

Visit to USC,
learning QuickPIC code
(26th April, 7th May '04)

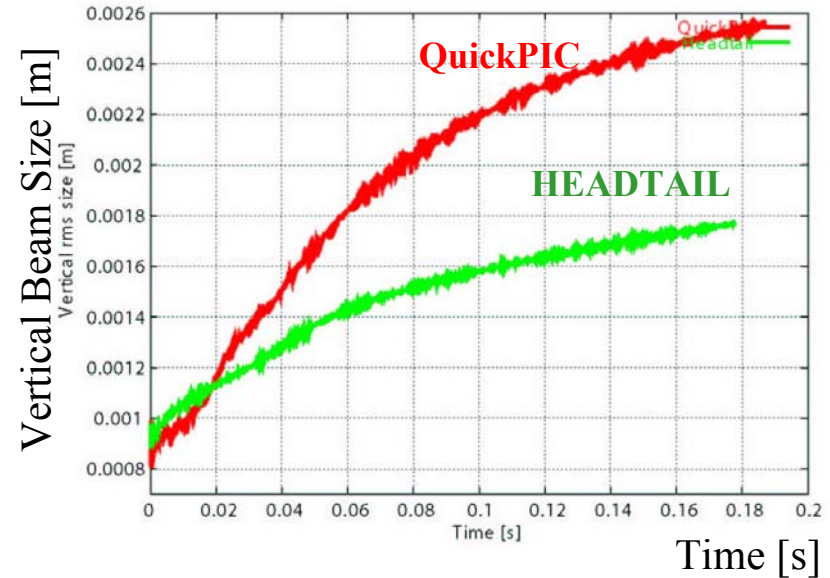
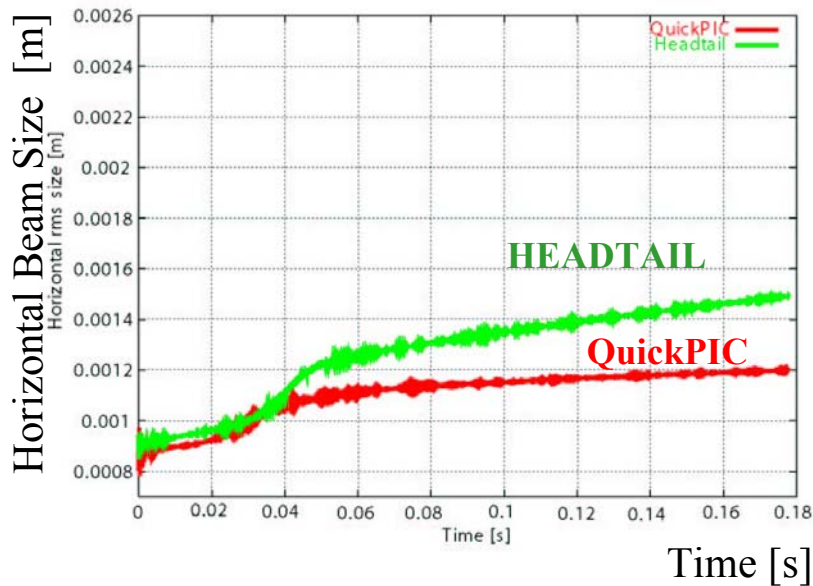
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The QuickPIC code

- QuickPIC is a 3D parallel PIC code, originally developed for **plasma wakefield accelerator** research
- In 2002 (collaboration with **G.Rumolo**) **Synchrotron and Betatron** motions has been added. **Chromaticity** effect is also taken into account.

Benchmark HEADTAIL with QuickPIC code (1 kick approx)



Horizontal (right) and vertical (left) beam size vs. time, for LHC parameters at injection.

For purpose of comparison in both HEADTAIL and QuickPIC the electron cloud has been modeled using 1 IP per turn.

Description of the Simulation Model (QuickPIC)

Maxwell equations in Lorentz gauge

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \mathbf{A} = \frac{4\pi}{c} \mathbf{j}$$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \phi = 4\pi\rho$$

$$\mathbf{j} = \mathbf{j}_b + \mathbf{j}_e \approx \mathbf{j}_b = c\rho_b \hat{\mathbf{z}}$$

Quasi-static approx.
 $\xrightarrow{\hspace{2cm}}$
 $\phi, A = \varphi, A(z - ct)$

Reduced Maxwell equations

$$-\nabla_{\perp}^2 \mathbf{A} = \frac{4\pi}{c} \mathbf{j}$$

$$-\nabla_{\perp}^2 \phi = 4\pi\rho$$

Local-- ϕ, A at any z -slice depend only on ρ, j at that slice!

$$(\mathbf{A} = A_{\parallel} \hat{\mathbf{z}})$$

$$\Psi = \phi - A_{\parallel}$$

Forces :

