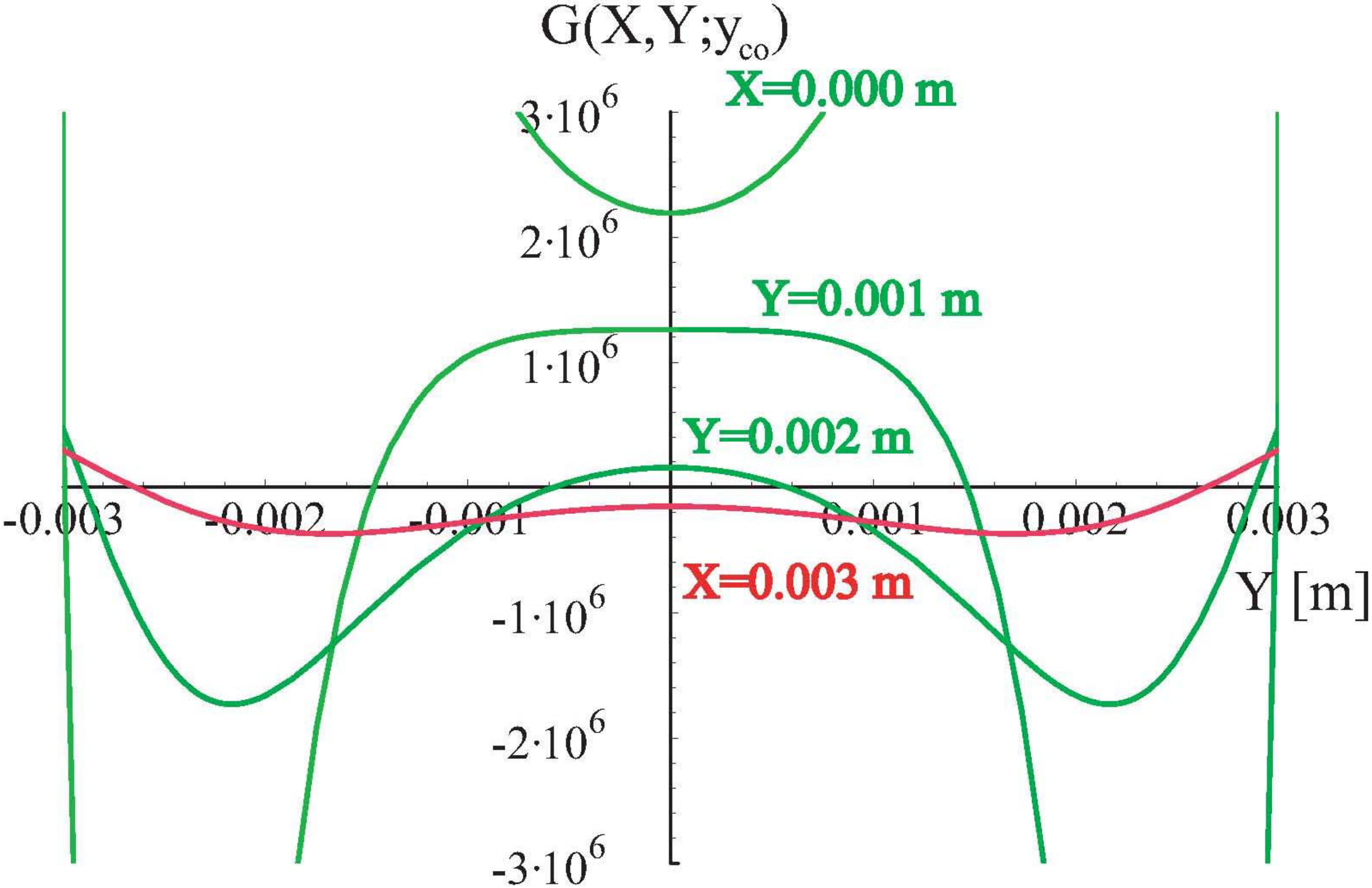


the function G(X,Y)



