

# TRANSVERSE RESISTIVE WALL IMPEDANCE FOR SIS100 (AT GSI) AND LHC COLLIMATOR

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**Slava is writing a Technical Proposal:  
“The Transverse Damping Systems for SIS100&SIS300”**

## ◆ **Comparison between Zotter2005 and Balbekov1985\***

\* V.I. Balbekov, Space Charged Effects in UNK, in The IX All Union Conference on Particle Accelerators, JINR, Dubna, vol. 2, p. 360, 1985 (in Russian)

## Beam and machine parameters for SIS100

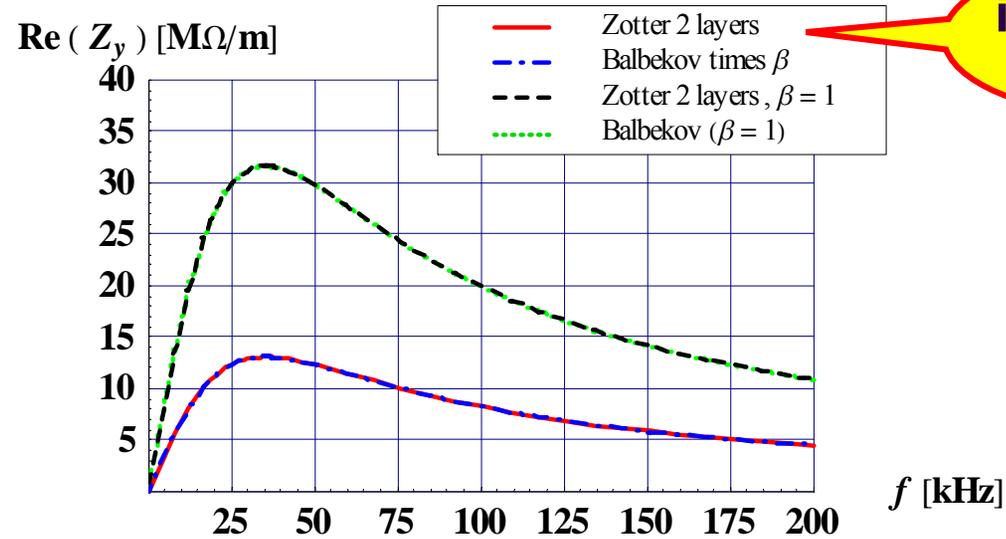
⇒ **Computation for  $^{238}\text{U}^{28+}$  at injection, assuming (first) a round pipe with radius 32 mm (=vertical half axis)**

Machine circumference [m]	1083.6
Beam rigidity [Tm]	12
$\beta$	0.413
$\gamma$	1.098
$f_{rev}$ [kHz]	114.4
$\rho$ [ $\Omega\text{m}$ ] $\Rightarrow$ SS	$9 \cdot 10^{-7}$
Vacuum chamber thickness [mm]	0.2 or 0.3
Horizontal half axis [mm]	32 (65 normally)
Vertical half axis [mm]	32

