LHC Project Note

July 11, 2003

Resistive wall impedance of an LHC collimator

Hiroshi Tsutsui*

Keywords: beam impedance instability collective effect

Summary

The resistive wall impedance of an LHC collimator of full-gap $2b = 3$ mm is calculated analytically and by electro-magnetic field simulation code.

1 Introduction

The beam coupling impedance of collimators is potentially large because they have small apertures. The impedance consists of resistive wall and geometrical contributions. In this note, the resistive wall impedance is evaluated by two different methods, field matching and numerical wire method using the electro-magnetic simulation code HFSS [1].

The methods are described in Section 2, and they are applied for some cases in Section 3.

2 Methods

2.1 Field matching

The model is shown in Fig. 1. The analytic expression of the electro-magnetic field in each region is already given in [2, 3], but the number of unknown coefficients in each region should be increased from two to four in order to include metallic coating on graphite collimators.

* on leave from Sumitomo Heavy Industries, Ltd., Japan

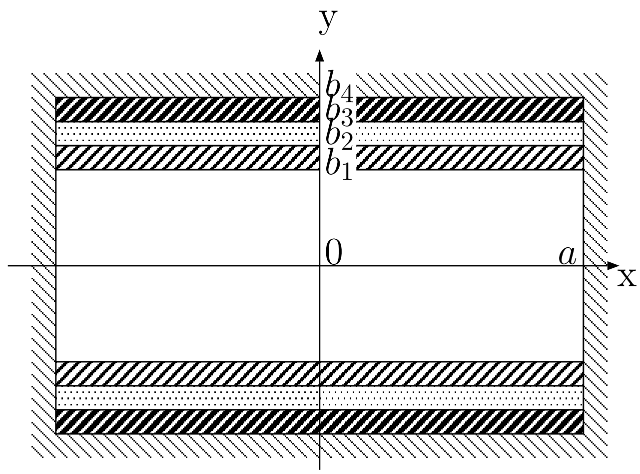

Figure 1: A simplified 2D model for resistive wall impedance calculation. The nominal beam is centered at $(x, y) = (0, 0)$. The i-th material is placed at $(-a < x < a, b_i < |y| < b_{i+1}, 1 \leq i \leq n)$. Outside is surrounded by perfect conductor. The length in z-direction is infinite.

The electro-magnetic field components in the i-th region are

$$
\begin{aligned}
E_z &= \sum_n [C_n \sin(k_{yn}(y - b_i)) - E_n \cos(k_{yn}(y - b_i)] \cos(k_{xn}x), \\
E_x &= \frac{j}{\epsilon_r \mu_r - 1} \sum_n \{ \frac{k_{xn}}{k} [C_n \sin(k_{yn}(y - b_i)) - E_n \cos(k_{yn}(y - b_i))] \\
&\quad + \mu_r \frac{k_{yn}}{k} [D_n \sin(k_{yn}(y - b_i)) - F_n \cos(k_{yn}(y - b_i))] \} \sin(k_{xn}x), \\
E_y &= \frac{j}{\epsilon_r \mu_r - 1} \sum_n \{ \frac{-k_{yn}}{k} [C_n \cos(k_{yn}(y - b_i)) + E_n \sin(k_{yn}(y - b_i))] \\
&\quad + \mu_r \frac{k_{xn}}{k} [D_n \cos(k_{yn}(y - b_i)) + F_n \sin(k_{yn}(y - b_i))] \} \cos(k_{xn}x), \\
Z_0 H_z &= \sum_n [D_n \cos(k_{yn}(y - b_i)) + F_n \sin(k_{yn}(y - b_i))] \sin(k_{xn}x), \\
Z_0 H_x &= \frac{j}{\epsilon_r \mu_r - 1} \sum_n \{ \epsilon_r \frac{k_{yn}}{k} [C_n \cos(k_{yn}(y - b_i)) + E_n \sin(k_{yn}(y - b_i))] \\
&\quad - \frac{k_{xn}}{k} [D_n \cos(k_{yn}(y - b_i)) + F_n \sin(k_{yn}(y - b_i))] \} \cos(k_{xn}x), \\
Z_0 H_y &= \frac{j}{\epsilon_r \mu_r - 1} \sum_n \{ \epsilon_r \frac{k_{xn}}{k} [C_n \sin(k_{yn}(y - b_i)) - E_n \cos(k_{yn}(y - b_i))] \\
&\quad + \frac{k_{yn}}{k} [D_n \sin(k_{yn}(y - b_i)) - F_n \cos(k_{yn}(y - b_i))] \} \sin(k_{xn}x), \qquad (1)
\end{aligned}
$$

where C_n, D_n, E_n, F_n are the unknown coefficients, k_{xn}, k_{yn} are the wave numbers, ϵ_r, μ_r are the relative permittivity and permeability, $Z_0 = 377\ \Omega$ is the vacuum impedance.

Then, four field components E_z, E_x, $\epsilon_r E_y$, and H_z are matched at each boundary. The other two boundary conditions are abundant. Finally, the coupling impedances per unit length are obtained from some of the coefficients. The calculation was made with Mathematica [4].

2.2 HFSS

Coaxial wire method and double wire method [5] can be used in electro-magnetic simulation code HFSS to evaluate the coupling impedances.