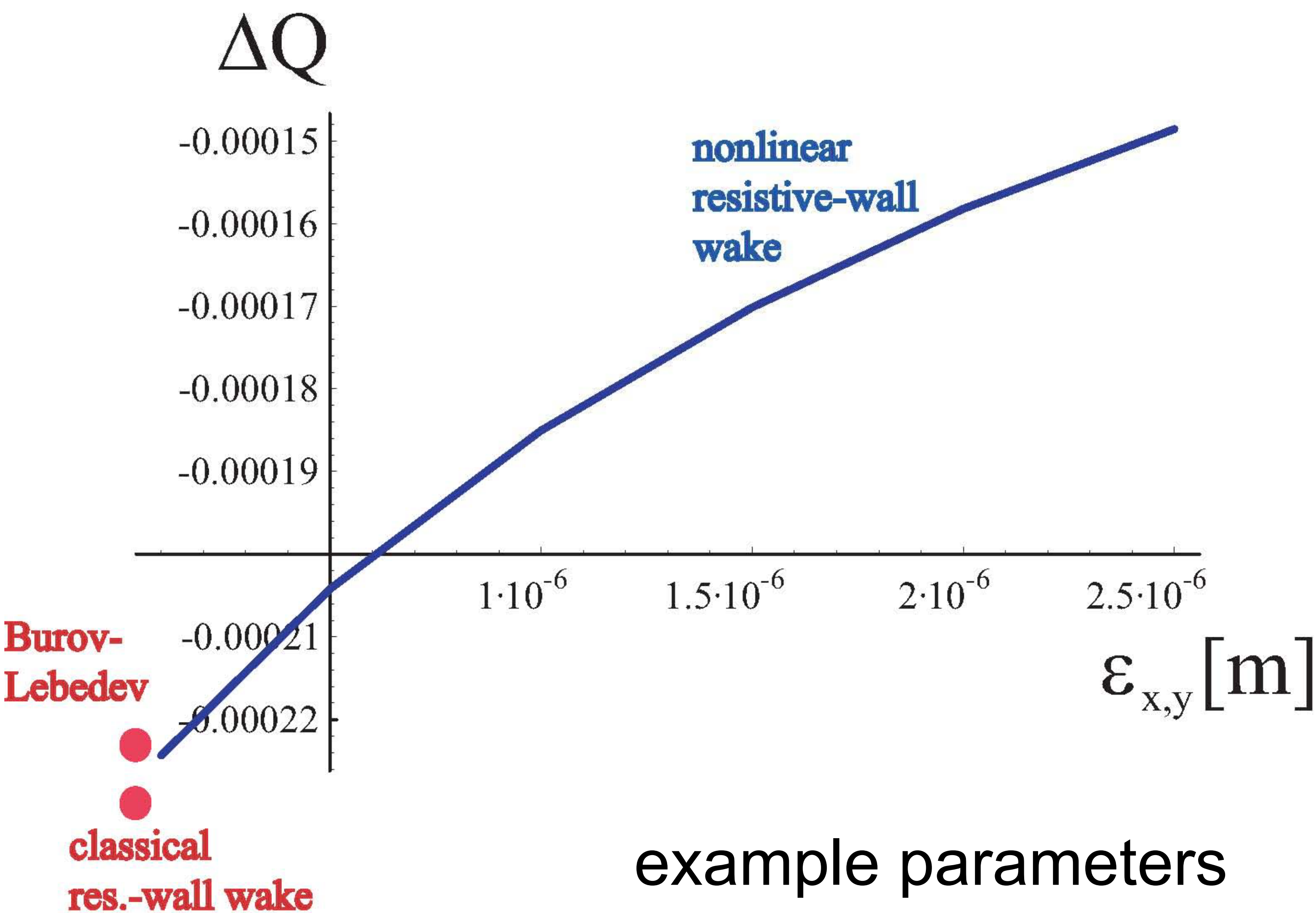
note: change of sign for large X

the function G(X,Y)



