REVIEW OF G. STUPAKOV’S 2005 PRSTAB PAPER
“Resistive wall impedance of an insert”

Elias Métral

- **What is Stupakov’s paper (already published) about?**
- **What is Gluckstern-Zotter’s paper (not yet published) about?**

G. STUPAKOV'S APPROACH

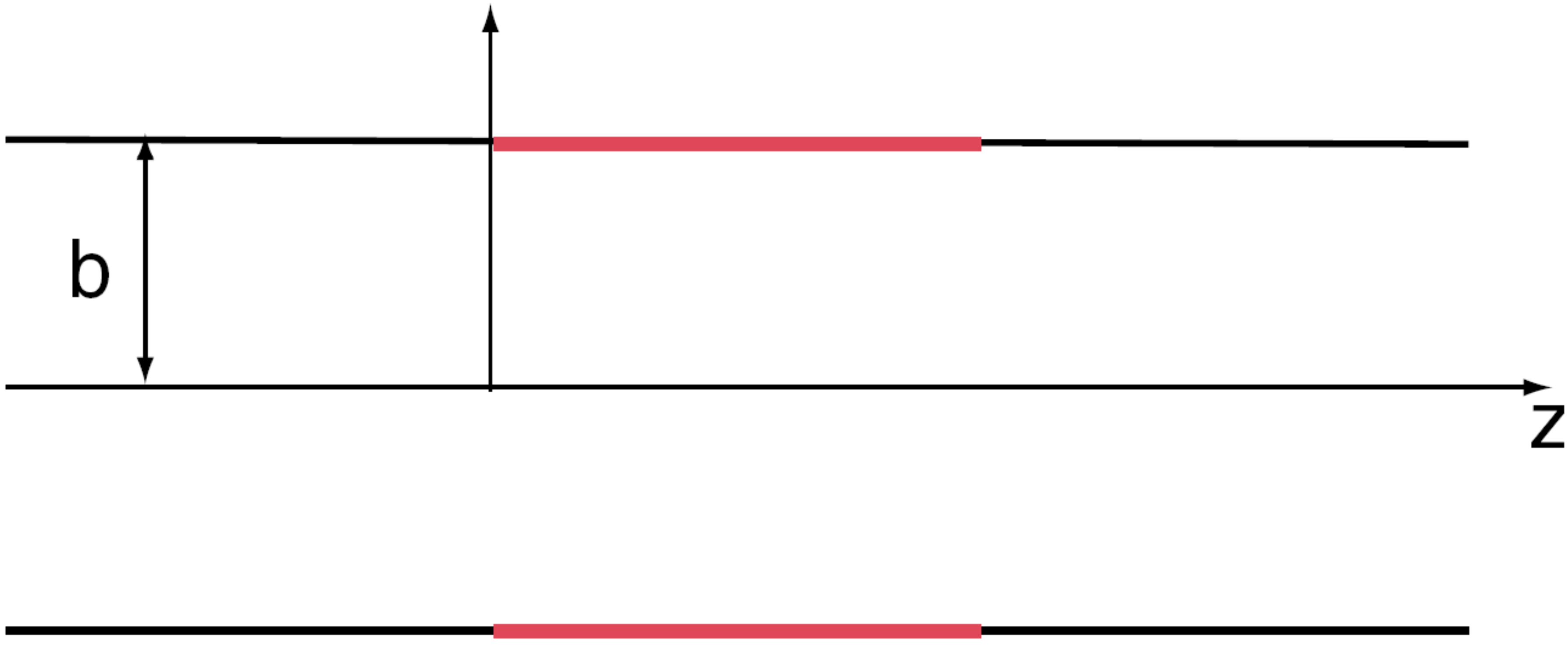

FIG. 1. (Color) Round pipe with an insert of length L (shown in red) having the wall conductivity σ. The rest of the pipe (shown in black) has an infinite conductivity.

