NEW DEFINITION OF THE STOP-BAND FOR THE INTENSITY DEPENDENT EMITTANCE TRANSFER BASED ON THE ENVELOPE EQUATIONS

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- **2D Theory**
- **Comparison with 2D and 3D simulations by I. Hofmann et al.**
- **Measurements made in the PS in 2003**

2D THEORY (1/4)

KAPCHINSKIJ AND VLADIMIRSKIJ (KV) ENVELOPE EQUATIONS

$$a'' + K_x\, a - \frac{2\,K_{sc}}{a+b} - \frac{\varepsilon_{x00}^2}{a^3} = 0$$

$$b'' + K_y\, b - \frac{2\,K_{sc}}{a+b} - \frac{\varepsilon_{y00}^2}{b^3} = 0$$

These KV envelope equations have been solved for small perturbations on top of equilibrium beam sizes

2D THEORY (2/4)

TRANSVERSE EMITTANCES IN THE PRESENCE OF SPACE CHARGE

$$\varepsilon_{x,y} = \varepsilon_{x0,y0} \mp \left(\varepsilon_{x0} - \varepsilon_{y0}\right) \frac{\left|C\right|^2 / 2}{\Delta^2 + \left|C\right|^2 + \Delta\sqrt{\Delta^2 + \left|C\right|^2}}$$

$$\left|C\right| = \left|\Delta Q_{inc,x0}\right| \times \left(1 + \frac{b_0}{a_0}\right)^{-1}$$

Equilibrium (in the presence of space charge but far from the resonance) beam sizes

Incoherent small-amplitude space-charge tune shift

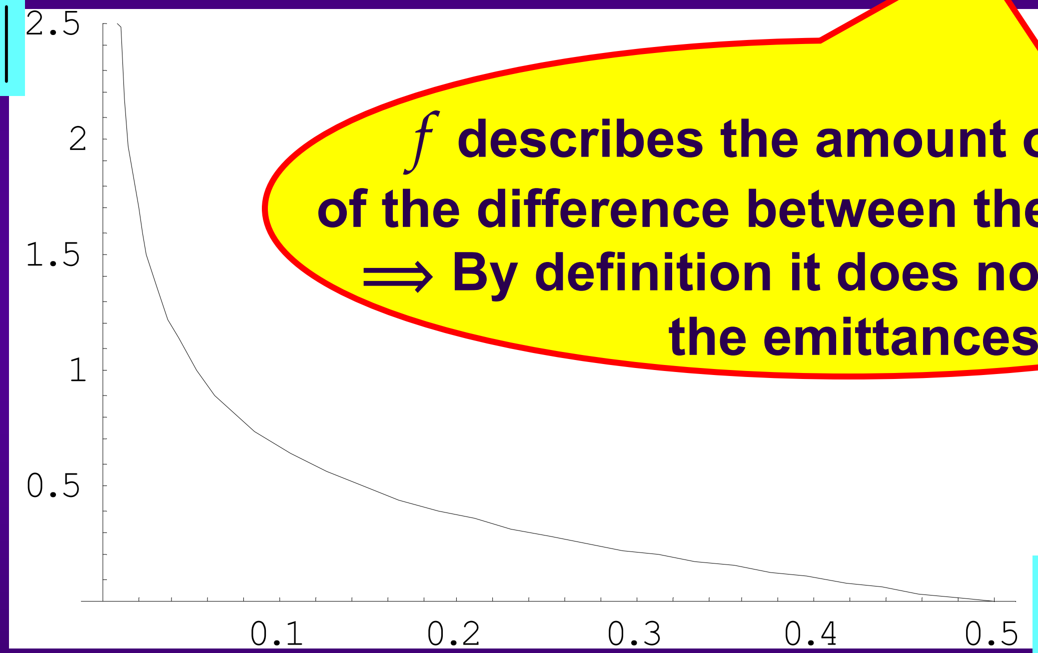
$$\Delta = 2Q_v - 2Q_h$$

Symmetrical stop-band predicted

2D THEORY (3/4) : "OLD" DEFINITION OF THE STOP-BAND

$$\delta_{half\ stop\ band} = \left| Q_v - Q_h \right|_{SC\ coupling} = \left| C \right| \times \frac{\left| 1 - 2f \right|}{4\sqrt{f\left(1-f\right)}}$$