The log formula for deriving longitudinal impedance is

$$Z_{\parallel} = -2Z_{ch} \log\left(\frac{S_{21}}{S_{REF}}\right), \tag{2}$$

where Z_{ch}, S_{21}, S_{REF} are the characteristic impedance, the scattering parameter of DUT, and $\exp(-j\omega l/c)$, respectively.

The transverse impedance can be approximated by the log formula as

$$Z_{\perp} = -\frac{2cZ_{ch}}{\omega d^2} \log\left(\frac{S_{21}}{S_{REF}}\right), \tag{3}$$

where d is the distance between the two wires.

The characteristic impedance and the scattering parameters are calculated by HFSS. Then, the coupling impedances are calculated by using the above equations.

3 Resistive wall impedance of a collimator

3.1 Graphite

Graphite (with conductivity $\sigma = 7 \times 10^4$ /Ωm) is considered to be used as the collimator material. The material has relative dielectric constant of around 10, but this is neglected in this note since the effect is negligible. The parameters used are $a = 25$ mm, $b_1 = 1.5$ mm, and $b_2 = 10$ mm. The model used in the HFSS simulation is shown in Fig. 2.

Figures 3-5 show the result. Field matching and HFSS simulations give similar results. Since HFSS forbids the calculations at low frequency, there are no points below 1 MHz. At low frequency, the transverse impedances have constant imaginary parts. The values are $Z_{hor}/L = j\pi Z_0/(48b_1^2)$, and $Z_{ver}/L = j\pi Z_0/(24b_1^2)$, which are independent of material properties.

3.2 Graphite with copper coating

Next, a model with low conductivity copper ($\sigma = 2.5 \times 10^7$ /Ωm) coated on the graphite is simulated. The thickness of the coating is $\xi = 1$ μm, 10 μm. Since the skin depth of metal is $1 \sim 100$ μm order for frequency range of 1 MHz $\sim$ 1 GHz, smaller size of meshes at metal surface is needed, which is very difficult. So, impedance boundary condition in HFSS is used

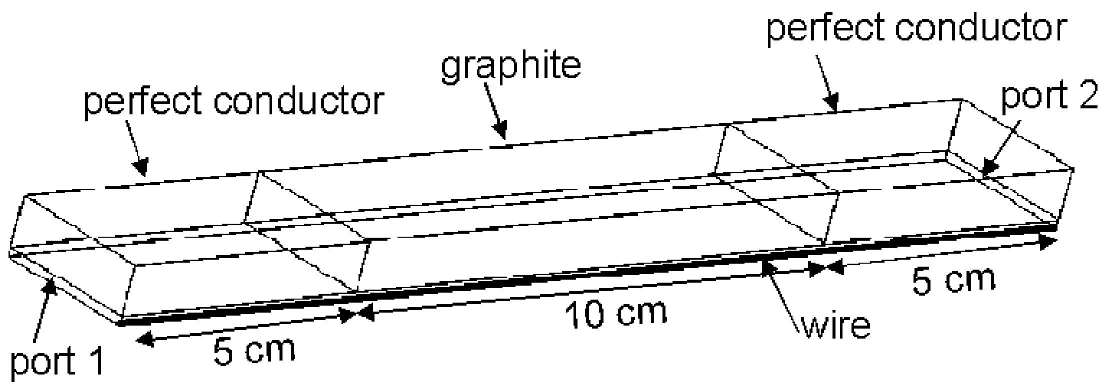

Figure 2: A quarter geometry used for HFSS simulation. The characteristic impedance of the ports and the scattering parameter from port 1 to 2 are calculated in HFSS.

to avoid this problem. But this boundary condition is valid only in the frequency region that the skin depth is larger than the thickness of the metal. In this case the condition is

$$f < \frac{1}{\pi \mu_0 \sigma \xi^2} = \begin{cases} 10 \text{ GHz} & (\xi = 1\ \mu\text{m}) \\ 100 \text{ MHz} & (\xi = 10\ \mu\text{m}) \end{cases} \tag{4}$$

Figures 6-8 show the result. The two methods give similar results.

3.3 Other materials

Beryllium ($\sigma = 2.5 \times 10^7$ /Ωm), and Ceramics ($\epsilon_r = 9.2$) cases are calculated analytically, as shown in Fig. 9.

If Beryllium is used, the impedance at high frequency is smaller by more than one order of magnitude than for the graphite case.

If Ceramics is used, the real part of the impedance is negligible, but the imaginary part is quite high.

4 Transverse effective impedance

The transverse effective impedance is calculated using the formula below:

$$(Z_{ver}/L)_{eff} = \frac{\sum_p (Z_{ver}/L) h(\omega_p)}{\sum_p h(\omega_p)}, \tag{5}$$

where the spectral density $h(\omega)$ is $\exp(-\omega^2 \sigma_\tau^2)$, and $\omega_p = \omega_0(pk_b + \mu + Q_\beta)$. Assumed values of the parameters are $\sigma_\tau = 0.25$ nsec, $\omega_0/2\pi = 11.245$ kHz, $k_b = 3564$, $Q_\beta = 59.3$, $a = 25$ mm, $b_2 = b_1 + \xi$, $b_3 = b_2 + 10$ mm. This definition is different from L. Vos's, which is without denominator ($\sum_p h(\omega_p) \to 1$). Result is shown in Fig. 10. Maximum values of the effective impedances over the coupled-bunch modes are plotted. The result may vary by changing k_b.