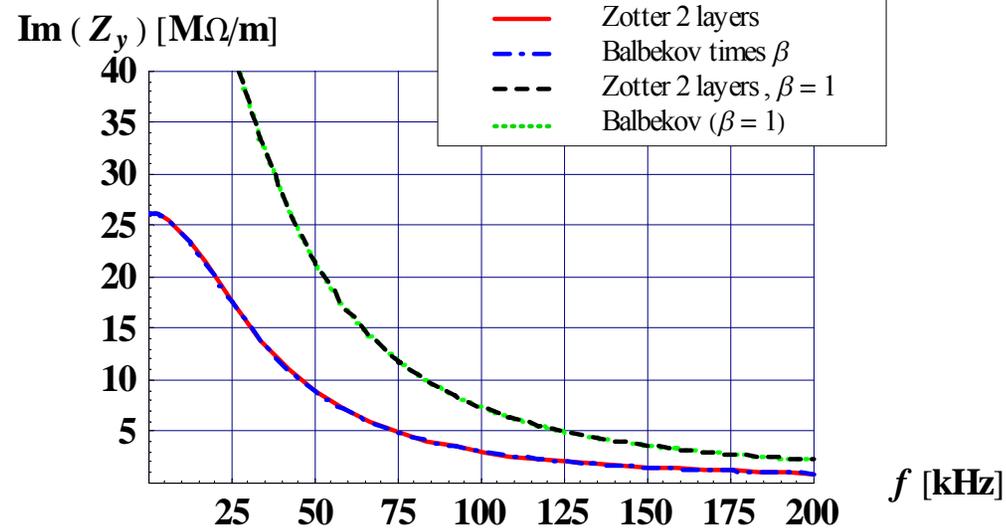
Vacuum chamber thickness = **0.2 mm**

... assuming the whole machine circumference with the same resistivity

Skin depth = 0.2 mm  
 $\Leftrightarrow f = 5.7 \text{ MHz}$



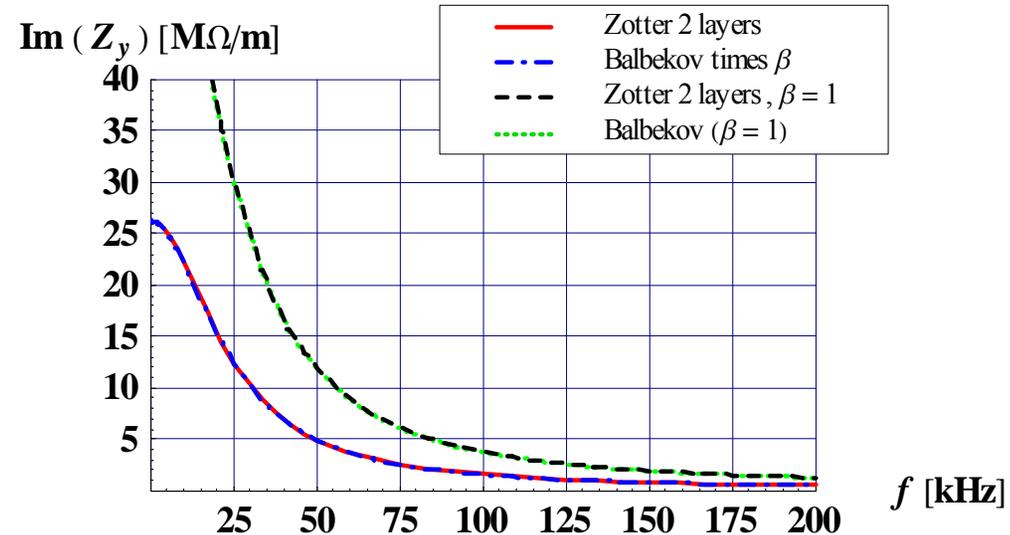
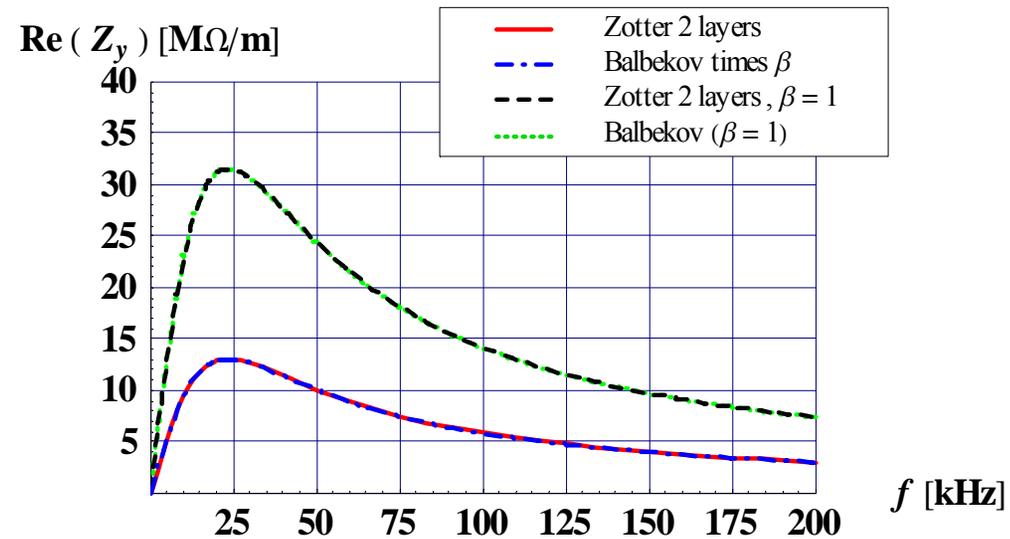
i.e. vacuum outside