Resistive wall impedance of an insert

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The standard theoretical formulas for resistive wall impedance are usually derived in a model which assumes an infinitely long pipe. In practice, one often has to deal with resistive inserts with a conductivity different from the rest of the pipe. To address this case, we calculate the resistive wall impedance when the wall conductivity varies along the axis of the pipe. We show that at not very high frequencies the impedance of an insert per unit length is given by the same formulas as for an infinitely long pipe.

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I. INTRODUCTION

The standard theoretical formulas for resistive wall impedance (see, e.g., [1]) are derived in a model of an infinitely long pipe. The longitudinal impedance per unit length of the pipe, in the Gaussian system of units, is given by the following equation:

$$Z_{\text{long pipe}}(\omega) = \frac{1-i}{cb}\sqrt{\frac{\omega}{2\pi\sigma}}, \qquad (1)$$

where ω is the frequency, b is the pipe radius, and σ is the conductivity of the pipe wall. Equation (1) is valid for not very large frequencies,

$$\omega \ll \omega_0, \qquad (2)$$

where $\omega_0 = (4\pi c^2\sigma/b^2)^{1/3}$. This condition is usually sat-

radius b. Our goal is to calculate the longitudinal impedance of such pipe at frequency ω. We assume that the skin depth in the metal is small compared to the pipe radius and the wall thickness, and use the Leontovich boundary condition relating the longitudinal electric field on the metal surface E_z with the magnetic field H_ϕ [4]

$$E_z = -\zeta H_\phi, \qquad (3)$$

where $\zeta = (1-i)\sqrt{\omega/8\pi\sigma}$ (we use the Gaussian system units throughout the paper). As is known, for impedance calculation, the beam can be represented as a filament current $I(z,t)$ on the axis of the pipe,

$$I(z,t) = I_0 e^{-i\omega t + ikz}, \qquad (4)$$

with $k = \omega/c$. The impedance is then given by the Fourier

Using the identity

$$\sum_{m=1}^{\infty} \frac{1}{\mu_m J_1(\mu_m)} = \frac{1}{2}, \quad (14)$$

we arrive at the following expression for the impedance

$$Z = \frac{2L\zeta}{bc} = L\frac{1-i}{cb}\sqrt{\frac{\omega}{2\pi\sigma}}, \quad (15)$$

which is exactly equal to $LZ_{\mathrm{long\ pipe}}(\omega)$. We see that the impedance per unit length (of the resistive part) is given by the same equation (1) as in the case of an infinitely long pipe.

III. GENERALIZATION FOR ARBITRARY CROSS SECTION AND TRANSVERSE IMPEDANCE

The result of the previous section can be derived in a simpler way, which also allows generalization to the case of arbitrary cross section of the pipe. We now assume that

transverse direction. Through the Panofsky-Wenzel theorem [7] this longitudinal impedance is directly related to the transverse one. Hence, we conclude that the transverse impedance per unit length of an insert, as well as the longitudinal one, will also be given by the formula derived in the limit of an infinitely long pipe.

