Transverse impedance of trapped modes in LHC collimator

A.Grudiev

RLC meeting

11.02.05

Shunt impedance

Accelerator definition: *r*

$$V = \int_0^L E_z e^{j\frac{\omega z}{c}} dz$$

$$r = \frac{V^2}{P}; Q = \frac{\omega_0 U}{P}; \quad \frac{r}{Q} = \frac{V^2}{\omega_0 U}$$

$$V = 2qk_l; k_l = \frac{U}{q^2}; \quad k_l = \frac{V^2}{4U}$$

$$k_l = \frac{\omega_0}{4}\frac{r}{Q}$$

RLC-circuit definition: *R*

$$Z_l(\omega) = \int_0^\infty W_l(\tau) e^{j\omega\tau} d\tau$$

$$k_l = \frac{1}{\pi}\int_0^\infty \Re\{Z_l(\omega)\} d\omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}}; Q = R\sqrt{\frac{C}{L}}$$

$$Z_l(\omega) = \frac{R}{1 + jQ(\omega/\omega_0 - \omega_0/\omega)}$$

$$k_l = \frac{\omega_0}{2}\frac{R}{Q}$$

$$\boxed{r = 2R}$$

Use loss(kick) factor instead of impedance

Kick factor of transverse modes

Two formula have been used in order to check numerical accuracy:

$$k_y = \frac{c\left(\int_0^L \partial E_z/\partial y\, e^{j\frac{\omega z}{c}}\, dz\right)^2}{4U\omega}; \quad k_y^{F_y} = \frac{\omega\left(\int_0^L (E_y + cB_x)e^{j\frac{\omega z}{c}}\, dz\right)^2}{4Uc}; \quad \Delta k_y = \frac{k_y - k_y^{F_y}}{k_y}$$

Table of transverse mode parameters: gap = 5 and 2.5 mm

gap = 5 mm

f[GHz]	Q	r_y[kΩ/m]	k_y[V/nC/mm]	Δk_y[%]
0.605	140	6.681	0.045	0.6
1.226	934	151.708	0.313	-0.8
1.228	961	352.507	0.708	0.5
1.295	808	184.502	0.464	2.8
1.306	570	2.005	0.007	7.0
1.595	172	59.568	0.868	1.9
1.611	171	0.065	0.001	-68.6
1.636	170	398.170	6.019	2.5
1.672	169	254.117	3.949	0.6
1.717	168	121.536	1.951	4.1
1.772	167	875.904	14.599	2.1
1.835	165	565.628	9.881	1.4
1.906	164	10.287	0.188	0.6
1.983	164	288.204	5.474	-0.6

gap =2.5 mm

f[GHz]	Q	r_y[kΩ/m]	k_y[V/nC/mm]	Δk_y[%]
0.607	140	6.716	0.046	-1.1
1.237	940	114.625	0.237	-0.8
1.238	990	582.435	1.144	0.2
1.297	830	218.303	0.536	-2.5
1.311	600	0.653	0.002	-15.4
1.591	88	86.352	2.452	1.6
1.606	88	8.134	0.233	14.1
1.632	87	660.196	19.453	2.2
1.668	86	229.258	6.985	0.6
1.714	86	366.255	11.466	7.2
1.769	85	1342.815	43.898	-0.4
1.834	84	619.615	21.250	1.0
1.906	83	6.370	0.230	10.5
1.986	83	618.253	23.237	-9.7

Estimate of tune shift due to l=0 head-tail mode in SPS

$$Z_1^y = \sum_m \frac{f_m}{f}\frac{R_m^y}{1+iQ_m\left(f_m/f - f/f_m\right)}; \quad R_m^y = \left(\frac{r_y}{Q}\right)_m \frac{Q_m}{2}$$

$$Z_{1\ eff}^y = \frac{\sum_p Z_1^y(f_p)h_l(f_p-f_\xi)}{\sum_p h_l(f_p-f_\xi)}; \quad h_l(f_p) = \left(\frac{2\pi f_p\sigma_z}{c}\right)^{2l} e^{-\left(\frac{2\pi f_p \sigma_z}{c}\right)^2}$$