Vacuum chamber  
thickness = **0.3 mm**

... assuming the whole  
machine circumference  
with the same resistivity

Skin depth = 0.3 mm  
 $\Leftrightarrow f = 2.5 \text{ MHz}$



# LHC graphite collimator

$$L = 1 \text{ m}$$

$$b = 2 \text{ mm (half gap)}$$

$$\rho_C = 10 \mu\Omega\text{m}$$

$$d_C = 2.5 \text{ cm}$$

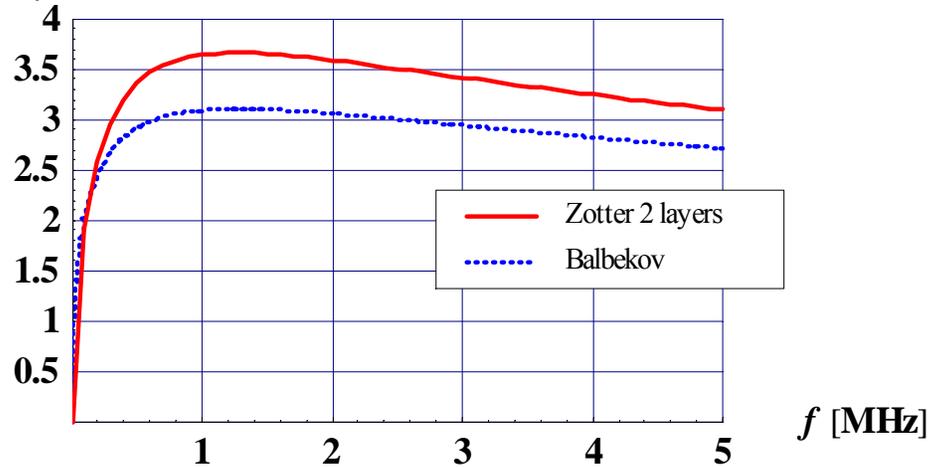
$$\gamma = 7462.69$$

$$\beta = 1$$

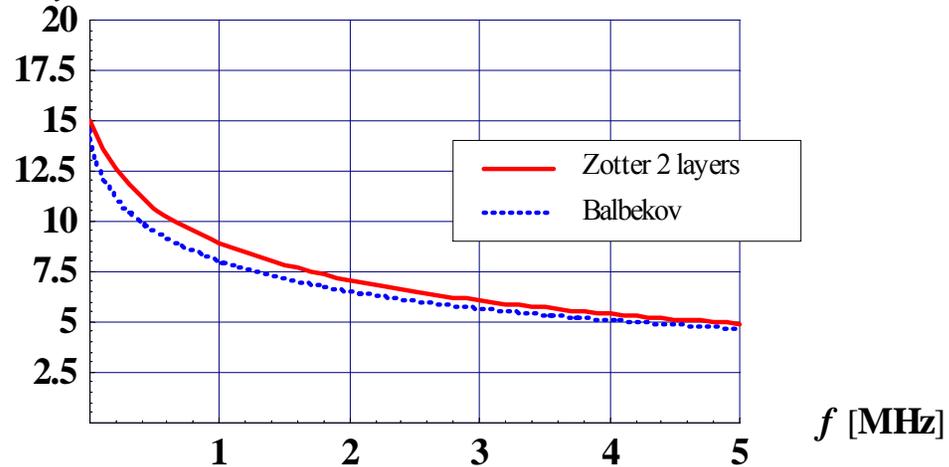
$$\text{Skin depth} = 2.5 \text{ cm}$$

$$\Leftrightarrow f = 4 \text{ kHz}$$

Re ( $Z_y$ ) [ $\text{M}\Omega/\text{m}$ ]