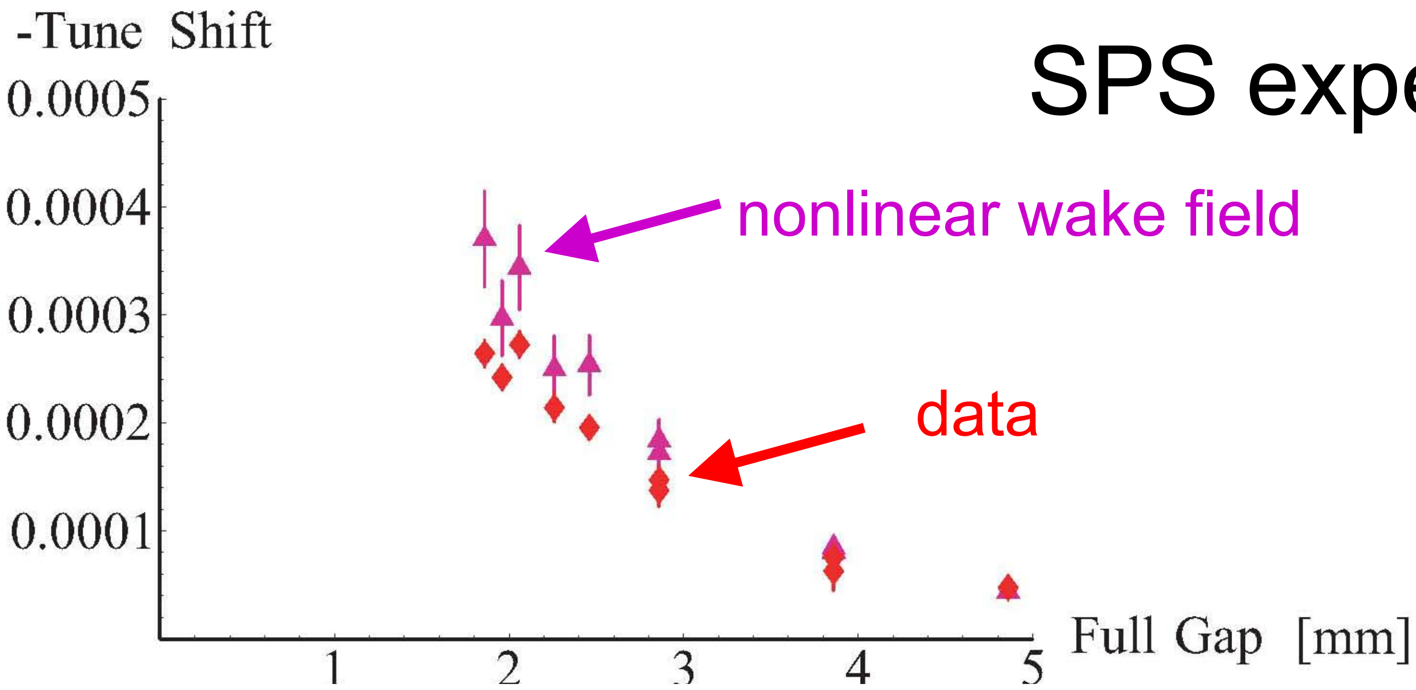
note: divergence for Y→2b

variation of coherent tune shift with emittance