$\delta_{half\ stop\ band} / \left| C \right|$



f

f describes the amount of transfer of the difference between the 2 emittances $\Longrightarrow$ By definition it does not depend on the emittances

Time scale $N^1_{turns} = \frac{1}{\left| C \right|}$

Average over 1 period of the oscillating term

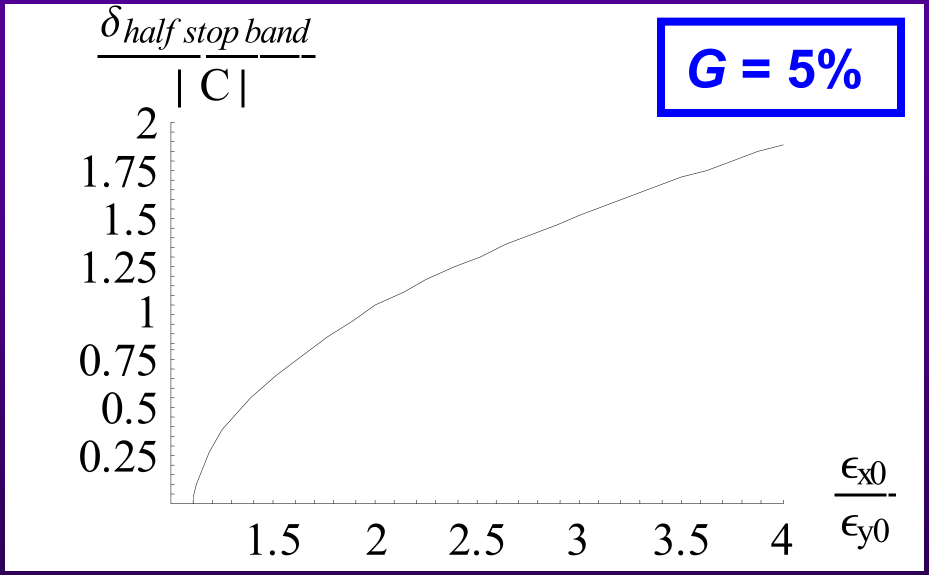
2D THEORY (4/4) : NEW DEFINITION OF THE STOP-BAND

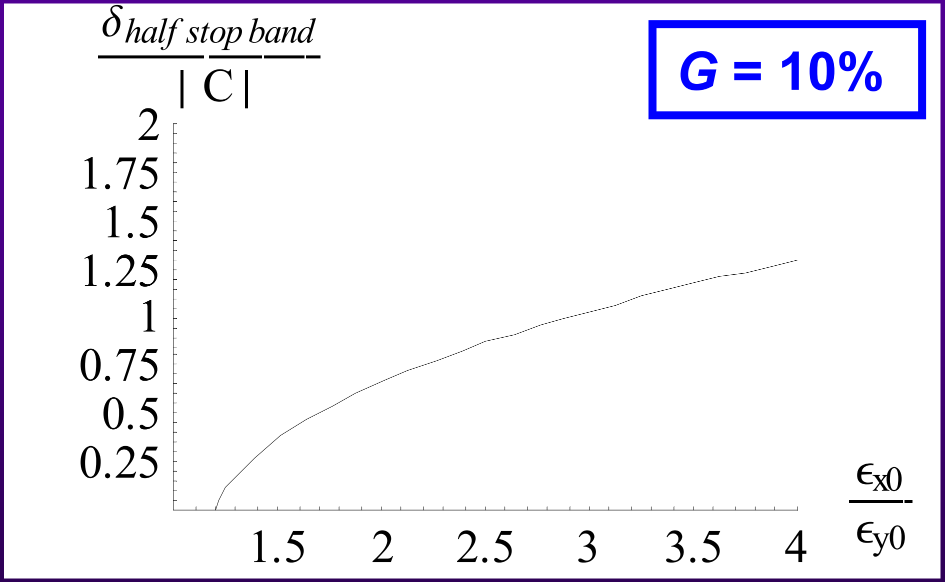
$$\varepsilon_y = \varepsilon_{y0}\,(1+G)$$

$$G = f\,(R-1)$$

$$R = \frac{\varepsilon_{x0}}{\varepsilon_{y0}}$$

$$\delta_{half\ stop\ band} = \left| Q_v - Q_h \right|_{SC\ coupling} = \left| C \right| \times \frac{(R-1-2G)}{4\sqrt{(R-1-G)\,G}}$$





