A Simple Demonstration of Child's Law for Positive Ions

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THE equation for the current between two electrodes at a potential difference when the current is space-charge limited was first set forth by Child for positive ions. This equation, known as Child's law, is obtained by combining Poisson's equation, which defines the potential distribution V due to a charge of density ρ,

$$d^2V/dx^2=-4\pi\rho,$$

the energy equation, giving the speed v which a particle of mass m and charge e attains on falling through a potential difference V,

$$Ve=\tfrac{1}{2}mv^2,$$

and the equation defining current density,

$$i=\rho v,$$

and then integrating twice the resulting equation,

$$\frac{d^2V}{dx^2}=4\pi i\left(\frac{m}{2e}\right)^{\frac{1}{2}}\frac{1}{(V)^{\frac{1}{2}}};$$

this gives

$$i=\frac{1}{9\pi}\left(\frac{2e}{m}\right)^{\frac{1}{2}}\frac{V^{\frac{3}{2}}}{d^2}, \qquad (1)$$

where d is the distance between electrodes. Although Eq. (1) is valid only for parallel, plane electrodes, a similar equation results for cylindrical electrodes, namely

$$i=\frac{2\sqrt{2}}{9}\left(\frac{e}{m}\right)^{\frac{1}{2}}\frac{V^{\frac{3}{2}}}{r_s f(r_s/r_c)}, \qquad (2)$$

where r_s and r_c are the radii of the outer and inner electrodes, respectively, and the function $f(r_s/r_c)$ approximates unity if $r_s \gg r_c$.

Although derived for positive ions, this three-halves power law is equally valid for electron currents, provided m is made the electronic mass. The usual method of experimentally verifying this relation for electron currents is to measure the space-charge limited current of a thermionic tube for various applied voltages. For tungsten filaments, this method is satisfactory, but for thoriated tungsten and oxide coated filaments, the results tend to deviate from those expected.

The use of a three-electrode gas discharge tube such as the 885 permits, however, a simple and convincing demonstration of the three-halves power law for positive ions. It is well known that the discharge in such a tube can be extinguished by making the grid sufficiently negative. On initiation of the arc, the grid is immersed in the plasma and a positive ion sheath forms around the negative grid wires. The thickness of the sheath depends upon the amount of negative voltage applied to the grid and on the arc current density. However, at any fixed value of arc current (anode current), as the grid is made more and more negative, the sheath becomes thicker and thicker, until a point is reached where the sheaths on adjacent grid wires begin to overlap, thus effectively "throttling" the plasma and interrupting the arc. Since the spacing between grid wires is fixed, the sheath thickness *at the instant of arc extinction* is constant and independent of arc current density. For high densities, the applied grid voltage will, of necessity, be more negative than for low densities, but the sheath *thickness* will be the same for both.

One boundary of the positive ion sheath will be the grid wire, the other will be the plasma. The potential distribution curve at the plasma boundary cannot bend abruptly, as there can

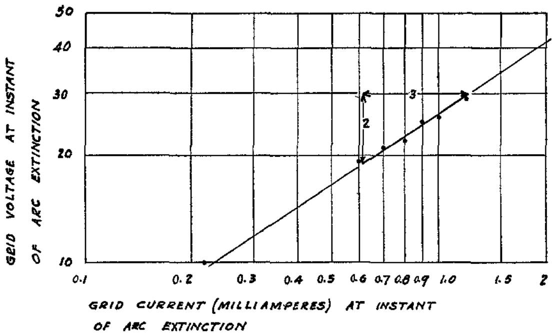

FIG. 1. Evidence that the grid current and voltage at the instant of arc extinction in a gas triode conform to Child's law.

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be no surface charge there, and the conditions at this boundary will, therefore, correspond to those at the cathode of a space-charge limited *electron* current. The current through the sheath will accordingly be a space-charge limited positive ion current and will conform to Child's law. For constant sheath thickness, the current density or current will depend only upon the voltage and will be proportional to the three-halves power of this voltage.

If the grid current and grid voltage at the instant of arc extinction are measured for various values of the anode current and these values plotted, the plotted points should lie along the curve

$$I_g = K V_g^{\frac{3}{2}},$$

where K is a proportionality constant. If the logarithms of these values are plotted, they should lie along a straight line of slope $\frac{3}{2}$. From Fig. 1, which shows the plot of a run made on an argon-filled gas triode, this is observed to be the case.

Lens Testing on a Student Spectrometer

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UNDERGRADUATE courses in optics and photography usually include mention of several lens aberrations, but due to the complicated mathematics involved in all except chromatic aberration, the subject is dropped without further treatment. The writer has found, however, that photography enthusiasts, particularly if they own expensive lenses, are intensely interested in measuring the astigmatism, field curvature and other errors of their lenses, though they may not understand all the reasons for such errors.

Two pieces (Fig. 1) have been constructed to replace the prism table and collimator of a student spectrometer and thereby make of it a simple, reasonably accurate lens testing device. Spectrometer attachments of a similar design and using the same principles were constructed several years ago by Professor I. S. Bowen for use in the optics laboratory of the California Institute of Technology, but no report concerning them has been made by him. The prism table is replaced by a nodal slide consisting of a sleeve for the central post, an attached horizontal rectangular plate with side ways, and two grooved uprights mounted on a second horizontal plate machined to fit the ways. A thin metal sheet cut to fit between the grooved uprights is drilled to take a particular camera lens and shutter; this sheet alone must be replaced in the study of different lenses. A simple lens may be fastened to any sheet with a few drops of collodion or shellac; a camera lens in its shutter requires its special sleeve nut to fasten it securely to the upright sheet.

The collimator replacement, herein called the *naometer*, consists of a slotted tube carrying the scale and slide of a vernier caliper. The slot of the present equipment allows measurement of focal distances between 3.6 and 17.5 cm. Cross hairs, vertical and horizontal, are shellacked to a small ring soldered to the slide of the vernier caliper. Much of the tube is cut away in order that the cross hairs may be well illuminated. To insure that the vernier reads directly the distance from the cross hairs to the axis of rotation of the nodal slide, care must be taken to fix the naometer accurately in the collimator sling. Fig. 2 shows a Gaertner student spectrometer with the two pieces attached. It is unfortunate that the central post of this spectrometer was so tall, for it necessitated trimming in order to place even moderate sized camera shutters sufficiently low to align the axes of lens and telescope.

After the telescope is focused for parallel rays and aligned roughly with the naometer, the lens plate with lens attached is lowered into its