FAST VERTICAL SINGLE-BUNCH INSTABILITY DUE TO THE BROAD-BAND E$^-$CLOUD INDUCED IMPEDANCE : STABILIZATION BY CHROMATICITY

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- **In the SPS at injection**
- **In the LHC at injection and top energy**

Formalism used (1/3)

$$N_{b,th} = \frac{8\pi Q_{y0} |\eta| \varepsilon_l}{e\beta^2 c} \times \frac{f_r}{|Z_y|} \times \left(1 + \frac{f_{\xi_y}}{f_r} \right)$$

- cf paper "Effect of bunch length, chromaticity, and linear coupling on the transverse mode-coupling instability due to the electron cloud", E. Metral, CERN/PS 2002-009 (AE), ECLOUD'02 Workshop, April 15-18 2002, CERN.
- This is the same formula as the one used by Ohmi, Zimmermann and Perevedentsev in "Study of the Fast Head-Tail Instability Caused by Electron Cloud", CERN-SL-2001-011 AP, 2001.
- In the paper "Wake field of the e^{-}cloud", SLAC-PUB-9025, 2001, S. Heifets confirmed the dependence of the wake on the beam parameters found above with tracking simulations.

Formalism used (2/3)

- **Scaling of the peak impedance and resonance frequency with the beam parameters**

$$\left| Z_y \right| = \left| Z_{y0} \right| \times \frac{\sigma_z}{\sigma_{z0}} \times \frac{\sigma_{y0} \left(\sigma_{x0} + \sigma_{y0} \right)}{\sigma_y \left(\sigma_x + \sigma_y \right)}$$

$$f_r = f_{r0} \times \sqrt{\frac{\sigma_{z0}}{\sigma_z}} \times \sqrt{\frac{N_b}{N_{b0}}} \times \sqrt{\frac{\sigma_{y0} \left(\sigma_{x0} + \sigma_{y0} \right)}{\sigma_y \left(\sigma_x + \sigma_y \right)}}$$

Formalism used (3/3)

$\Rightarrow$

$$N_{b,th} = \frac{f^2}{4\,\tau_b^3} \times \left(1 + \sqrt{1 + \frac{4\,g\,\xi_y\,\tau_b^2}{f}} \right)^2$$

$$f = \frac{8\,\pi\,Q_{y0}\,|\eta|\,\varepsilon_l\,f_{r0}\,\tau_{b0}^{3/2}}{e\,\beta^2\,c\,|Z_{y0}|\sqrt{N_{b0}}} \times \sqrt{\frac{\sigma_y\left(\sigma_x + \sigma_y\right)}{\sigma_{y0}\left(\sigma_{x0} + \sigma_{y0}\right)}}$$

$$g = \frac{Q_{y0}\,f_0\,\sqrt{N_{b0}}}{\eta\,f_{r0}\,\tau_{b0}^{1/2}} \times \sqrt{\frac{\sigma_y\left(\sigma_x + \sigma_y\right)}{\sigma_{y0}\left(\sigma_{x0} + \sigma_{y0}\right)}}$$

In the SPS at injection (1/2)

Beam parameters used to evaluate the e^-cloud induced impedance

Bunch population	N_{b0} = 7.5×10^{10} p/b
Bunch length	σ_{z0} = 30 cm
Horizontal beam size	σ_{x0} = 5 mm
Vertical beam size	σ_{y0} = 3 mm