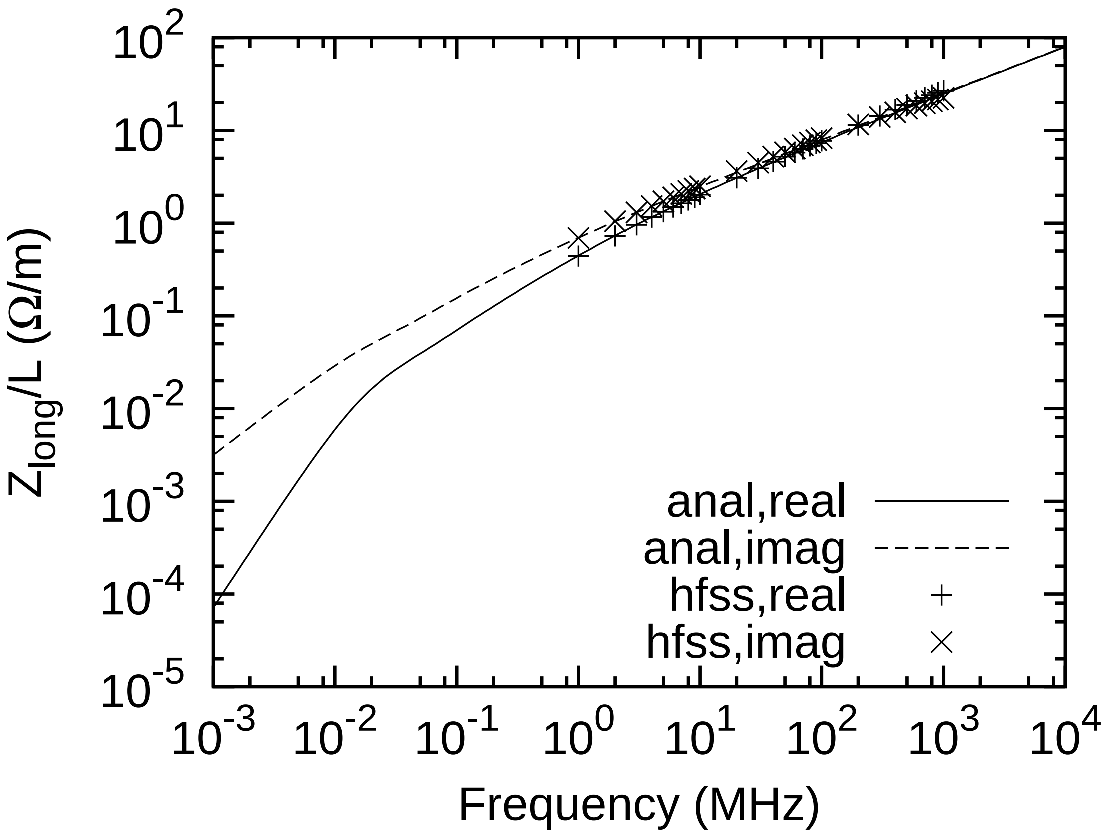

Figure 3: Resistive wall longitudinal impedance per unit length. Material is graphite ($\sigma = 7\times10^4$ /Ωm). Parameters are $a = 25$ mm, $b_1 = 1.5$ mm, $b_2 = 10$ mm. Lines are by analytical calculations. Symbols are by HFSS simulations.

5 Discussion of results and outlook

The resistive wall part of the collimator impedance is evaluated by two independent methods. The first method is the analytical calculation. A simple 2D structure, ultra-relativistic beam, are assumed and the electro-magnetic field is calculated by field matching. One weak point of this method is that the real collimator has finite longitudinal length. The second method is the HFSS simulation with thin metal wire. With this method, the simulation of a finite length structure is not difficult. But it is known that this method is dangerous when the impedance is comparable or larger than the characteristic impedance, or when DUT has resonant structure. Similarity of the results by two methods shows that these methods are probably valid for this case.

Then the vertical effective impedance is calculated. The definition of it is different from L. Vos's, which is without the denominator. Since the sum of the spectral density is

$$\sum_{p=-\infty}^{\infty} \exp(-\omega_p^2\sigma_\tau^2) \approx \int_{-\infty}^{\infty} dp \exp(-\omega_p^2\sigma_\tau^2) = \frac{\sqrt{\pi}}{\omega_0 k_b \sigma_\tau} = 28.2, \tag{6}$$

the effective impedance in this note is 28.2 times smaller than his.