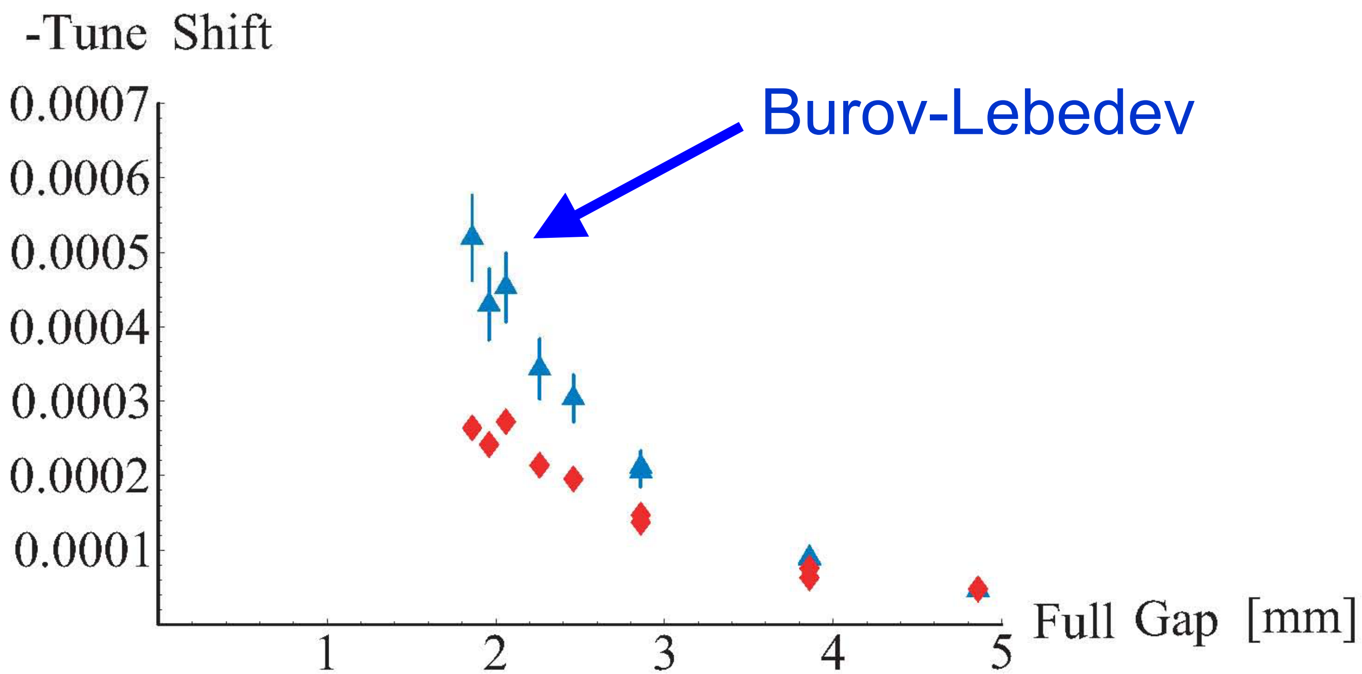


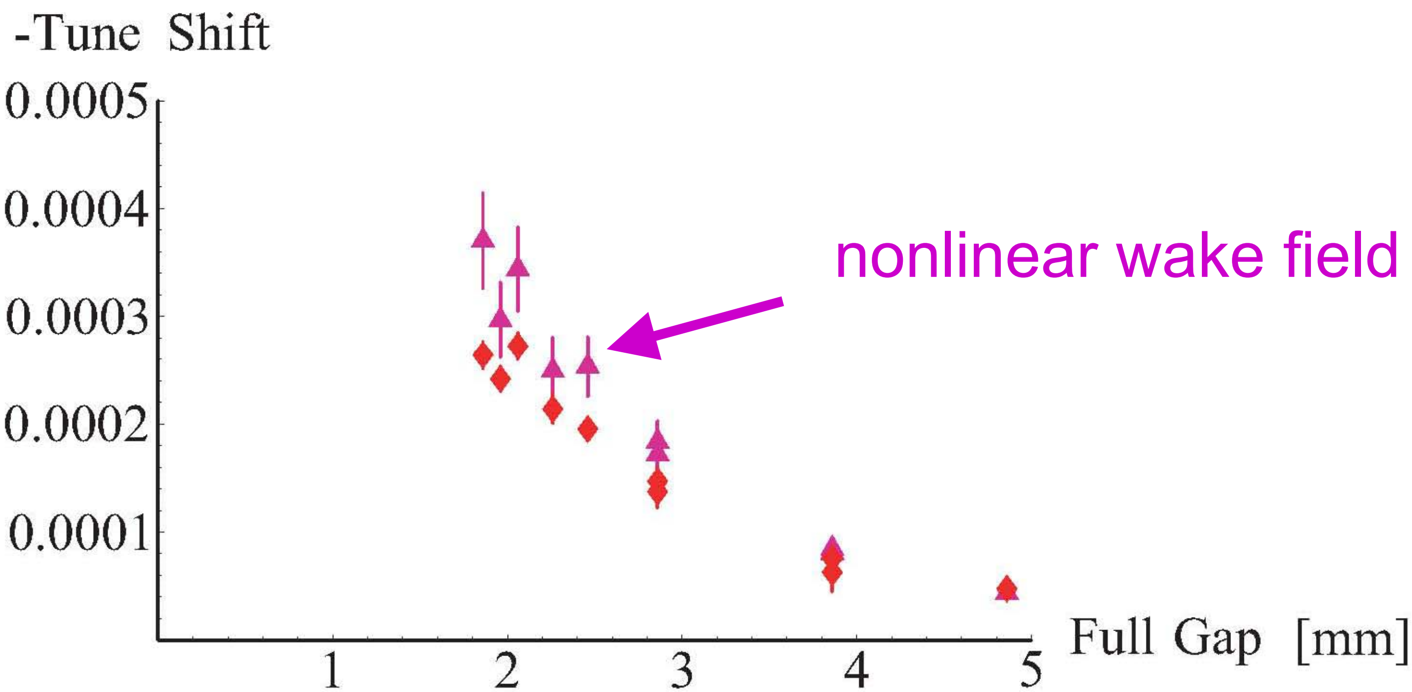
example parameters

SPS experiment

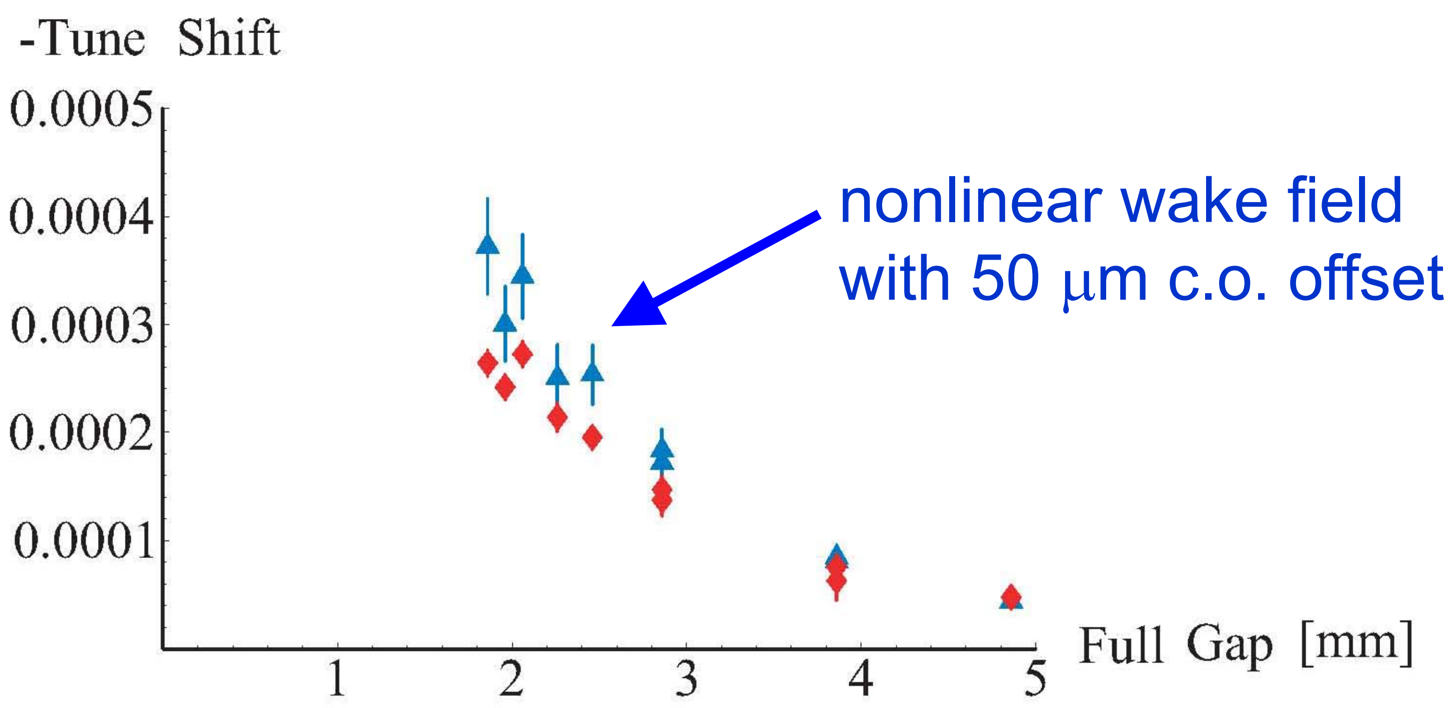