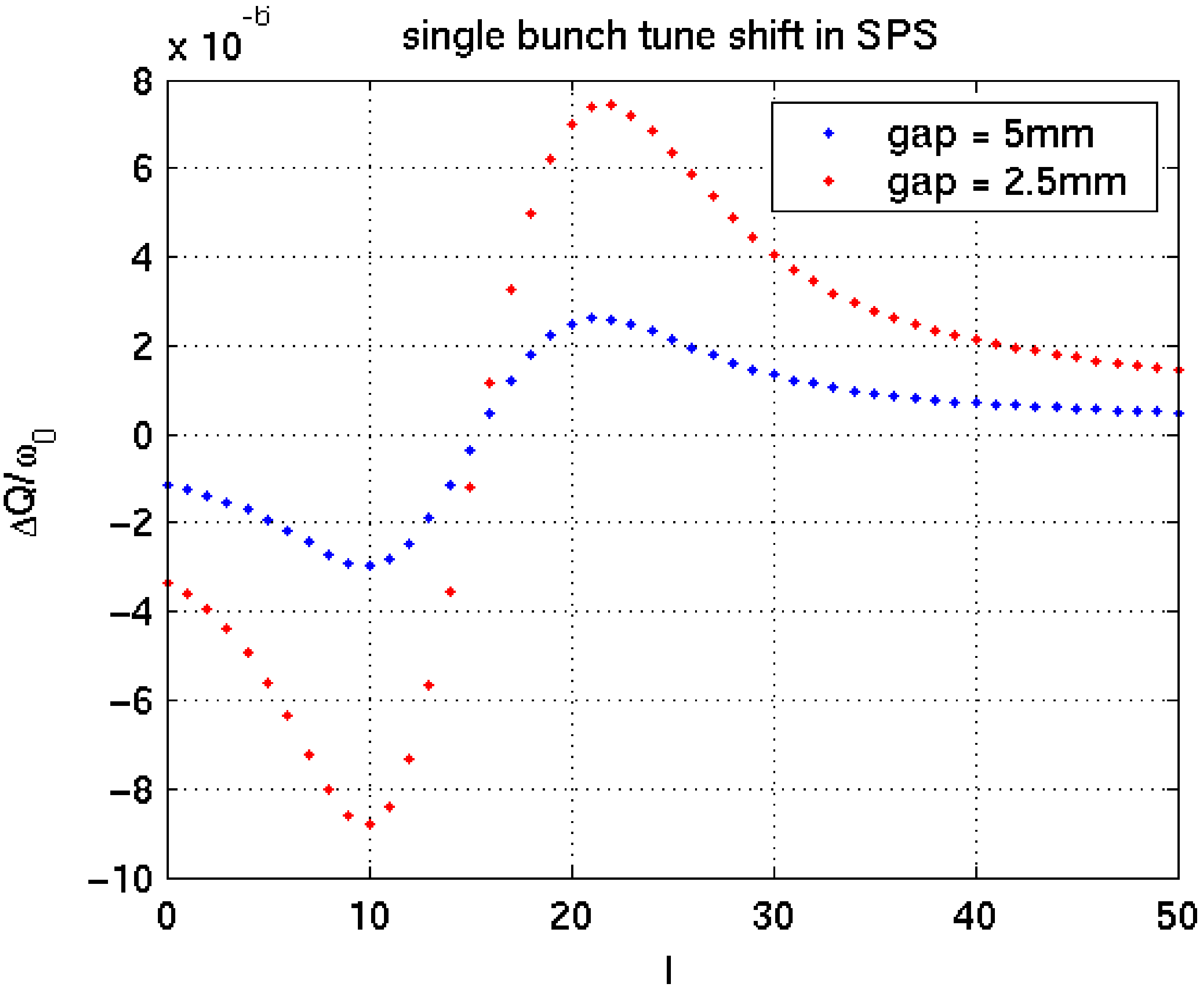
$$f_p = pf_0 + f_\beta; \quad f_\beta = Q_\beta f_0; \quad f_\xi = \xi f_\beta/\eta; \quad \sigma_z = 105mm$$

$$\eta = 5.55\times10^{-4}; \quad Q_\beta = 26.13; \quad \xi = 0; \quad f_0 = 43.4kHz$$

$$\Delta Q = \frac{\Omega-\omega_\beta}{\omega_0} \approx \frac{f_0}{\omega_0}\frac{N_p e c^2}{2\,E/e\,\omega_\beta 2\sqrt{\pi}\sigma_z}\Im\left\{Z_{1\ eff}^y\right\} = -1.15\times10^{-6};$$

$$E/e = 270GeV; \quad N_p = 8.5\times10^{10}; \quad \Im\left\{Z_{1\ eff}^y\right\} = 8.4\,k\Omega/m$$

Tune shift due higher order head-tail modes



Transverse impedance calculated using Gdfidl (red) and reconstructed from HFSS data (blue)