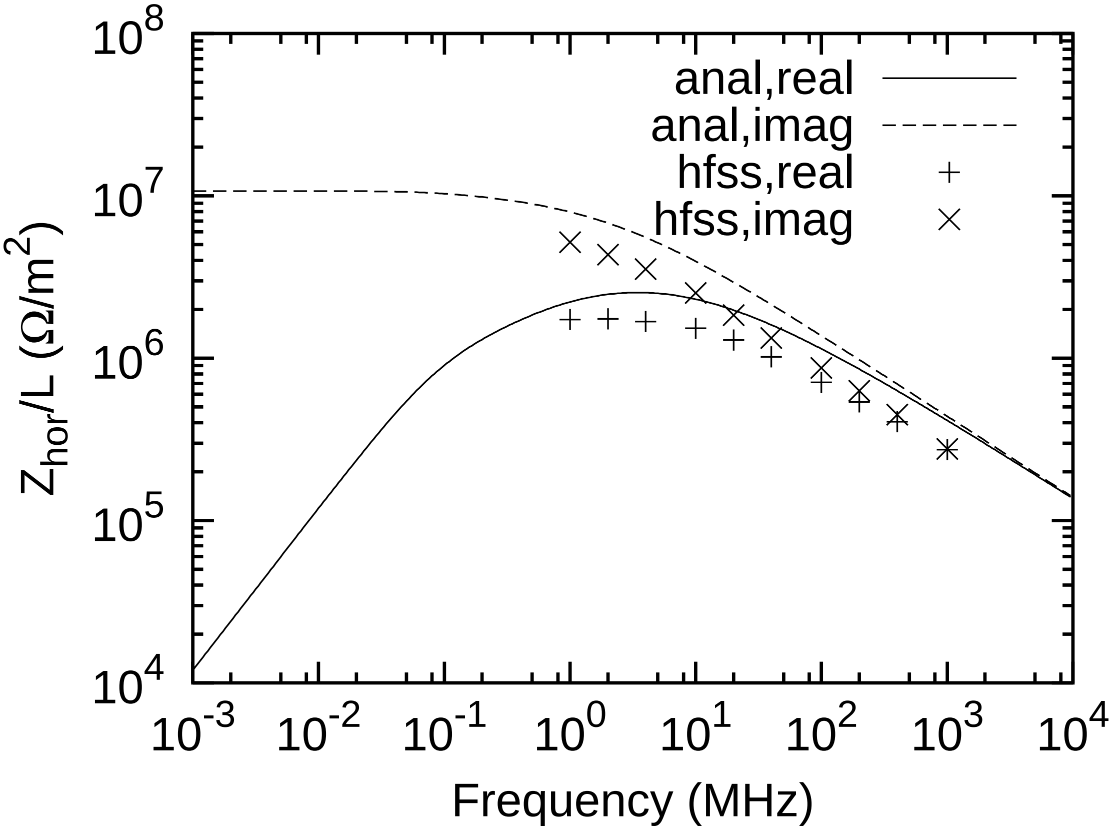

Figure 4: Resistive wall horizontal impedance per unit length. Material is graphite ($\sigma = 7 \times 10^4$ /Ωm). Parameters are $a = 25$ mm, $b_1 = 1.5$ mm, $b_2 = 10$ mm. Lines are by analytical calculations. Symbols are by HFSS simulations.

As shown in Fig. 10, the graphite has big effective impedance because of bad conductivity. By the metal coating the impedance become smaller. Coating of 1 μm and 10 μm give almost the same effective impedance, since the coating will reduce the impedance at 10 MHz or larger frequency significantly, but the impedance at low frequency does not change. So, only the impedance at 8 kHz ($= (1 - \Delta Q_\beta) f_0$) contributes to the effective impedance.

Geometric part of the collimator impedance is also important. There are some structures suggested by F. Caspers[6]. For the evaluation of the geometric impedance, it may be dangerous to use wire methods since there are some resonant structures inside the collimator tank. Maybe eigenmode expansion or use of the current sources are more appropriate methods in frequency domain analysis.

It may be difficult to measure the collimator impedance in test bench using the wire methods. For the resistive wall part, the longitudinal impedance is around 0.1 Ω at 100 MHz. This corresponds to 4×10^{-3} dB in amplitude and 0.03° in phase, assuming $Z_{ch} = 100$ Ω. The transverse impedance of 1 MΩ/m at 100 MHz corresponds to 9×10^{-2} dB in amplitude and 0.6° in phase, which are difficult to measure. The geometric part is also difficult to measure because there should be resonances inside tank.