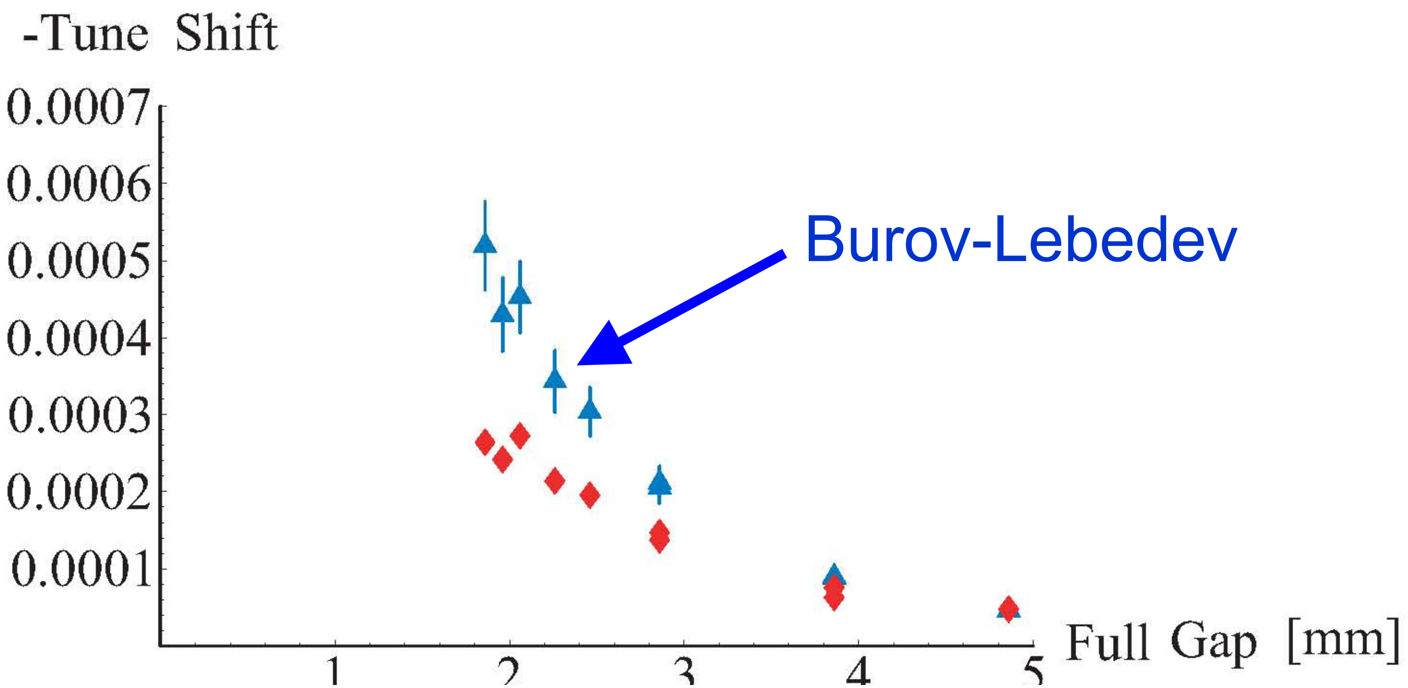
factor
2 difference
at small
gaps
for B-L,
becomes
20%
for the
nonlinear
wake field

