# Stability of the longitudinal coherent modes (Part 2)

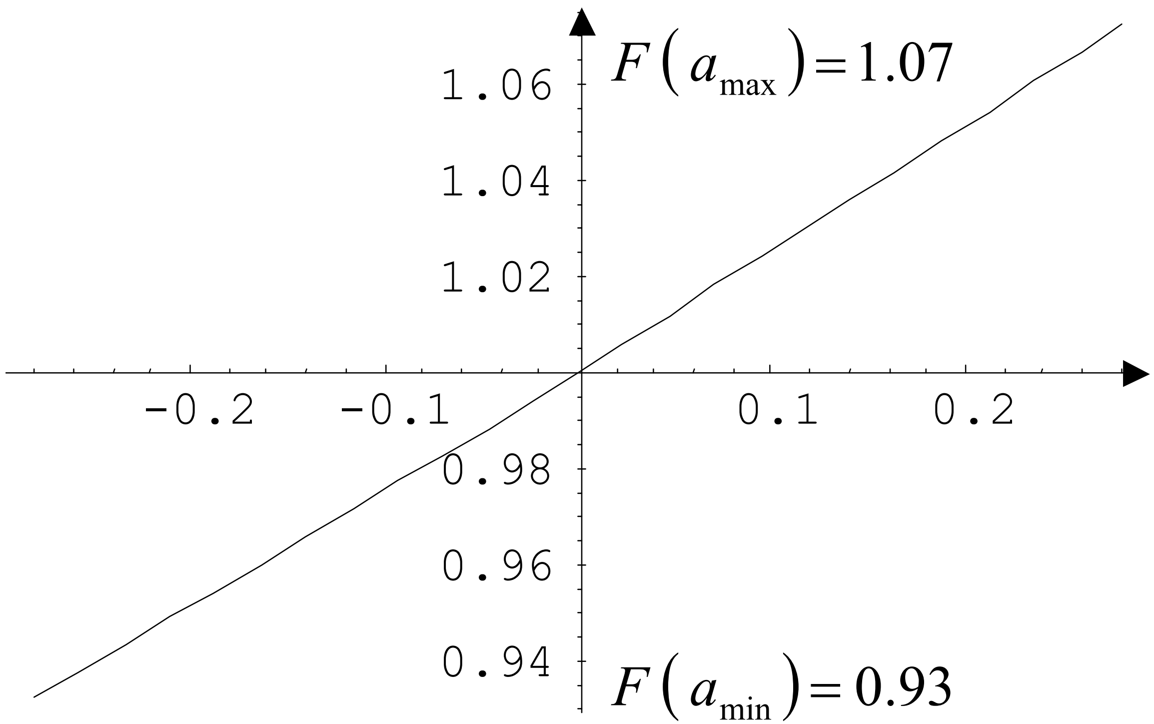
- General factor F (see LCE meeting on 31/01/03)

- Stability diagrams
  - Parabolic distribution
  - Gaussian distribution
  - Distribution used by Sacherer for his stability criterion

- "Elliptical" distribution

# General factor F

- Neglecting the synchronous phase shift and considering the same effective impedance for mode 00 and 11