Im ( $Z_y$ ) [ $\text{M}\Omega/\text{m}$ ]



# LHC graphite collimator

$$L = 1 \text{ m}$$

$$b = 2 \text{ mm (half gap)}$$

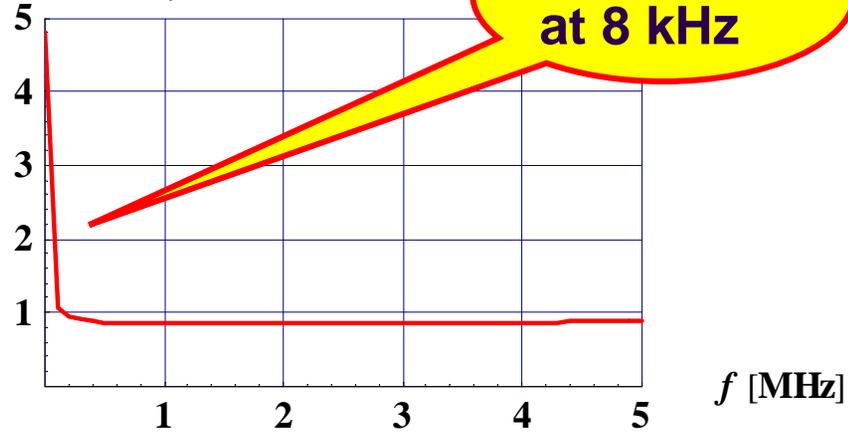
$$\rho_C = 10 \mu\Omega\text{m}$$

$$d_C = 2.5 \text{ cm}$$

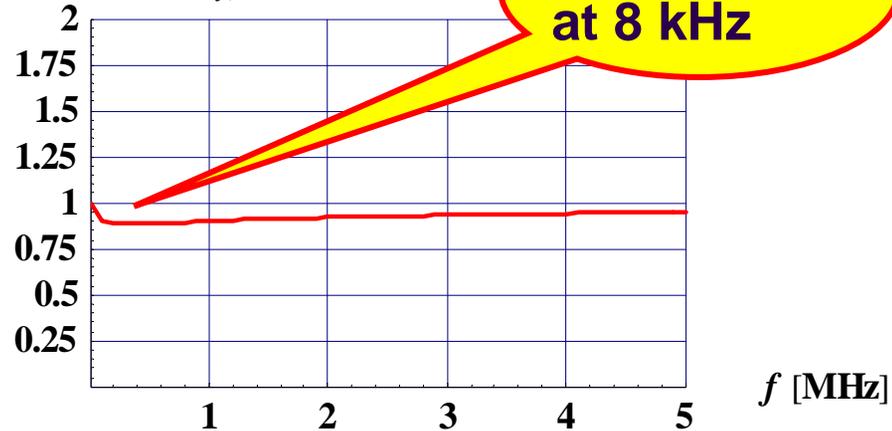
$$\gamma = 7462.69$$

$$\beta = 1$$

$$\text{Re} ( Z_{y, \text{Balbekov}} ) / \text{Re} ( Z_{y, \text{Zotter}} )$$



$$\text{Im} ( Z_{y, \text{Balbekov}} ) / \text{Im} ( Z_{y, \text{Zotter}} )$$



## APPENDIX 1: BALBEKOV'S FORMULA FOR A ROUND PIPE

$$Z_y = \frac{j L Z_0}{2 \pi b^2} \times \frac{1}{1 + \frac{b}{(1-j)\delta \coth \left[ (1+j) \frac{d}{\delta} \right]}}$$

“Inductive-bypass” effect due to this term ( $\Rightarrow$  2 impedances in // as Vos2003)

where

$L$  = Length of the object

$Z_0 = 120 \pi$

$b$  = Half gap

$\delta$  = Skin depth

$d$  = Thickness of the vacuum chamber

**APPENDIX 2: Draft paper  
 “Damping of Transverse  
 Instabilities for Coasting  
 Beams” by V. Kornilov,  
 O. Boine-Frankenheim,  
 I. Hofmann and B. Doliwa  
 ⇒ Given to me (for  
 comments) by Slava**