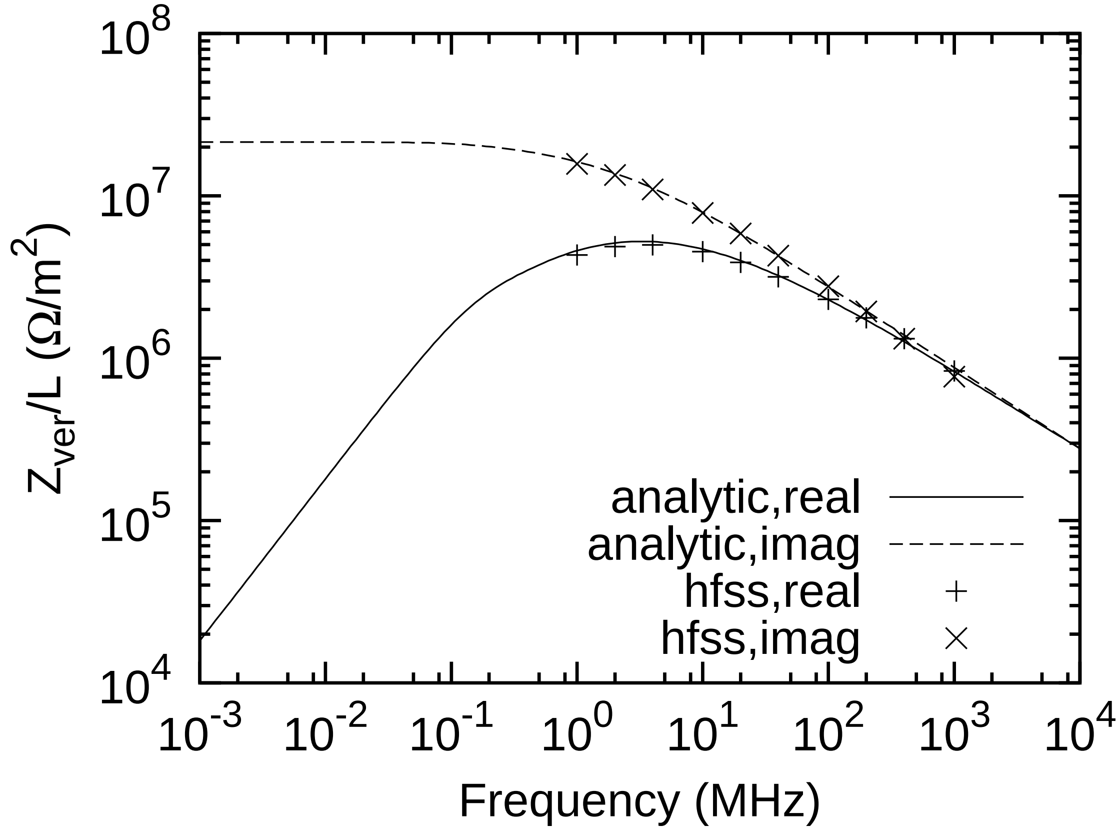

Figure 5: Resistive wall vertical impedance per unit length. Material is graphite ($\sigma = 7 \times 10^4$ /Ωm). Parameters are a = 25 mm, b_1 = 1.5 mm, b_2 = 10 mm. Lines are by analytical calculations. Symbols are by HFSS simulations.

6 Conclusion

Two different methods are used for obtaining the resistive wall impedance of the collimator. They give similar values. Also the vertical effective impedance is calculated.

7 Acknowledgments

The author would like to thank F. Caspers, F. Ruggiero, and L. Vos for valuable discussions.

References

[1] see http://www.ansoft.com

[2] H. Tsutsui, *Some Simplified Models of Ferrite Kicker Magnet for Calculation of Longitudinal Coupling Impedance*, CERN-SL-2000-004 AP (2000).

