Minutes of a meeting
on the resistive-wall impedance
March 01, 2004

- **Participants**: F. Caspers, E. Metral, F. Ruggiero.

- **Reminder from the Handbook of Accelerator Physics and Engineering (F. Caspers, p. 574) and from the paper "Interpretation of coupling impedance bench measurements", PRSTAB, 7, 012001 (2004), by H. Hahn:**

Coupling impedance values of accelerator components can be obtained from standard bench measurements based on the coaxial wire method (proposed in the paper by A. Faltens et al., Proc. 8th Int. Conf. High Energy Acc. (1971), p. 338). The basic concept of bench measurements relies on simulating the beam by a wire for longitudinal or twin wire for transverse measurements inserted into a "Device Under Test" (DUT).

The coaxial wire method assumes that an ultra-relativistic beam has a very similar EM field distribution (TEM field) to that of a short pulse on a coaxial line. For non ultra-relativistic beams the wire measurement results need therefore corrections. The standard formulas used to interpret the measured data were all derived in the framework of transmission line theory. The field configuration on an ideal transmission line is a TEM wave with purely transverse components. The finite wall conductivity, or a geometrical wall disturbance, changes the field into a mode with a local axial component of the electric field responsible for the interaction with the beam. The assumption in the transmission line theory is, however, that the analysis can be performed with ideal walls and the real situation is handled by appropriately modifying the characteristic impedance and propagation constant. At a sufficient distance away from the device, the pure TEM mode is re-established but with modified amplitude and phase of the scattering coefficients. The coupling impedance follows from the interpretation of the scattering coefficients from a network analyzer. In multi-port junctions the so-called "scattering matrix" describes the linear relation between

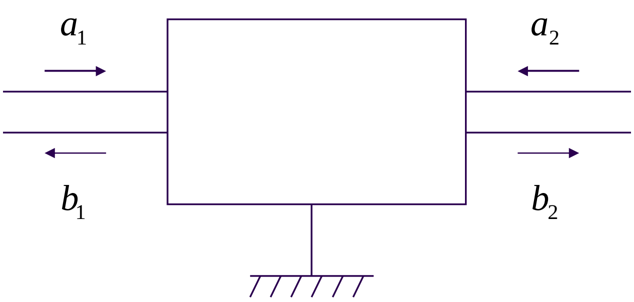

Figure 1: Sketch of a two-port case. "*a*" stands for direct (forward) wave, and "*b*" reflected (backward) wave.

forward and backward waves at the different ports (see Fig. 1 for the example of the two-port case: the concept of impedance is replaced by the concept of reflection factor).

For the two-port case, the scattering matrix is defined by

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \tag{1}$$

while in the general case it is given by

$$[b] = [S][a], \tag{2}$$

where $[S]$ is the scattering matrix (the terms S_{ij} are complex dimensionless quantities). This scattering matrix is universally used and can be easily measured. Note that for reciprocal circuits $[S]$ is symmetric. For reciprocal and lossless circuits, $[S][S]^* = [I]$.

In the case of a lumped impedance, the standard lumped formula (H. Hahn and F. Pedersen, BNL Report No. BNL 50870, 1978) has to be used. This yields for the transverse impedance

$$Z_T = \frac{2\,c}{\omega\,\Delta^2} \times Z_C \left(\frac{S_{21}^{\mathrm{REF}}}{S_{21}^{\mathrm{DUT}}} - 1 \right), \tag{3}$$

where c is the speed of light, Δ the distance between the two wires, and Z_C the characteristic impedance of the coaxial line. The forward transmission coefficient (from port 1 to port 2) of the DUT is S_{21}^{DUT}, and S_{21}^{REF} is for the reference measurement.