$\Rightarrow$

$$\left| Z_{y0} \right| = 20 \text{ M}\Omega/\text{m}$$

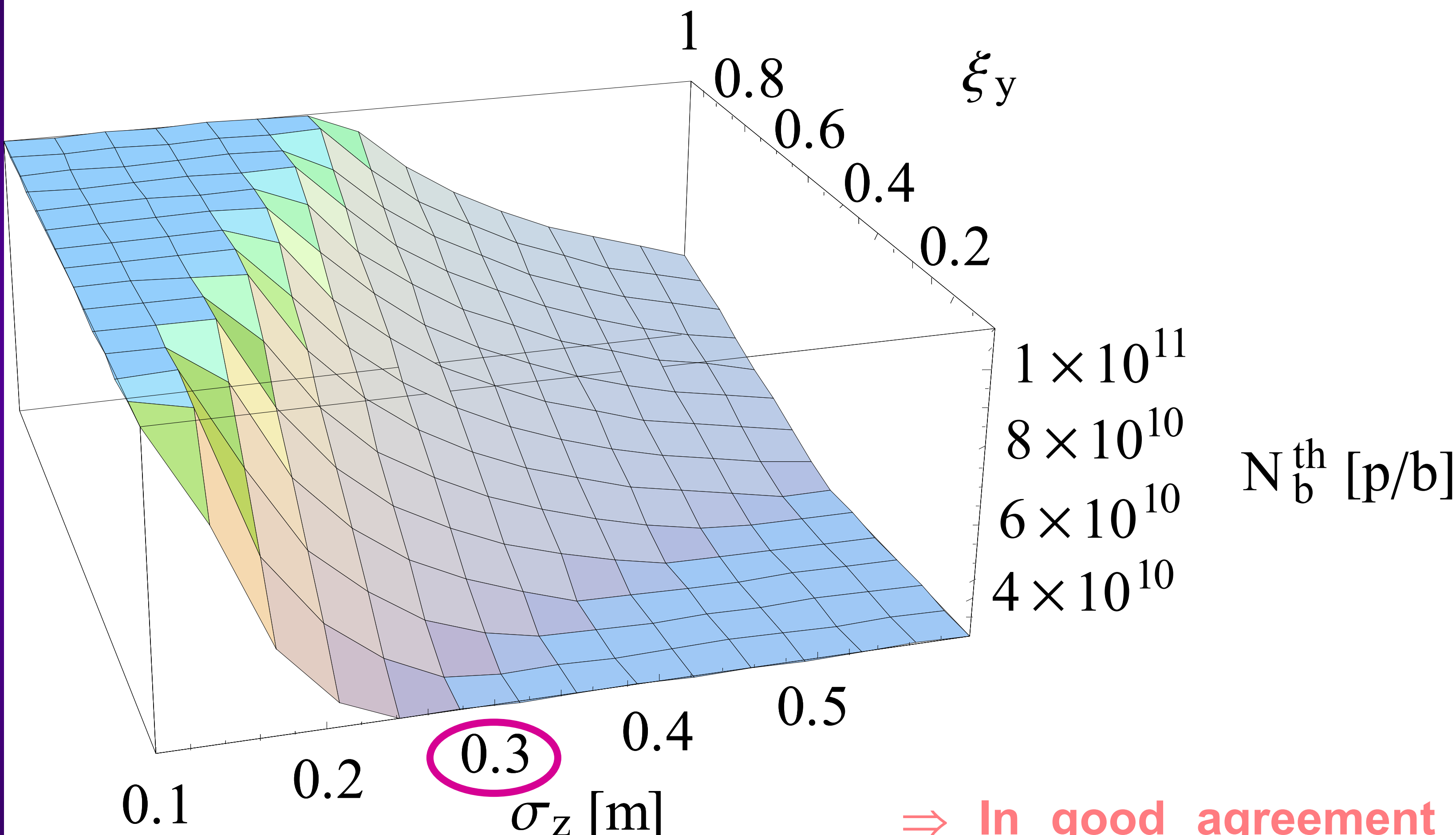
$$f_{r0} = 220 \text{ MHz}$$

$$Q \approx 1$$

Real beam parameters

Average machine radius	R = 1100 m
Slippage factor	η = 5.5×10^{-4}
Beam energy	E = 26 GeV
Bunch population	N_b = 11×10^{10} p/b
Longitudinal emittance	ϵ_l = 0.35 eVs
Bunch length	σ_z = 30 cm
Horizontal beam size	σ_x = 2.6 mm
Vertical beam size	σ_y = 1.9 mm
Vertical tune	Q_y ~ 26.7

In the SPS at injection (2/2)



⇒ **In good agreement with the observations**

In the LHC at injection (1/3)