COMPARISON WITH 2D AND 3D SIMULATIONS BY I. HOFMANN ET AL. (1/2)

$\Longrightarrow$ See paper "Dynamical Effects in Crossing of the Montague Resonance", Proc. 9th EPAC, Lucerne, Switzerland, 5-9 July 2004

Simulations were made until now in the static case

This is what was predicted analytically by the previous model

Abstract

In high-intensity accelerators space-charge-induced emittance coupling, known as Montague resonance, is known to occur for small tune split, where it can lead to emittance equilibration. We show here by simulation that new phenomena arise, if slow crossing of this resonance. In 2D coasting beams the crossing leads to practically pure exchange of emittances, in spite of the underlying nonlinearity, while the beam remains intrinsically self-matched. In 3D bunched beams an additional mixing effect by synchrotron motion is found, which suppresses complete exchange, depending on the speed of crossing.

COMPARISON WITH 2D AND 3D SIMULATIONS BY I. HOFMANN ET AL. (2/2)

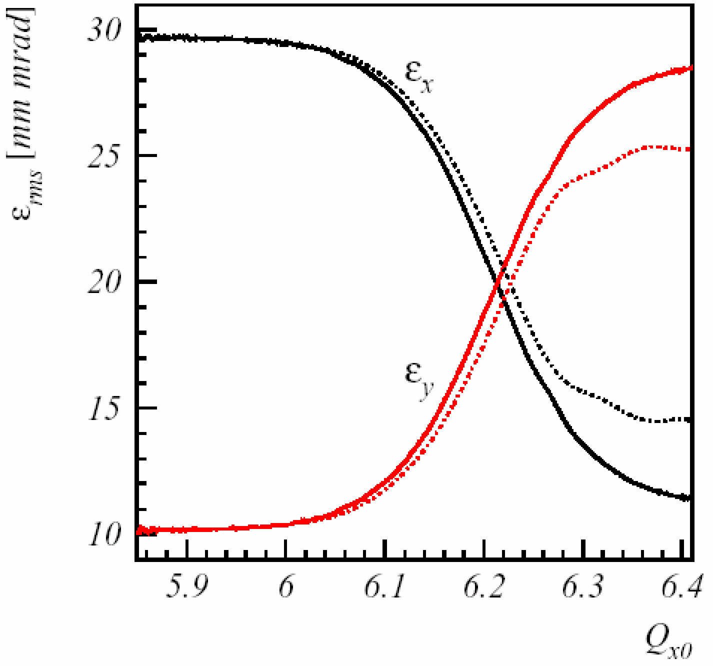


Figure 2: Rms emittance evolution in 2D for a tune ramp $Q_x = 5.85 \rightarrow 6.45$ over 30 and 100 turns.

