quadrupole wake

- wake proportional to quadrupole moment and acting on test particle like additional quadrupole (A.Chao & R.Cooper, 1982)

  $\sim N_s[\langle x^2\rangle_s - \langle y^2\rangle_s](xu_x - yu_y)_{test}$ or

  $\sim N_s[2 \langle xy\rangle_s](yu_x + xu_y)_{test}$
- by extension any wake proportional to the offset of the test particle acting like a quadrupole, e.g.,

  $\sim N_s(xu_x - yu_y)_{test}$

  (for example, J. Irwin, in NLC ZDR SLAC Report 1996, p. 597; S. Heifets, A. Wagner, B. Zotter, SLAC/AP110, 1998)

[note: dipole horizontal wake $\sim N_s u_{x,test}$]

deflection by flat collimator

$$\Delta y'(\tau) = \frac{\pi^2}{3}\left(\frac{1}{2}\frac{N r_p}{\gamma \sigma_z}\right)\frac{L}{g}\sqrt{\frac{g}{z_0}\sigma_z}$$

(*)

$$\times \frac{1}{g^2}\underbrace{\left[\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{d\tau'}{\sqrt{\tau'}}\, e^{-\frac{(\tau'+\tau)^2}{2}}\right]}_{f_R(\tau)}\left(y_0 + \frac{1}{2}y(\tau)\right)$$

y_0: dipole wake ("source offset")

$\frac{1}{2}y(\tau)$: quadrupole wake

A. Piwinski DESY-HERA-92-04

K. Yokoya Part. Acc. 41, 1993

J. Irwin NLC ZDR p. 594

(*) assume beam is centered in gap $y_0 = 0$

$$\Delta y'(\tau) = C\, f_R(\tau)\, y(\tau)$$

where $C = \frac{\pi^2}{3}\left(\frac{1}{2}\frac{N r_p}{\gamma \sigma_z}\right)\frac{L}{g^3}\sqrt{\frac{g}{z_0}\sigma_z}\,\frac{1}{2}$

emittance $\varepsilon^2 = \langle y^2\rangle\langle (y'+\Delta y')^2\rangle - \langle y(y'+\Delta y')\rangle^2$

$$= \varepsilon_0^2 + \Delta\varepsilon^2$$

where $\Delta\varepsilon^2 = C^2\sigma_y^4\underbrace{\left(\langle f_R^2\rangle - \langle f_R\rangle^2\right)}_{\approx 0.109}$

$$\Delta\varepsilon^{\text{real}} \approx \frac{1}{2}\frac{\Delta\varepsilon^2}{\varepsilon_0}$$

take: $L = 1\,m$, $g = 1\,mm$ — 1 collimator

$\sigma_z = 7.7\,cm$, $N = 1.1 \times 10^{11}$

$\rho = 2.9 \times 10^{-8}\ \Omega m$ (Al)

$\Rightarrow C \simeq 6 \times 10^{-7}$

$\sigma_y = 300\,\mu m$

$\Rightarrow \Delta\varepsilon^2 = 2.9 \times 10^{-28}\ m$

$$\Delta\varepsilon^{real} = \frac{\Delta\varepsilon^2}{2\varepsilon_0} \simeq 3 \times 10^{-19}\ m$$

$$\frac{\Delta\varepsilon^{real}}{\varepsilon_0} \hat{=} 6 \times 10^{-10} \text{ per turn}$$

$\simeq 1$ after 10^9 turns

or after 10^8 turns for 10 pairs of jaws

less turns for more resistive material