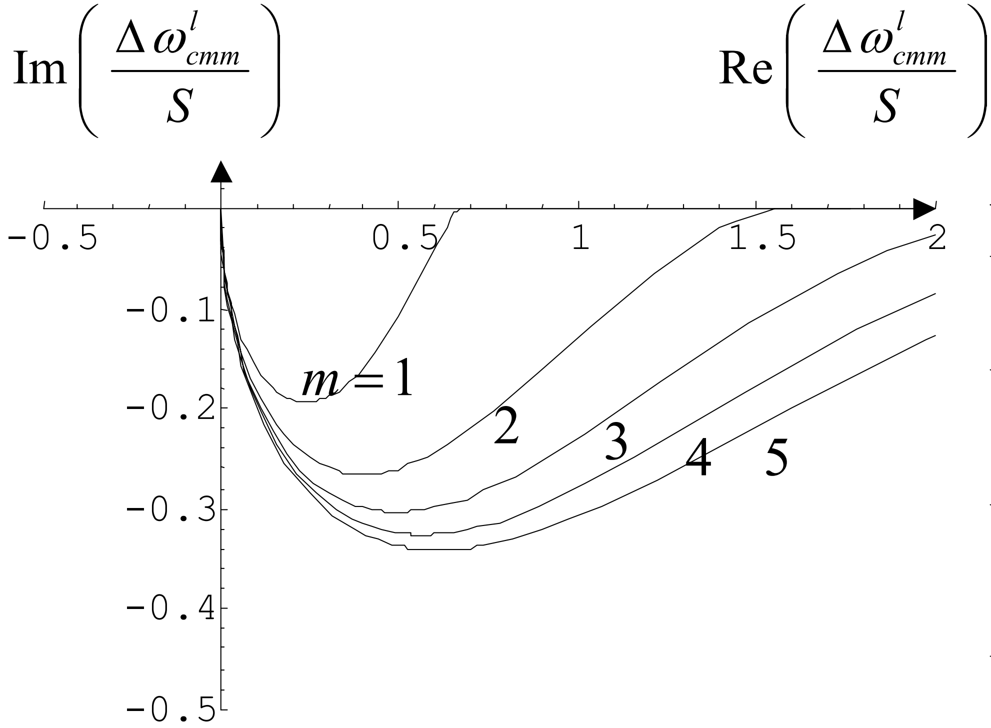

$$a_{\max} = -\,a_{\min} = \frac{9}{32} \approx 0.28$$

- Numerical application for LHC at top energy with the same parameters (see last LCE meeting) $F = 1.01$ ~~not 1.4~~

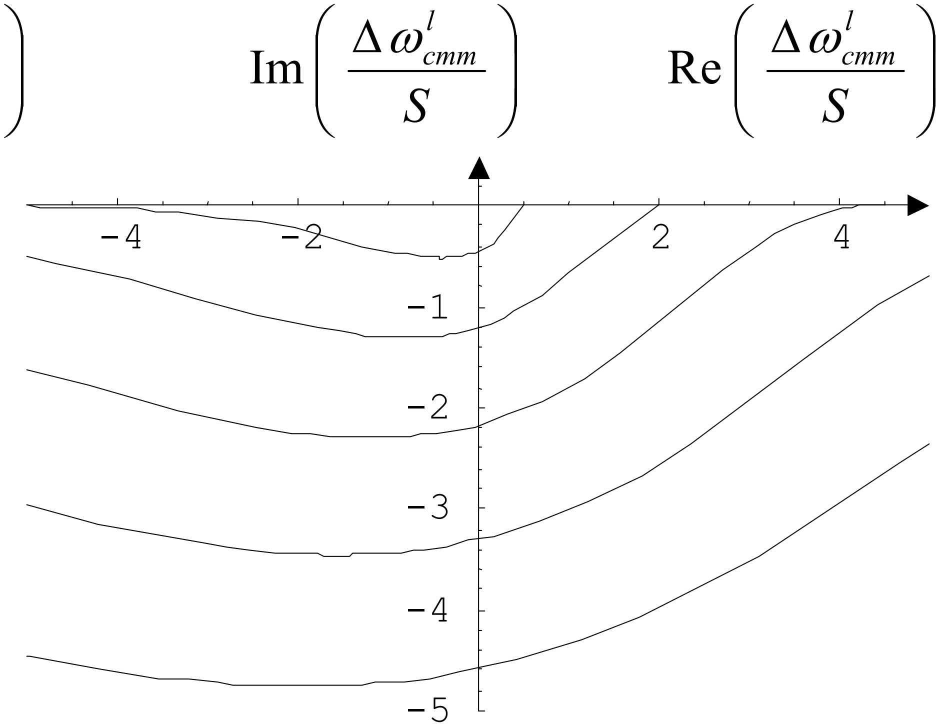
# Stability diagrams (1/3)