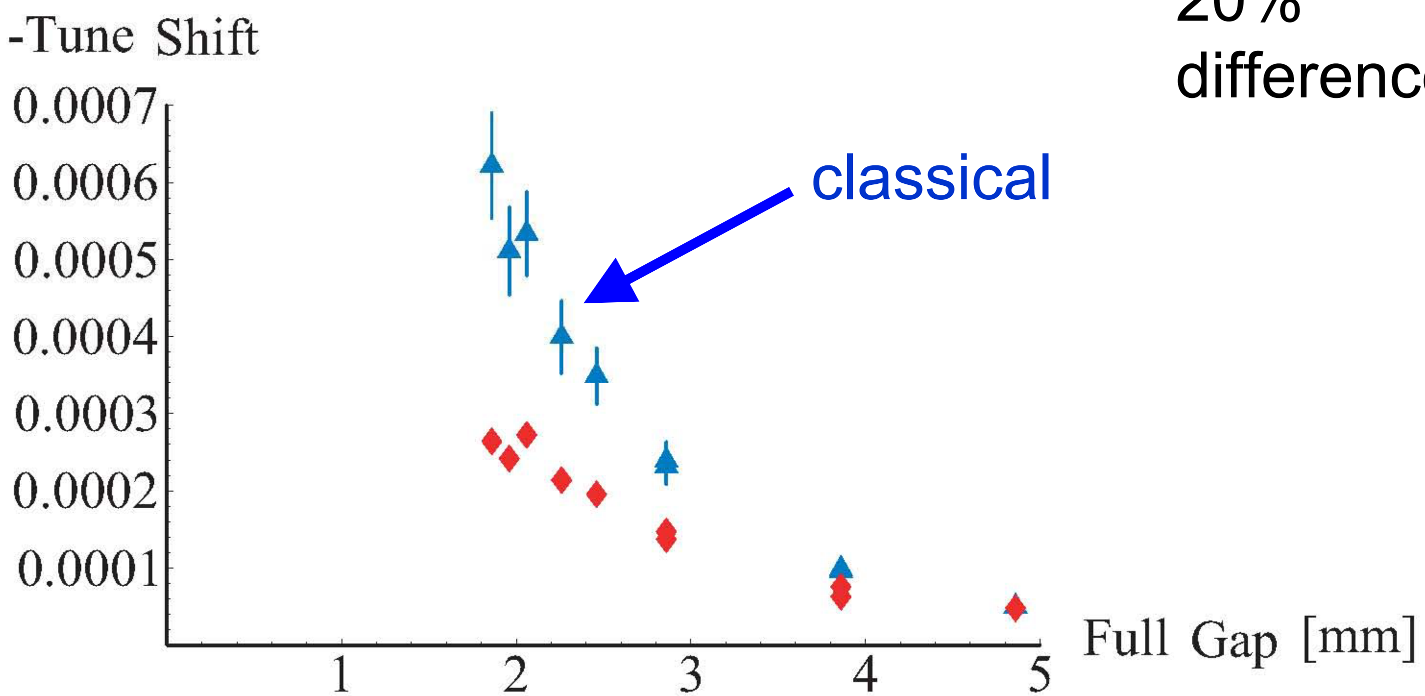




closed orbit offset of 50 μm increases χ^2 from 0.30 to 0.32







the classical prediction is 20% higher than the B-L one, which could explain the remaining 20% difference