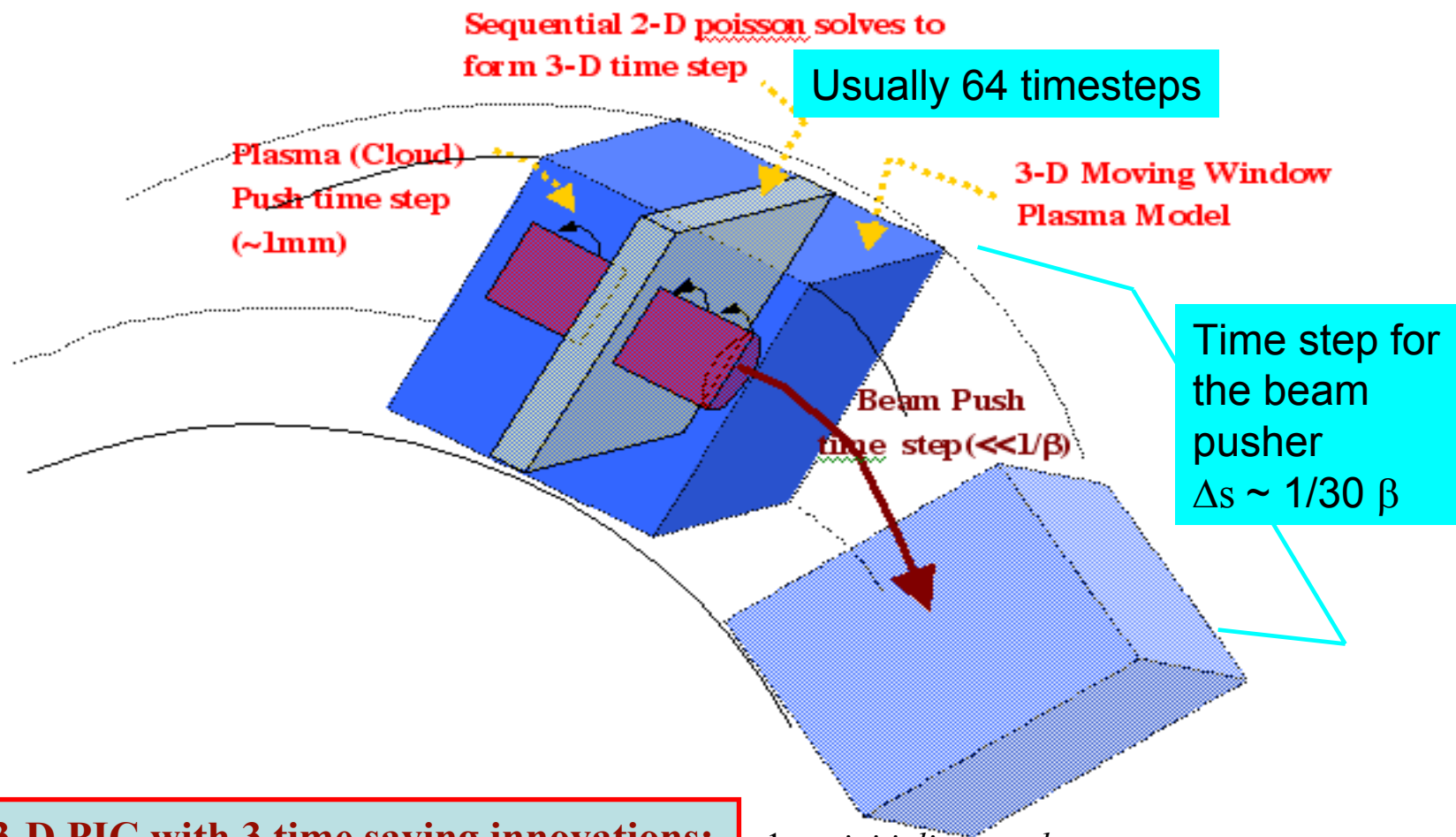
$$plasma : F_{e\perp} = -e\nabla_{\perp} \phi$$

$$beam : F_{b\perp} = -e\nabla_{\perp} \Psi$$

QuickPIC is a 3D PIC (particle In Cell) code with parallel processing

- It Uses Quasi- Static or frozen field approximation ($\beta \gg \sigma_z$)
- 3D Maxwell equations are reduced to 2D equations
- The need for solving 2D equations results in larger time steps of the 3D push, enabling a time savings of 2-3 orders of magnitude compared to traditional PIC models

Description of the Simulation Model, (Continued)



Explicit 3-D PIC with 3 time saving innovations:

1. Moving window follows the beam.
2. Separate time steps for beam and cloud
3. Use local nature of wake equations to form 3-D wakefields from a sequence of 2-D Poisson solves.