Used by Besnier

- Parabolic



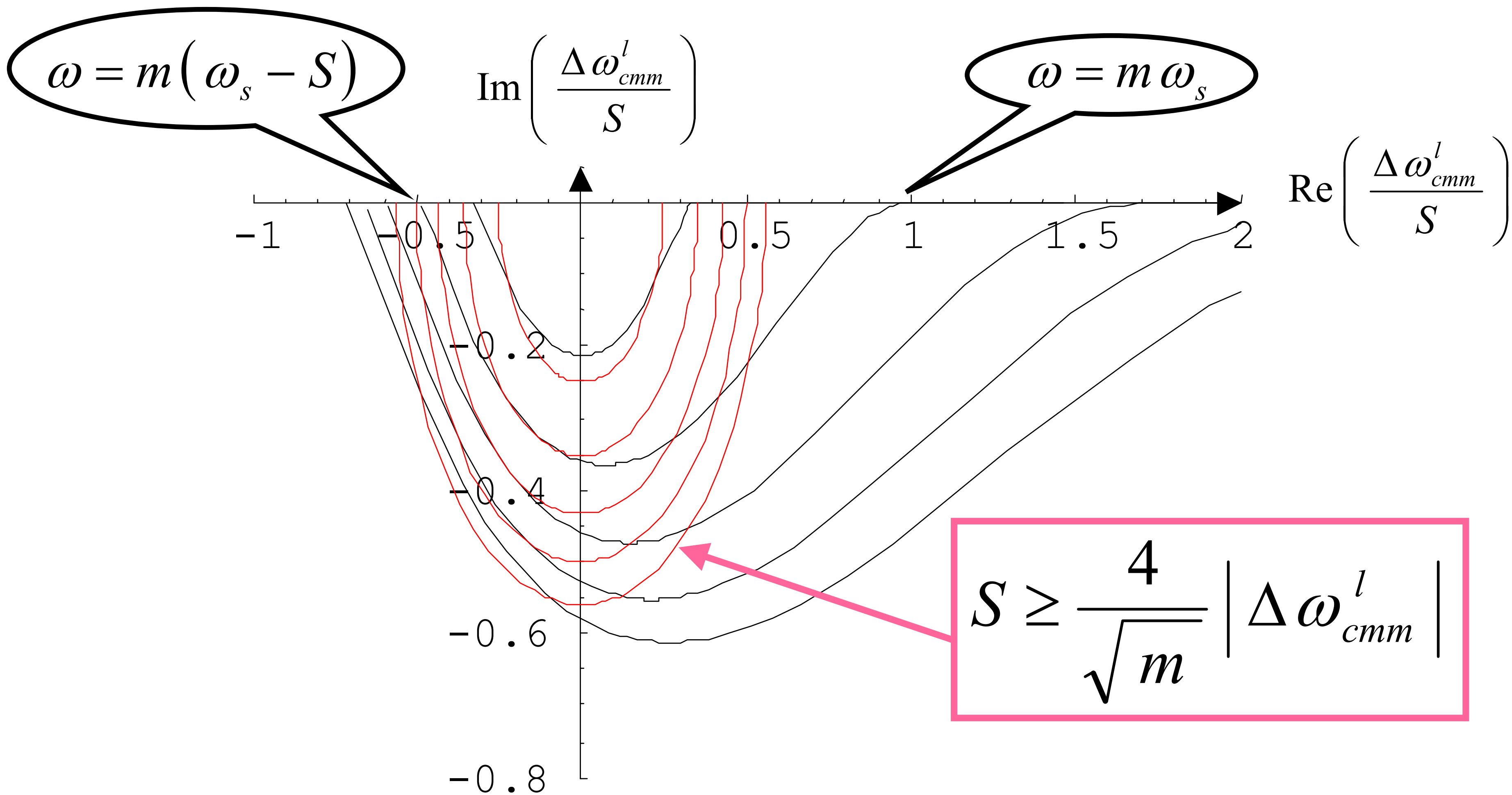

- Gaussian




$$\mathrm{Re}\left(\frac{\Delta\omega^{l}_{cmm}}{S}\right) > 0 \Leftrightarrow \text{capa. imp. BT and ind. imp. AT}$$

# Stability diagrams (2/3)

- Sacherer distribution $\Rightarrow g_0(r) \propto (1 - r^2)^2$



# Stability diagrams (3/3)

$$\omega = m\,\omega_s \quad \Rightarrow \quad \frac{\Delta\omega_{cmm}^{l}}{S} = \frac{m^2}{m+2}$$

$$\omega = m\left(\omega_s - S\right) \quad \Rightarrow \quad \frac{\Delta\omega_{cmm}^{l}}{S} = -\frac{m}{m+2}$$

$$\omega \approx m\left(\omega_s - \frac{m}{m+1}S\right) \quad \Rightarrow \quad \mathrm{Re}\left[\frac{\Delta\omega_{cmm}^{l}}{S}\right] = 0$$

# "Elliptical" distribution (1/2)

**Case of the dipole mode *m* = 1** $\Delta \omega_{c11}^{l} = U - jV$ $I_1^{-1}(\omega) = \Delta \omega_{c11}^{l}$

$$r^2 \frac{dg_0(r)}{dr} \propto \sqrt{1 - \left(2r^2 - 1\right)^2}$$

$$\Rightarrow \quad \omega = \left(\omega_s - \frac{S}{2}\right) + U \frac{S^2 + 16\left(U^2 + V^2\right)}{16\left(U^2 + V^2\right)} + jV \frac{S^2 - 16\left(U^2 + V^2\right)}{16\left(U^2 + V^2\right)}$$

$\Rightarrow$ Stability criterion $S \ge 4 \left| \Delta \omega_{c11}^{l} \right|$

**Sacherer criterion recovered analytically**

$$\mathrm{Re}(\omega) = \omega_{s0} + \Delta \omega_s^i + U - \frac{S}{2} + \frac{S^2}{16U}$$

$V << |U|$

**Generalization in the presence of frequency spread**

# "Elliptical" distribution (2/2)