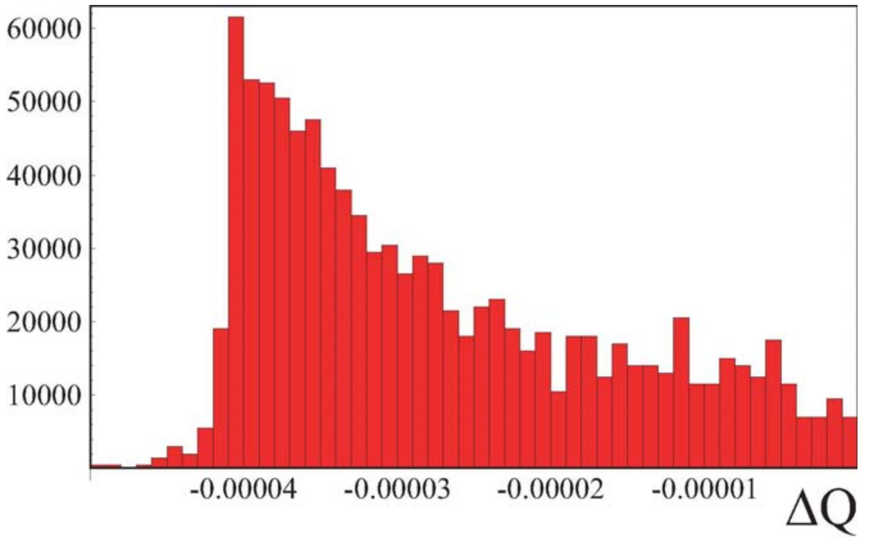
incoherent tune shift

$$\Delta Q_y = -\frac{1}{\left[\operatorname{erf}\left(\frac{b-y_{co}}{\sqrt{2}\sigma_y}\right)\right]^2}\frac{\beta}{4\pi}\kappa\langle f_R\rangle_\tau \int_{-\infty}^{\infty}\int_{-b}^{b}\frac{1}{\kappa f_R(\tau)}\frac{\partial(\Delta y'(x,y,x_0,y_0,\tau))}{\partial y}\frac{e^{-\frac{y_0^2}{2\sigma_y^2}-\frac{x_0^2}{2\sigma_x^2}}}{(2\pi)^2\sigma_x\sigma_y}dy_0 dx_0$$

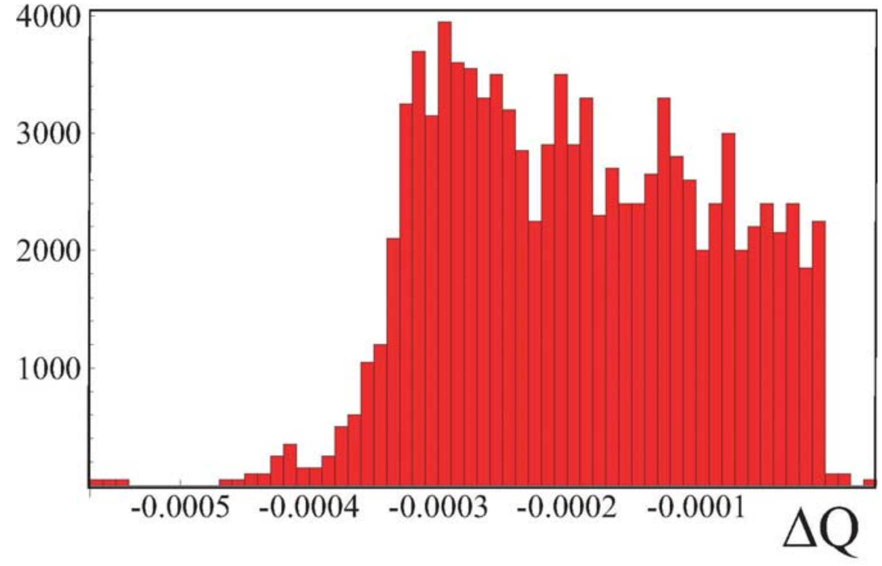
$$\frac{1}{\kappa f_R(\tau)}\frac{\partial(\Delta y'(x,y,x_0,y_0,\tau))}{\partial y} =$$

$$\frac{\pi^2}{4b^2}\left[\begin{array}{l} -\frac{2\sin y_-(y_-\cos y_- + \sin y_-)}{(\cos y_- - \cosh x_-)^2} + \frac{-2\cos y_- + y_-\sin y_-}{(\cos y_- - \cosh x_-)} + \frac{2\sin y_+(y_+\cos y_+ + \sin y_+)}{(\cos y_+ + \cosh x_-)^2} \\ +\frac{2\cos y_+ - y_+\sin y_+}{(\cos y_+ + \cosh x_-)} - \frac{\cos y_-(y_-\sin y_- + x_-\sinh x_-)}{(\cos y_- - \cosh x_-)^2} - \frac{2\sin^2 y_-(y_-\sin y_- + x_-\sinh x_-)}{(\cos y_- - \cosh x_-)^2} \\ +\frac{\cos y_+(y_+\sin y_+ - x_-\sinh x_-)}{(\cos y_+ + \cosh x_-)^2} + \frac{2\sin^2 y_+(y_+\sin y_+ - x_-\sinh x_-)}{(\cos y_+ + \cosh x_-)^2} \end{array}\right]$$

