DRIVING AND DETUNING WAKES: REVIEW OF BUROV-DANILOV THEORY

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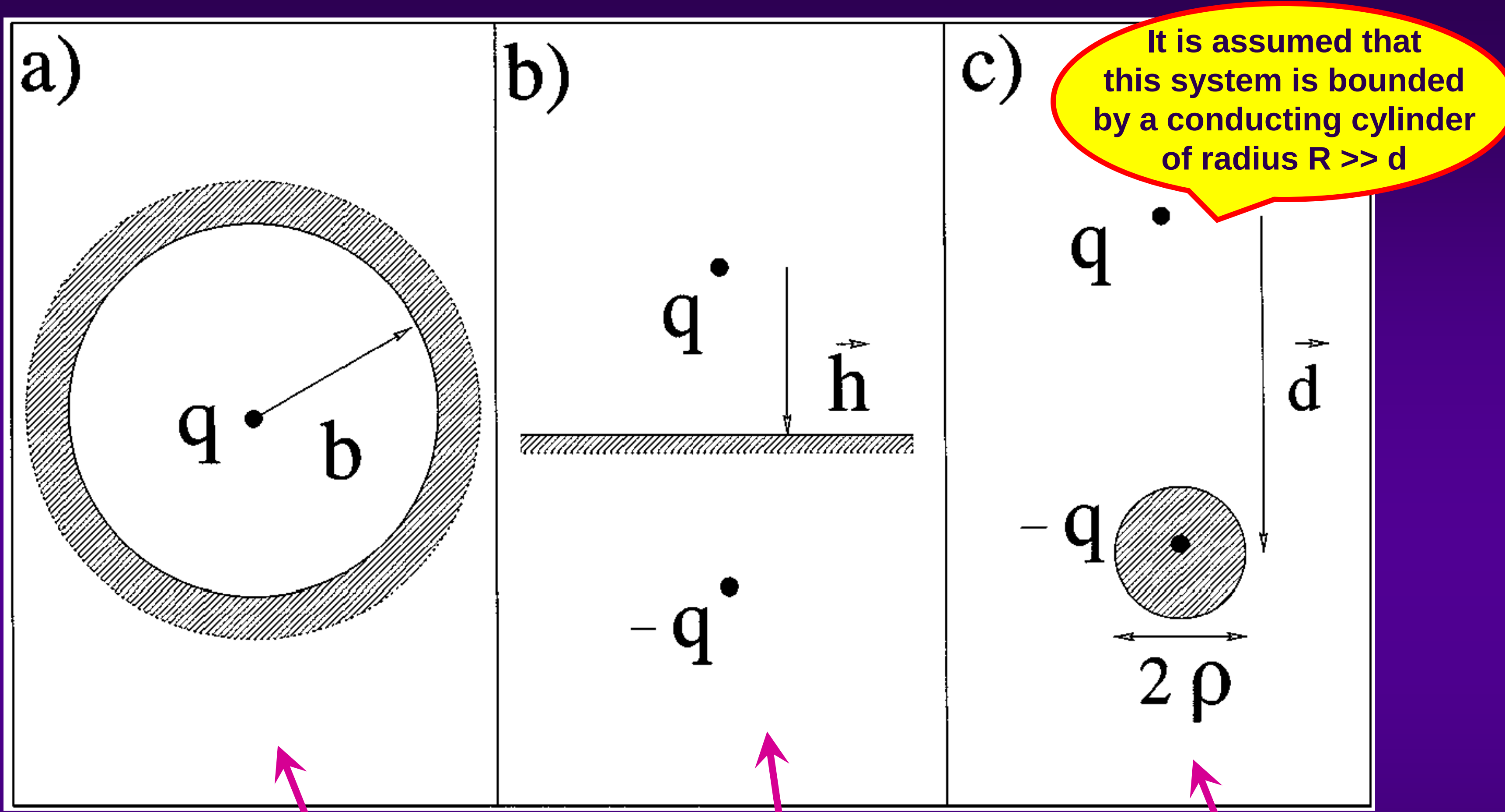
⇒ **From their PRL paper, Vol. 82, N. 11, 15 March 1999**

$$\int_L F_x \, ds = -q^2 x_0 W_x(z) + q^2 x D(z)$$

$$\int_L F_y \, ds = -q^2 y_0 W_y(z) - q^2 y D(z)$$

(CLASSICAL) RESISTIVE-WALL WAKE FUNCTIONS (1/2)