**No “inductive-bypass” term**

### 3 Impedances

The transverse impedance due to the space charges (sc) we evaluate using

$$Z_{sc}^{\perp} = iZ_0 \frac{R}{\beta\gamma^2} \left[ \frac{1}{a^2} - \frac{1}{b^2} \right], \quad (29)$$

in which we take into account the elliptic cross-sections replacing the radii as  $2a^2 \rightarrow R \left( \frac{\epsilon_{v,h}}{Q_{v,h}} + \sqrt{\frac{\epsilon_v \epsilon_h}{Q_v Q_h}} \right)$ .

The impedance stemming from a resistive wall with thickness  $d_{\text{pipe}}$  we estimate [2] as

$$Z_{rw}^{\perp} = \frac{2\beta c}{b^2 \Omega} Z_{rw}^{\parallel} = (1-i)Z_0 \frac{R\beta \delta_{\text{skin}}}{b^3} \coth \left\{ \frac{\Omega}{\beta c} \sqrt{\frac{1}{\gamma^2} - i \frac{4\pi\sigma\beta^2}{\Omega}} d_{\text{pipe}} \right\} \quad (30)$$

( $\delta_{\text{skin}} = c/\sqrt{2\pi\sigma\Omega}$ ,  $Z_0 = 4\pi/c$ ). Figure 4 shows this impedance in its dependencies on the coherent frequency  $\Omega$  (left-hand side) and on the wall thickness  $d_{\text{pipe}}$  (right-hand side). The parameters taken correspond to a  $U^{28+}$  coasting beam at the injection energy, see Table 3 and Table 4 in the Appendix. The right-hand side plot is made for the frequency 156 kHz, the corresponding skin length is  $\delta_{\text{skin}} = 1.1$  mm. For the left-hand side plot, the skin length

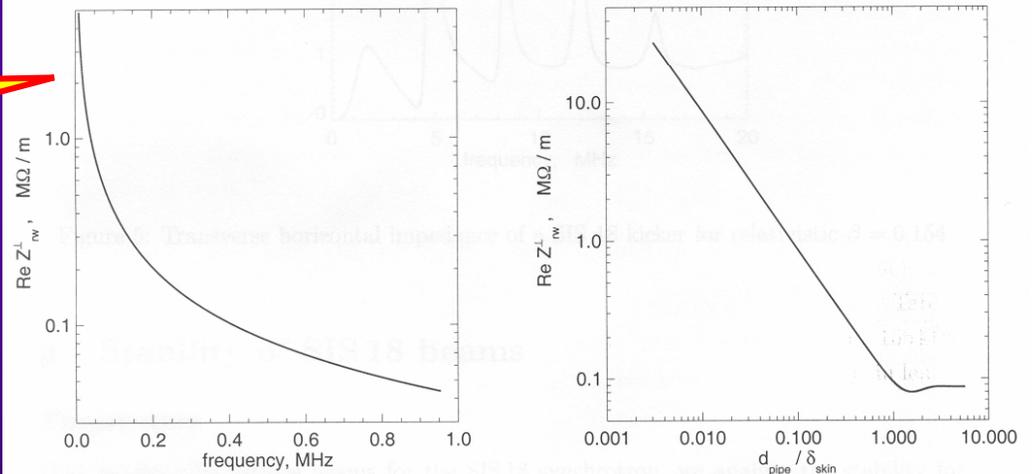


Figure 4: Transverse resistive wall impedance [Eq. (30)] as a function of the frequency (left) and of the wall thickness (right). Note the logarithmic scale for  $Z_{rw}^{\perp}$  and  $d_{\text{pipe}}$ .

is equal to  $\delta_{\text{skin}} = 0.3$  mm at the frequency 1.9 MHz, i.e. this is the thin-wall regime for such a pipe.