incoherent tune spread



b=3 mm

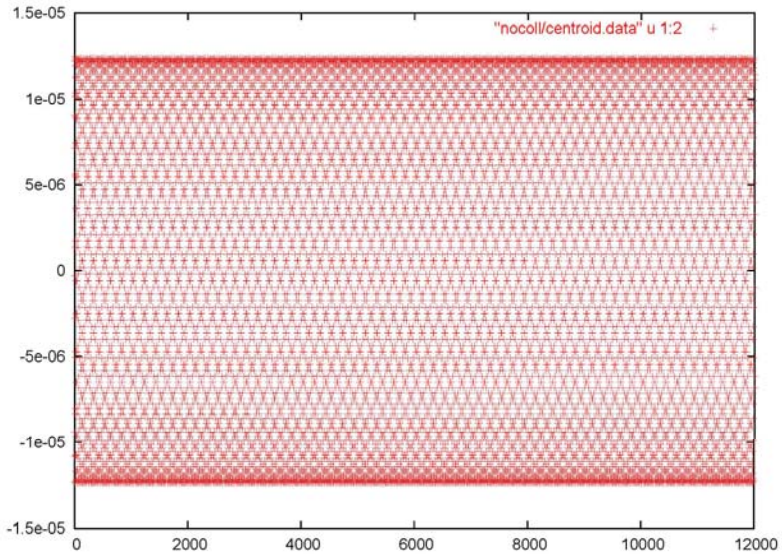


b=1.5 mm

Monte-Carlo evaluation of analytical formula

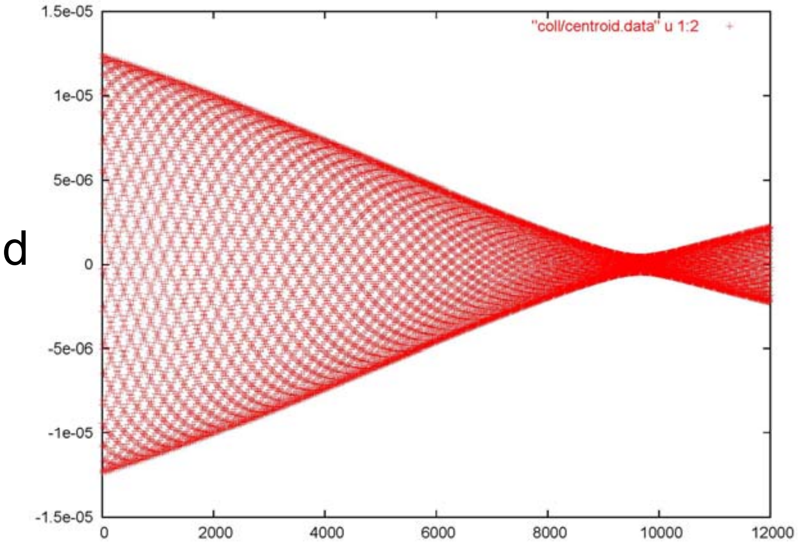
for example parameters

multi-particle tracking (2000 particles over 12000 turns)



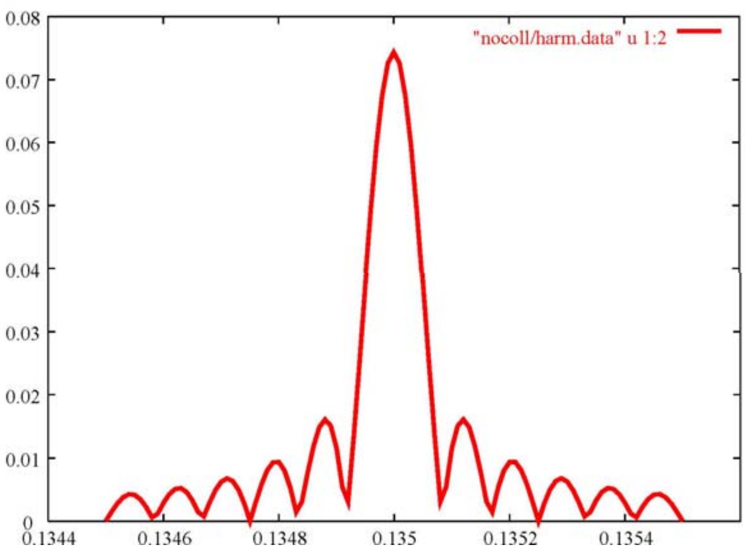
centroid

motion

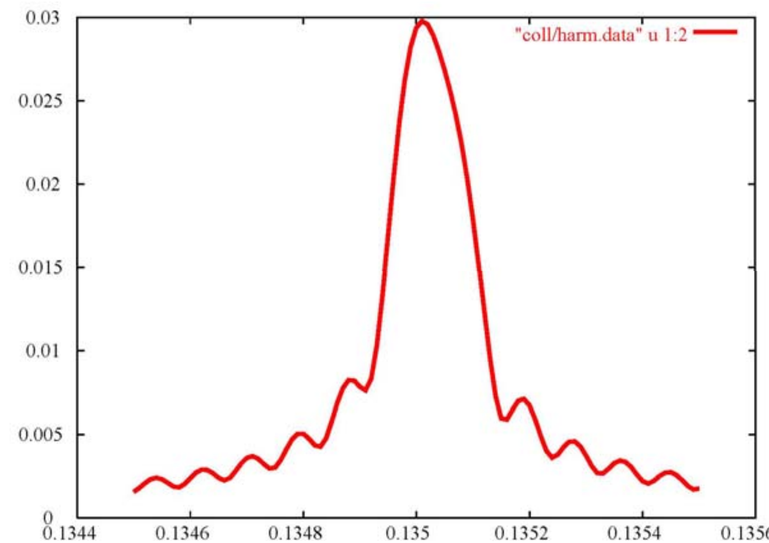


no collimator

b=1.5 mm



FFT



w/o synchrotron motion