$$W_x(z) = W_y(z) = -\frac{2}{\pi b^3}\sqrt{\frac{c}{\sigma z}}\,L, \qquad D(z) = 0$$

$$W_x(z) = W_y(z) = D(z) = -\frac{L}{2\pi h^3}\sqrt{\frac{c}{\sigma z}}$$

$$D(z) = -\frac{L}{\pi d^2 \rho \ln(R/\rho)}\sqrt{\frac{c}{\sigma z}}$$

$$W_x(z) = W_y(z) = -\frac{L\rho}{\pi d^4}\sqrt{\frac{c}{\sigma z}}$$

(CLASSICAL) RESISTIVE-WALL WAKE FUNCTIONS (2/2)

$$\kappa_x = D(z)/W_x(z)$$

$$\kappa_x = \begin{cases} 0, & \text{axial symmetry,} \\ 1, & \text{plane wall } \mathbf{n}_x \perp \mathbf{h}\,, \\ -1, & \text{plane wall } \mathbf{n}_x \parallel \mathbf{h}\,, \\ d^2/[\rho^2 \ln(R/\rho)], & \text{small cylinder } \mathbf{n}_x \perp \mathbf{d}\,, \\ -d^2/[\rho^2 \ln(R/\rho)], & \text{small cylinder } \mathbf{n}_x \parallel \mathbf{d}\,. \end{cases}$$

CONCLUSION for the force in the horizontal plane, i.e. $\perp \vec{h}$ or $\vec{d}$

$$\text{If } x = x_0 \quad \Rightarrow \quad \oint_L F_x \, ds = -q^2 \, x_0 \, W_x(z) \times \underbrace{(1 - \kappa_x)}_{F_x}$$

- **Case a)** $\Rightarrow$ $F_x = 1$
- **Case b)** $\Rightarrow$ $F_x = 0$
- **Case c)** $\Rightarrow$ $F_x = 1 - \dfrac{d^2}{\rho^2 \ln\left(\dfrac{R}{\rho}\right)} < 0$

$\Rightarrow$ **Explains why ≥ 0 or ≤ 0 (generalized, i.e. driving + detuning) impedances (Re and Im) can be measured with a single wire !**

Longitudinal and (generalized) transverse impedances measured with a single wire

- **Panofsky-Wenzel theorem**

(shown for non-symmetric structures in Zotter's book)

$$\frac{\partial}{\partial z} \int_L \mathbf{F}_\perp \, ds = \nabla_\perp \int_L F_\parallel \, ds$$

If $x = x_0$

If $y = y_0$

$$\int_L F_\parallel \, ds = f(z) - q^2 \left[W_x'(z) - D'(z) \right] x_0^2 - q^2 \left[W_y'(z) + D'(z) \right] y_0^2$$

$$\Rightarrow Z_\parallel^{\text{measured}}(f) = Z_{\parallel,0}(f) + Z_{\parallel,1x}(f)\, x_0^2 + Z_{\parallel,1y}(f)\, y_0^2$$

Same result recovered by Tsutsui in 2002 (CERN-SL-Note-2002-034 AP)

$$Z_x^{\text{driving+detuning}}(f) = \frac{c}{2\pi f} Z_{\parallel,1x}(f) \qquad Z_y^{\text{driving+detuning}}(f) = \frac{c}{2\pi f} Z_{\parallel,1y}(f)$$

(Driving or dipolar or classical) impedance for a 1-sided collimator compared to a 2-sided one

Linked to $W_{x,y}(z)$

$$Z_x^{1-\text{sided}} = \frac{6}{\pi^2} Z_x^{2-\text{sided}} \approx 0.6\, Z_x^{2-\text{sided}}$$

$$Z_y^{1-\text{sided}} = \frac{3}{\pi^2} Z_y^{2-\text{sided}} \approx 0.3\, Z_y^{2-\text{sided}}$$

$$Z_x^{1-\text{sided}} = Z_y^{1-\text{sided}} = \frac{Z_\perp^{\text{circular}}}{4}$$

$$Z_x^{2-\text{sided}} = \frac{\pi^2}{24} Z_\perp^{\text{circular}} \approx 0.4\, Z_\perp^{\text{circular}}$$

$$Z_y^{2-\text{sided}} = \frac{\pi^2}{12} Z_\perp^{\text{circular}} \approx 0.8\, Z_\perp^{\text{circular}}$$