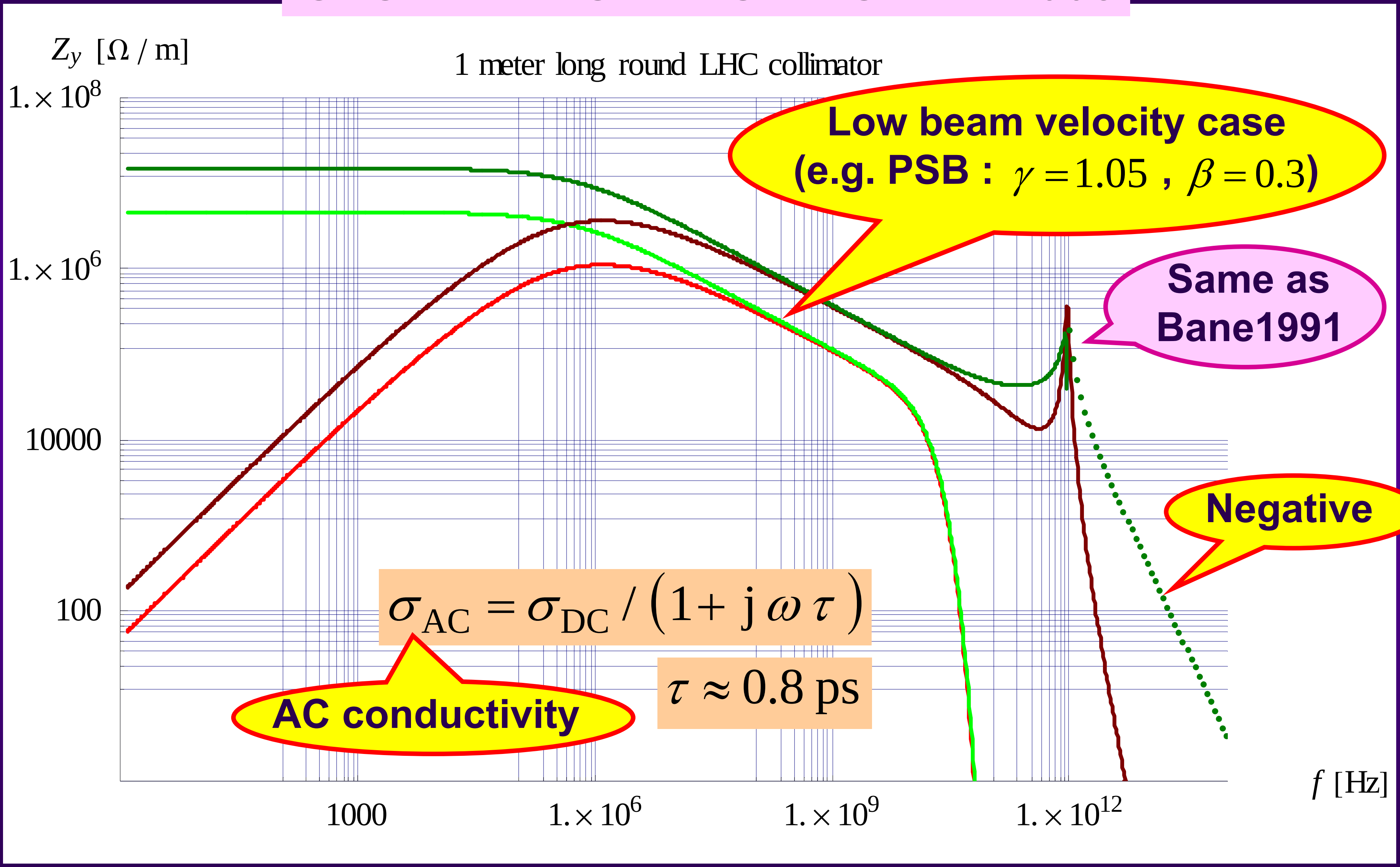
B. ZOTTER'S APPROACH FOR AN INFINITELY LONG BEAM PIPE

⇒ For a circular beam pipe

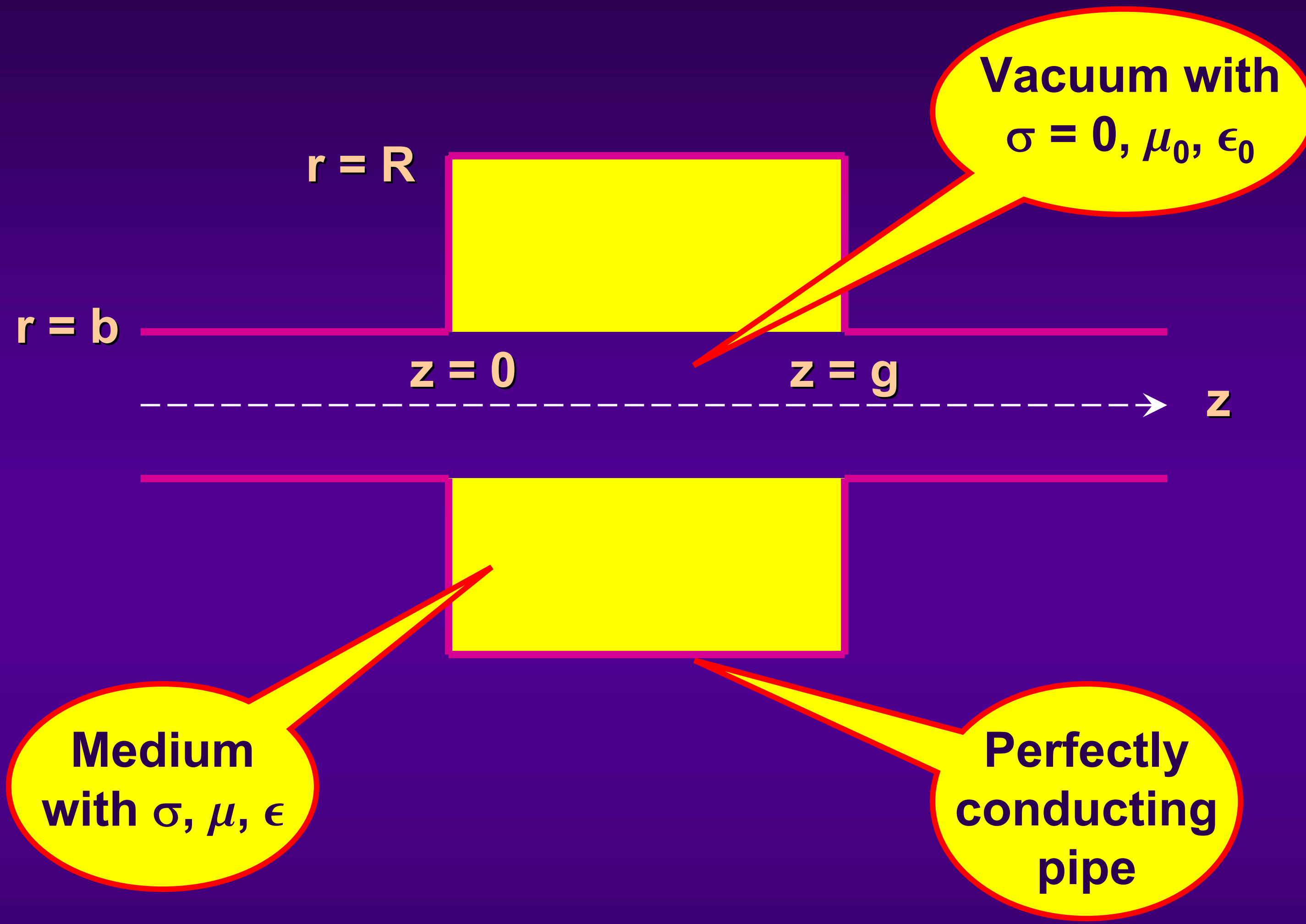
- Any number of layers
- Any beam velocity
- Any σ (conductivity), ε (permittivity) and μ (permeability)
- Any frequency ⇒ Unification of 3 regimes
  - Burov-Lebedev2002 (low-frequency regime)
  - "Thick-wall" (intermediate-frequency regime)
  - Bane1991 (high-frequency regime)

G. Stupakov made his analysis in this regime

GLOBAL PLOT FROM ZOTTER2005



GLUCKSTERN-ZOTTER'S APPROACH FOR THE FINITE LENGTH CASE



⟹ The idea is to look in particular at the low-frequency regime, which is important for the LHC collimators

…and, as said at previous RLC meetings, preliminary results seem to indicate that ~ the same conclusion as G. Stupakov could be obtained (but still numerical problems, scans made only over a certain range and for the particular case of a collimator…)