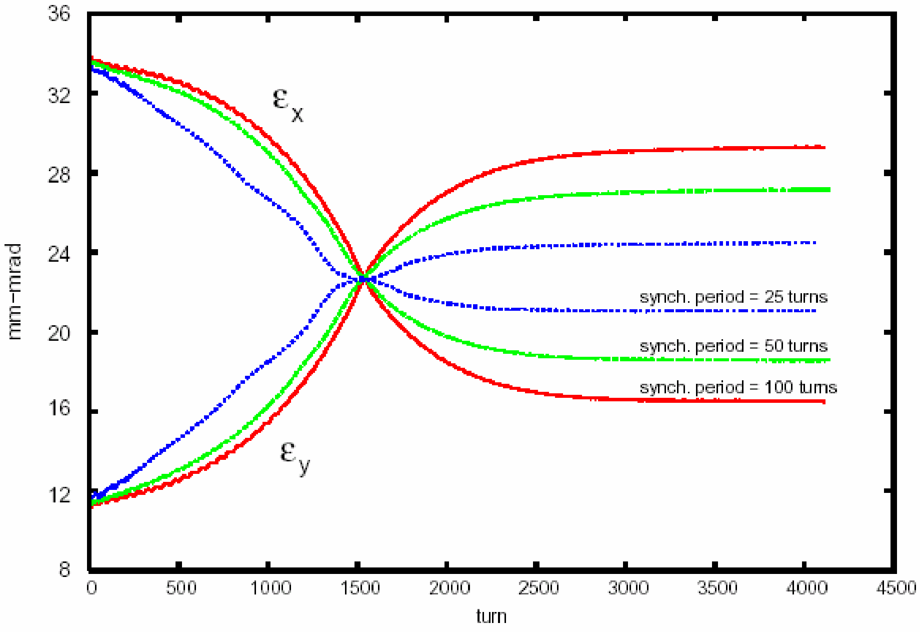


Figure 6: Rms emittances in 3D bunched beam for given tune ramp, but doubled and quadrupled synchrotron frequency.

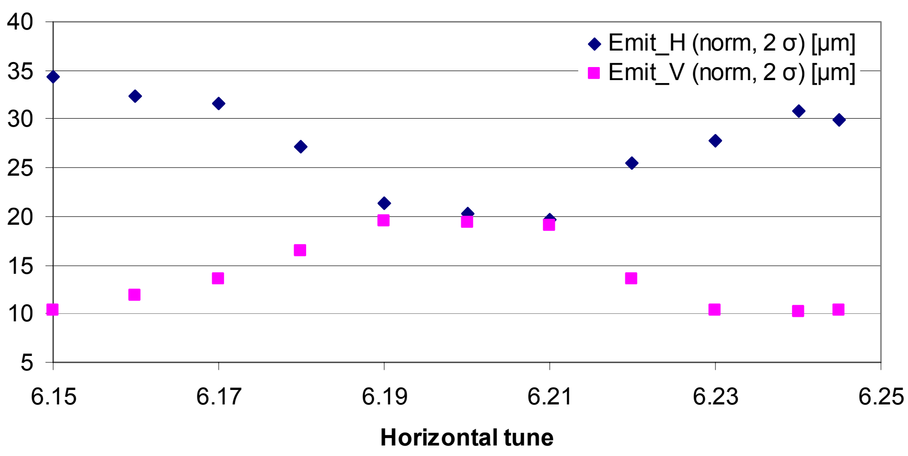
⟹ The crossing speed has to be slow compared to the time scale during which the coupling occurs
⟹ Full exchange, as predicted by the analytical model

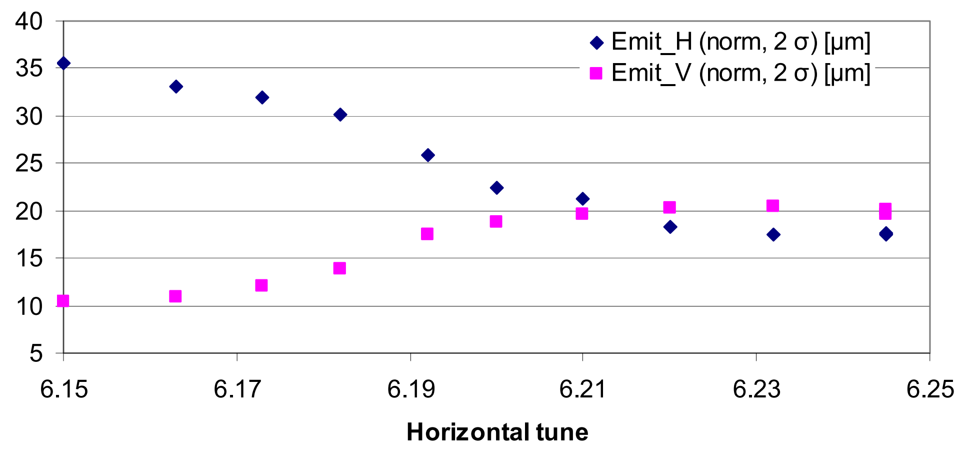
⟹ The crossing speed has to be fast compared to the synchrotron motion
⟹ Not taken into account in the present analytical model