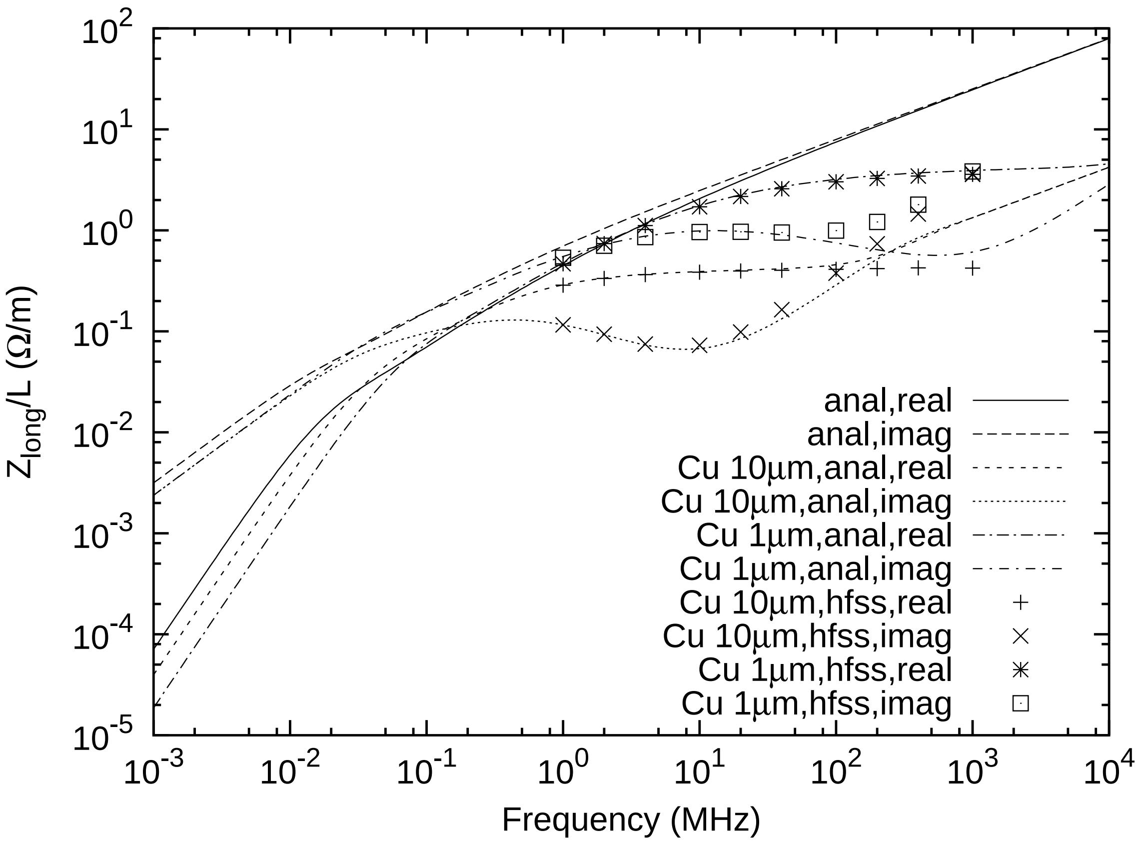

Figure 6: Resistive wall longitudinal impedance per unit length. Material is graphite ($\sigma = 7 \times 10^4$ /Ωm) with copper coating of thickness $\xi = 1$ μm and 10 μm. Parameters are $a = 25$ mm, $b_1 = 1.5$ mm, $b_2 = 1.5$ mm $+ \xi$, $b_3 = 10$ mm. Lines are by analytical calculations. Symbols are by HFSS simulations.

[3] H. Tsutsui and L. Vos, *Transverse Coupling Impedance of a Simplified Ferrite Kicker Magnet Model*, LHC Project Note 234 (2000).

[4] see http://www.wolfram.com.

The notebooks are in /afs/cern.ch/user/h/htsutsui/public/collimator.

[5] F. Caspers, *Bench Measurements* in *Handbook of Accelerator Physics and Engineering*, edited by A. W. Chao and M. Tigner, World Scientific (1998).

[6] F. Caspers, private communications.

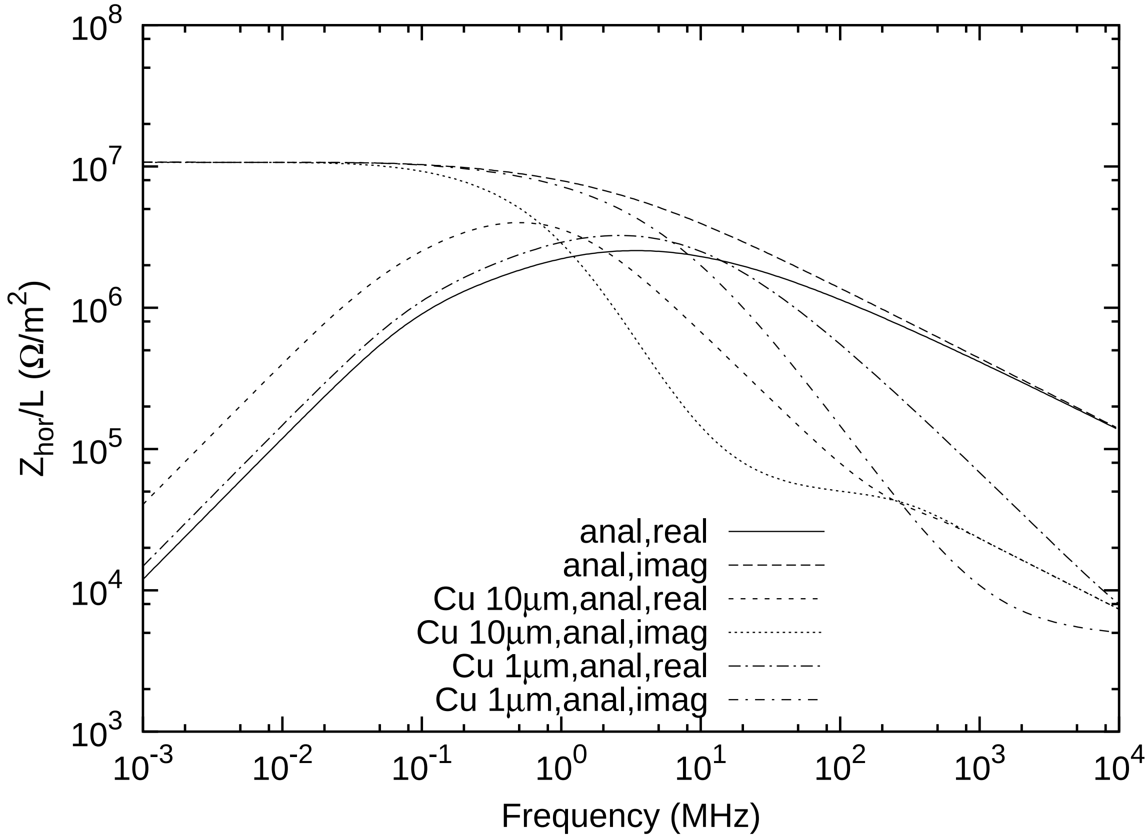

Figure 7: Resistive wall horizontal impedance per unit length. Material is graphite ($\sigma = 7 \times 10^4$ $/\Omega$m) with copper coating of thickness $\xi = 1$ μm and 10 μm. Parameters are $a = 25$ mm, $b_1 = 1.5$ mm, $b_2 = 1.5$ mm $+ \xi$, $b_3 = 10$ mm. Lines are by analytical calculations. HFSS simulations were not done.