1. initialize beam
2. solve $\nabla_{\perp}^2 \phi = \rho, \nabla_{\perp}^2 \psi = \rho_e \Rightarrow F_p, \psi$
3. push plasma, store ψ
4. step slab and repeat 2.
5. use ψ to giant step beam

Changes and modifications to QuickPIC

- **Adding Synchrotron and Betatron motions**

By adding external focusing forces in transverse and longitudinal directions

- **Adding Chromaticity effect**

By adding external magnetic field and modifying our 3-D magnetic pusher.

**Transverse Equations
of Motion**

$$\left\{ \begin{array}{l} \frac{d^2 x}{dt^2} + \left[Q_x + \Delta Q_x \left(\frac{\delta p}{p_0} \right) \right]^2 \omega_0^2 x = \frac{1}{m_p \gamma} F_{cl-x} \\ \frac{d^2 y}{dt^2} + \left[Q_y + \Delta Q_y \left(\frac{\delta p}{p_0} \right) \right]^2 \omega_0^2 y = \frac{1}{m_p \gamma} F_{cl-y} \end{array} \right.$$

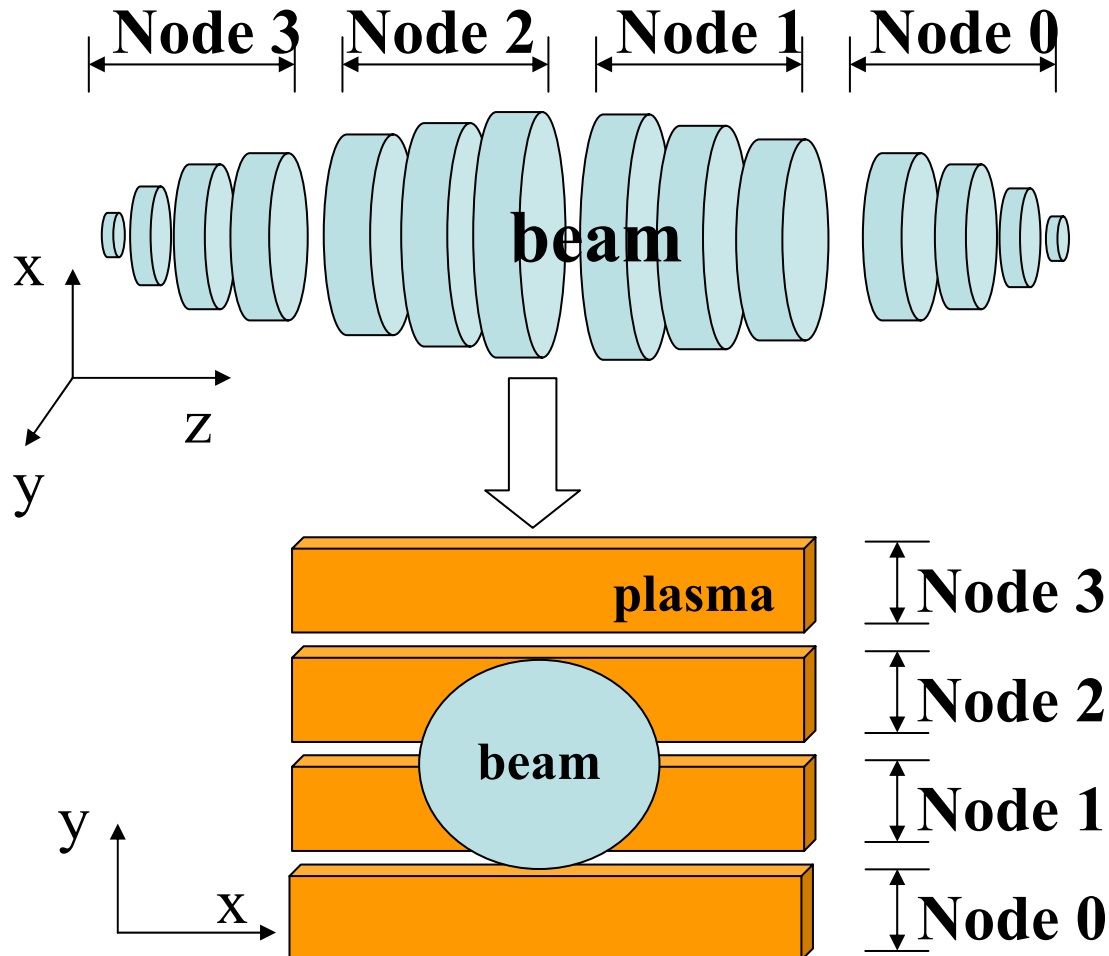
**Longitudinal Equations
of Motion**

$$\left\{ \begin{array}{l} \frac{dz}{dt} = -\eta c \left(\frac{\delta p}{p_0} \right) \\ \frac{d}{dt} \left(\frac{\delta p}{p_0} \right) = \frac{Q_s^2 \omega_0^2}{\eta c} z + \frac{1}{p_0} F_{cl-z} \end{array} \right.$$