MEASUREMENTS MADE IN THE PS IN 2003 (1/3)

⟹ See paper “Intensity Dependent Emittance Transfer Studies at the CERN Proton Synchrotron”, Proc. 9th EPAC, Lucerne, Switzerland, 5-9 July 2004

$Q_v = 6.21$ $\Delta Q_{inc,x0} = -0.054$

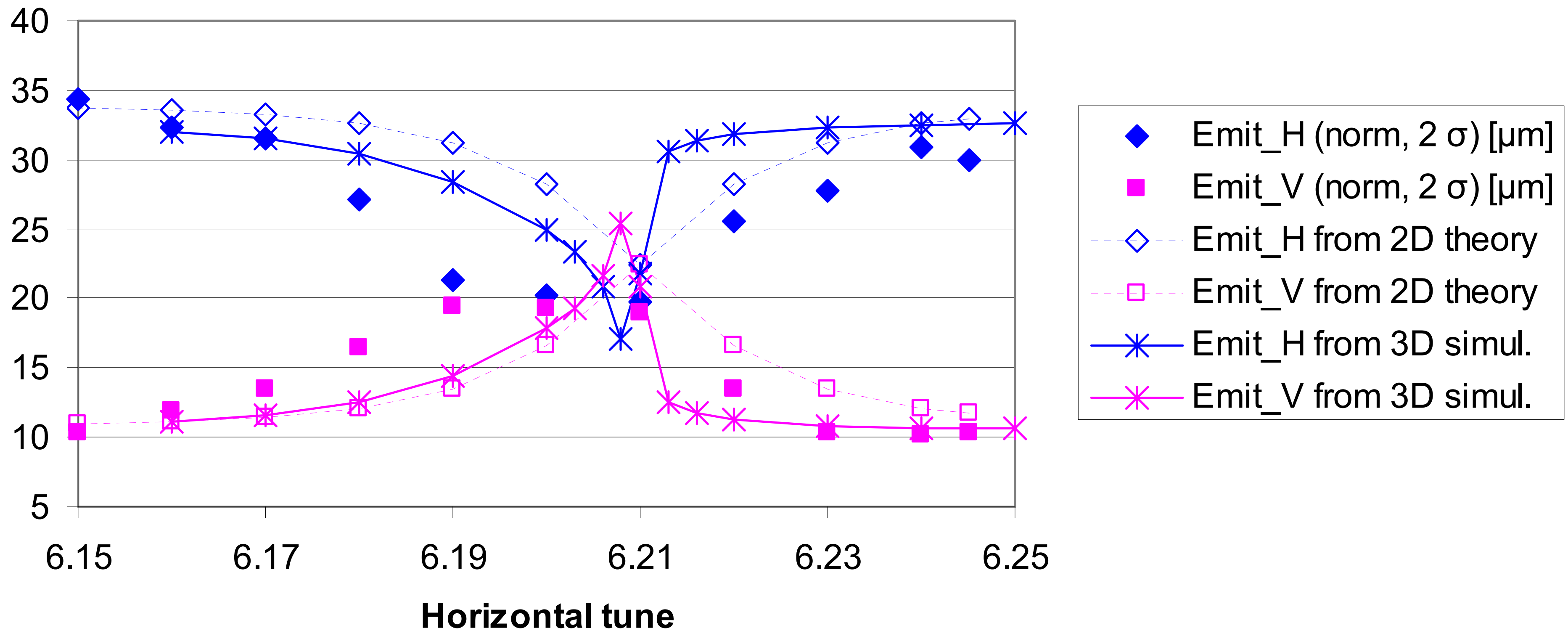




Static case (constant horizontal tune from injection to the measurement point)

Dynamic case (the horizontal tune was changed linearly from 6.15 to 6.25 in 100 ms)