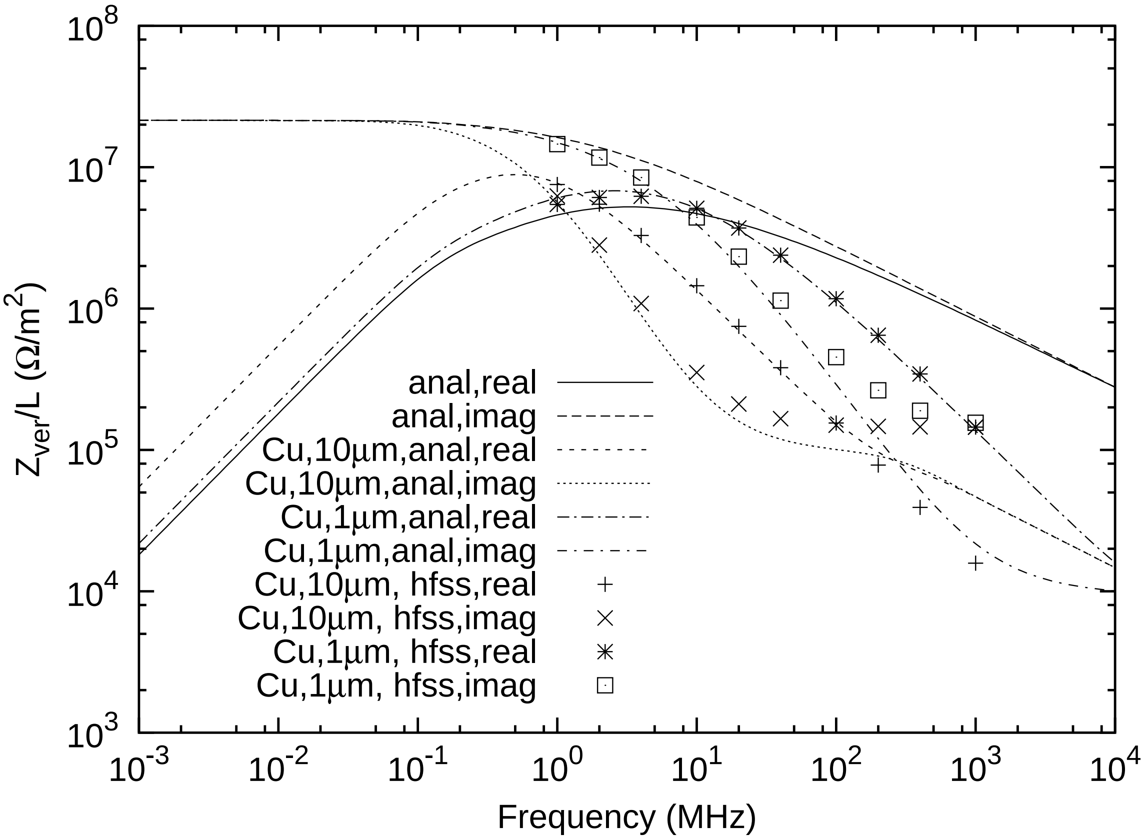

Figure 8: Resistive wall vertical impedance per unit length. Material is graphite ($\sigma = 7 \times 10^4$ /Ωm) with copper coating of thickness $\xi = 1$ μm and 10 μm. Parameters are $a = 25$ mm, $b_1 = 1.5$ mm, $b_2 = 1.5$ mm $+ \xi$, $b_3 = 10$ mm. Lines are by analytical calculations. Symbols are by HFSS simulations.