Q_x : Horizontal tune
 Q_y : Vertical tune
 ΔQ_x : Chromatic shift in x
 ΔQ_y : Chromatic shift in y
 $\delta p/p_0$: momentum spread
 γ : relativistic factor
 η : slippage factor
 F_{cl} : Total force by cloud

Parallelization

- QuickPIC uses 2 different domain decompositions:
 - Beam decomposition along Z
 - Cloud decomposition along Y
- MPI is used for communication between nodes



Overcoming Numerical Tune Shift

Solving harmonic Oscillator with Leap Frog pusher results in a tune shift (Quadratic in time step):

$$v\left(t + \frac{\Delta t}{2}\right) = v\left(t - \frac{\Delta t}{2}\right) - k_0 x(t) \Delta t$$

$$x(t + \Delta t) = x(t) + v\left(t + \frac{\Delta t}{2}\right) \Delta t$$

$$\omega = \frac{\sin^{-1} \omega_0 \frac{\Delta t}{2}}{\frac{\Delta t}{2}} \approx \omega_0 \left(1 - \frac{(\omega_0 \Delta t)^2}{24}\right)$$

Modified Pusher:

$$v\left(t + \frac{\Delta t}{2}\right) = v\left(t - \frac{\Delta t}{2}\right) - \left(\frac{\sin\left(\omega_0 \frac{\Delta t}{2}\right)}{\frac{\Delta t}{2}}\right)^2 x(t) \Delta t$$

$$\omega = \omega_0$$

The 3D pusher for the protons

- The equation of motion for the protons is solved (using a modified **leap frog method**)

$$\ddot{\mathbf{r}} = \tilde{\mathbf{F}}_{EC,\perp} + \frac{1}{m\gamma} \mathbf{F}_{ext,\perp} + \tilde{\mathbf{F}}_{RF}$$

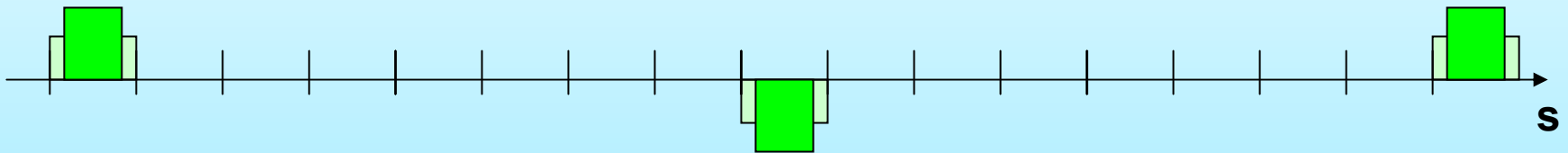
$\mathbf{F}_{EC,\perp}$ due to the electron cloud
 $\mathbf{F}_{ext,\perp}$ external focusing force
 \mathbf{F}_{RF} RF force, for the synchr. motion

- The **transverse external force**, responsible of the betatron oscillations, is assumed to be constant and uniform focusing in both planes:

$$\tilde{F}_{ext,u} = \left(\frac{Q_{x,y}}{R} \right)^2 u ; \quad u = x,y$$

The alternating gradient focusing

- SPS ring
- 216 quadrupoles over the ring ($\sim 6911\text{m}$)
 - chosen Δs , in order to have exactly 1728 steps over the ring
 - Focusing/Defocusing Quadrupoles every 8 time steps
 - in the other time steps we have Drifts



- The transverse external force now is:

$$\tilde{F}_{ext,u} = \frac{\pm 1/f}{\Delta s} u \quad \text{in the Quads}$$

$$\tilde{F}_{ext,u} = 0 \quad \text{in the Drifts}$$

What we have done

- Run w/o electron cloud, w/o synchrotron motion
- The only force ‘pushing’ the protons is the **quadrupoles focusing/defocusing**
- Got the envelope of the beam over a few time steps

**PROBLEM STILL
TO BE FIXED**

→ Matching the initial
beam distribution

EC single bunch instabilities; Future Plans

- QuickPIC:
 - Get the beam envelope and betatron tunes corresponding to the **FODO lattice** we have introduced
 - Introduce the FODO lattice for SPS and LHC
 - Consider the **motion of the electrons** into the Dipoles and Quadrupoles
- HEADTAIL:
 - **Merge** all changes and new features (by Frank, G.Rumolo and me) into a **single version** to be put in the web and update the **manual** in the web
 - Check Frank's new version **for LHC**: RF cavities only at a single location around the ring → **synchr.motion** treated in another way
 - Concentrate the IPs in one Betatron wavelength
 - Introduce the real Beta function also in HEADTAIL
 - **Benchmark** with SPS experiments
- Pinch effect, analytical work
- Etc... 😊