MEASUREMENTS MADE IN THE PS IN 2003 (2/3)

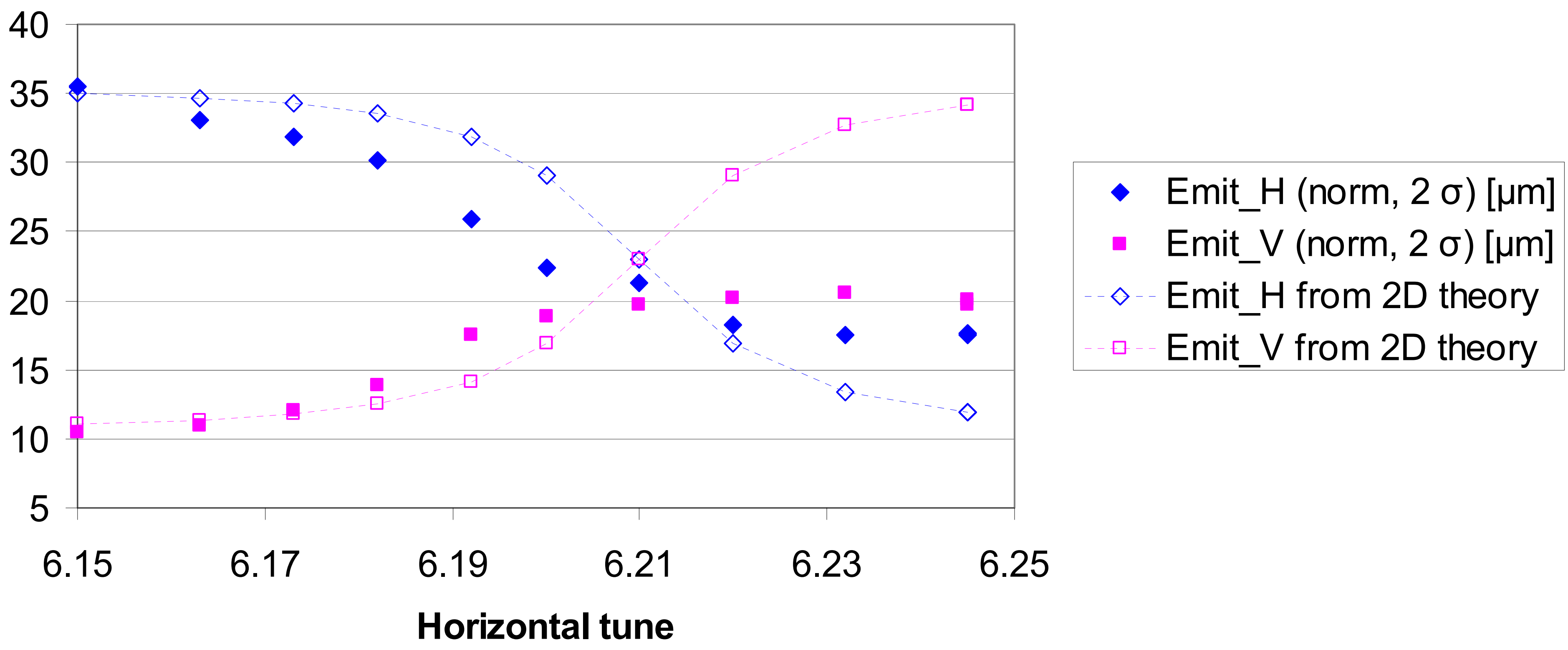


Measurements in the static case compared to the 2D analytical predictions and 3D simulations

$$\delta_{half\ stop\ band}^{5\%} \approx 0.06$$

$$\delta_{half\ stop\ band}^{10\%} \approx 0.04$$

MEASUREMENTS MADE IN THE PS IN 2003 (3/3)



Measurements in the dynamic case compared to the 2D analytical predictions ⟹ Longitudinal motion is missing. 3D simulations should be close (To be checked…)

$$\delta^{5\%}_{half\ stop\ band} \approx 0.06$$

$$\delta^{10\%}_{half\ stop\ band} \approx 0.04$$