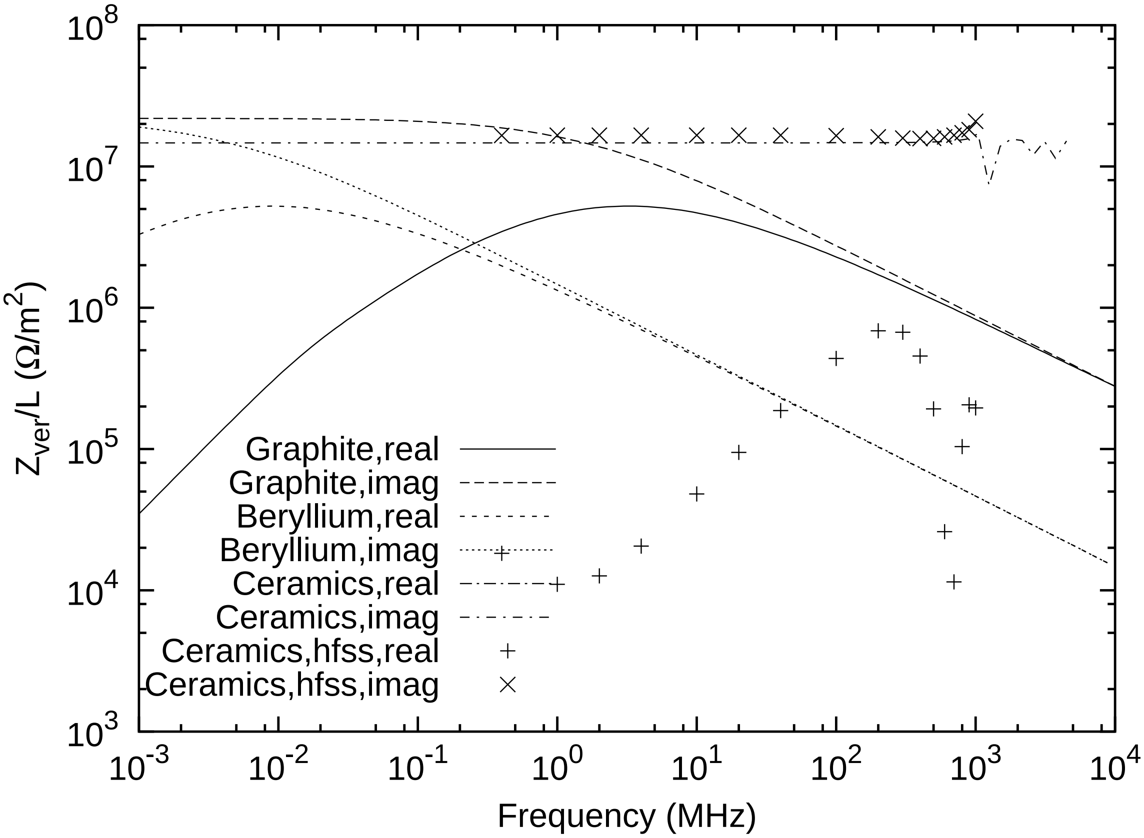

Figure 9: Vertical resistive wall impedance per unit length. Beryllium and ceramics are assumed for the collimator material. Parameters are $a = 25$ mm, $b_1 = 1.5$ mm, $b_2 = 30$ mm.

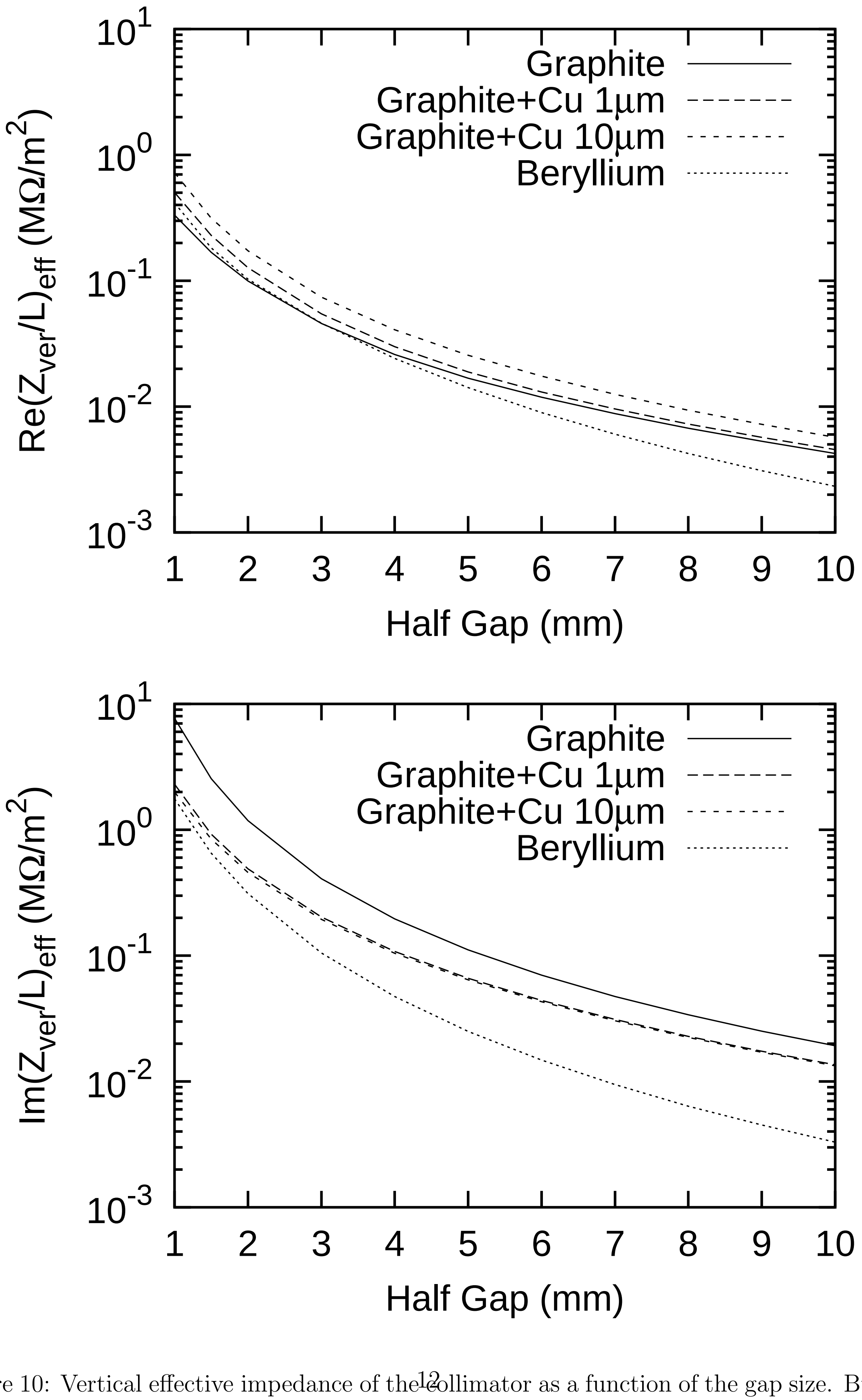

Figure 10: Vertical effective impedance of the collimator as a function of the gap size. Bunch length is 0.25 nsec.