In the case of a distributed impedance, the log formula (L.S. Walling et al., Nucl. Instrum. Methods Phys. Res. Sect. A **281**, 433 (1989)) has to be used. This yields for the transverse impedance

$$Z_T = -\frac{2\,c}{\omega\,\Delta^2} \times Z_C \;\mathrm{Log}\left(\frac{S_{21}^{\mathrm{DUT}}}{S_{21}^{\mathrm{REF}}} \right). \tag{4}$$

Note that the solution of this equation is not unambiguous and thus only the principal value of the complex logarithm should be taken. Note also that in principle Z_T may also be deduced from a measured S_{11} (input reflection coefficient seen at port 1) but it is more difficult in practice. Measurements of single lumped elements and even more so of distributed impedances are intrinsically perturbative and thus require the highest characteristic impedance of the reference tube and the smallest wire size, only limited by signal-to-noise ratio.

The most powerful tool to carry out bench type measurements is the VNA (Vector Network-Analyser) with time domain option. An array of complex data points (usually S_{21} in equidistant frequency steps) taken in the frequency domain including a “frequency domain” cable and connector calibration is converted via a Fourier transform algorithm (nowadays often Chirp-Z type) into an equivalent time-domain data.

Reminder from paper “Bench Measurements of Low Frequency Transverse Impedance”, CERN-AB-2003-051 (RF) by A. Mostacci et al.:

For frequencies below a few MHz the classical two wire transmission line method is subject to difficulties in sensitivity and measurements uncertainties $\Rightarrow$ for evaluation of the low frequency transverse impedance properties of e.g. the LHC dump kicker a modified version of the two wire transmission line has been used: it consists of a 10 turn loop of approximately 1m length and 2cm width.

Being I the beam current, the source of the differential wall current is the dipole moment $I\Delta$ per unit length of the beam. The same wall currents and magnetic (deflecting) field result if the beam is replaced by two parallel wires or more simply by a loop of length L, width Δ and current I. The magnetic field induces a voltage in the loop which increases its impedance (the current I is constant). This additional impedance is simply the variation of the loop impedance when inserted in the DUT with respect to the loop impedance. Assuming that the loop is coiled N times (to reduce the signal to noise ratio), the transverse coupling impedance is obtained from

$$Z_T = \frac{c}{\omega \Delta^2} \times \frac{Z^{DUT} - Z^{p.c.}}{N^2}, \tag{1}$$

where Z^{DUT} is the (measured) impedance of the loop when inserted in the DUT and $Z^{p.c.}$ the (measured) impedance of the loop inside a perfectly conducting pipe with the same geometry of the DUT (and not in free space!).

To reduce the signal-to-noise ratio (particularly important in this case since the measured signals are very small), the loop was coiled $N = 10$ times. In this way, one can increase the useful signal with a factor N^2 with a drawback of lowering the frequency of loop self resonances: the chosen number of turns is a compromise to keep the lowest self resonance above 1 MHz.

Conclusions

- Usually the two wire method is used to measure the transverse coupling impedance. But this is good only when we have a TEM like field. Indeed, field description as superposition of monopole (Zero-pole) mode + dipole + quadrupole etc. is only permissible if we have a TEM like field without significant longitudinal components. Therefore, this method should be used only in the case of a negligible longitudinal electric field ($E_L / E_T << 1$?), which is not the case for example with the kickers.
- One should keep in mind that the basic definition of the transverse impedance is the variation of longitudinal impedance vs. transverse displacement of the beam.

- For the collimators there is a mixture of several problems:
  - 3 D
  - bypass effects
  - redistribution of the image currents at very low frequencies (called inductive bypass by L. Vos)
  - …

  ⇒ F. Caspers recommends to make 3D simulations with HFFS.

- Note that H. Tsutsui already made some HFFS simulations but he used the coaxial wire method to evaluate the coupling impedance!

- Another problem raised by F. Caspers is the following: All the theories of transverse impedances are based on the image current. At very low frequency there is no image current (DC component) ⇒ What is the impedance?

- F. Caspers raised also the problem of the contact resistance between graphite and any metal, which are not known and need to be measured. This may be a critical point as first rough estimates by F. Caspers show that the impedance is not negligible! ⇒ To be followed.

Elias