Beam parameters used to evaluate the e^{-}cloud induced impedance

Bunch population	N_{b0} = 11.5×10^{10} p/b
Bunch length	σ_{z0} = 17.5 cm
Horizontal beam size	σ_{x0} = 1 mm
Vertical beam size	σ_{y0} = 1 mm

$$\Rightarrow \left| Z_{y0} \right| = 223\ \mathrm{M\Omega/m}$$

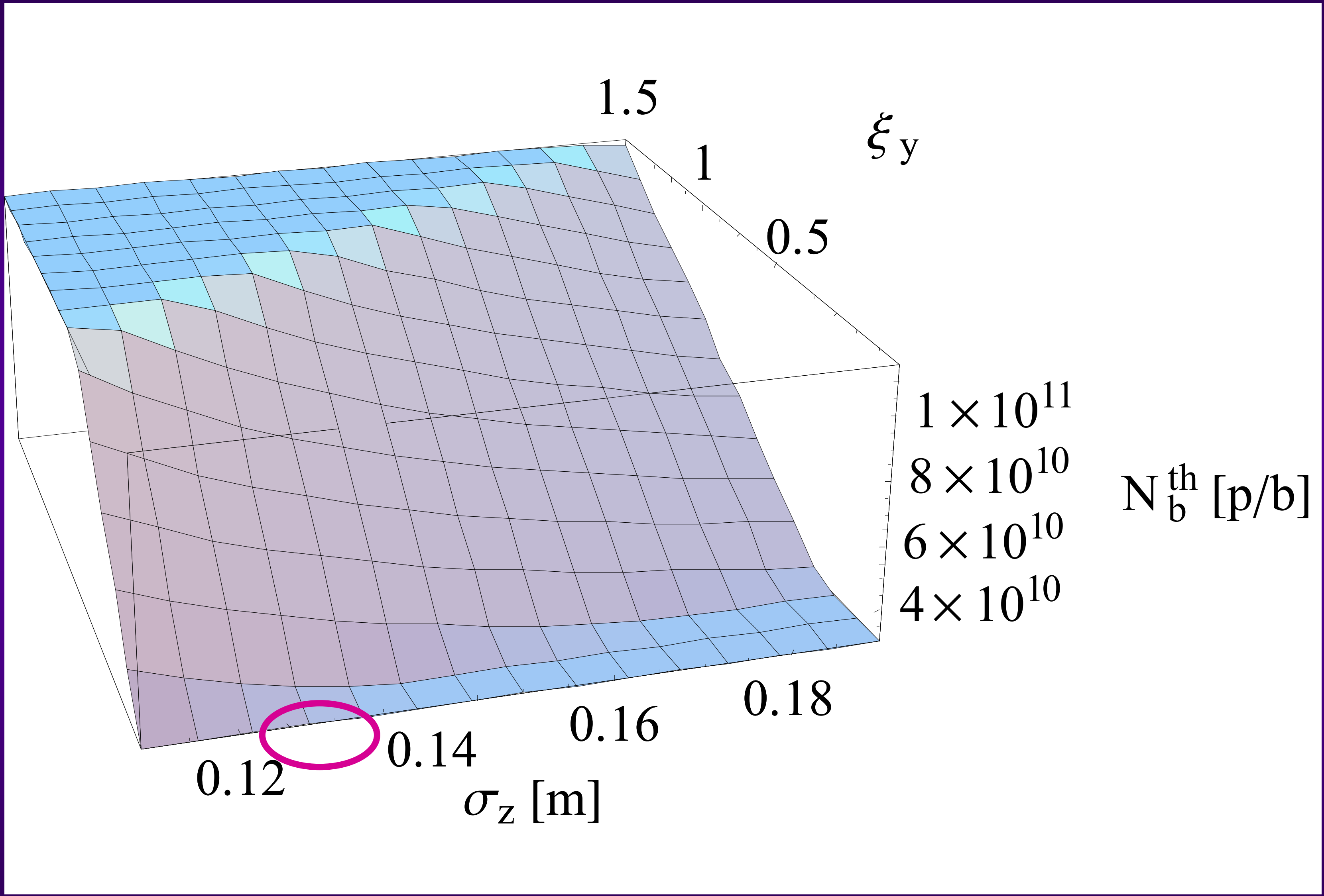
$$f_{r0} = 0.91\ \mathrm{GHz}$$

$$Q \approx 1$$

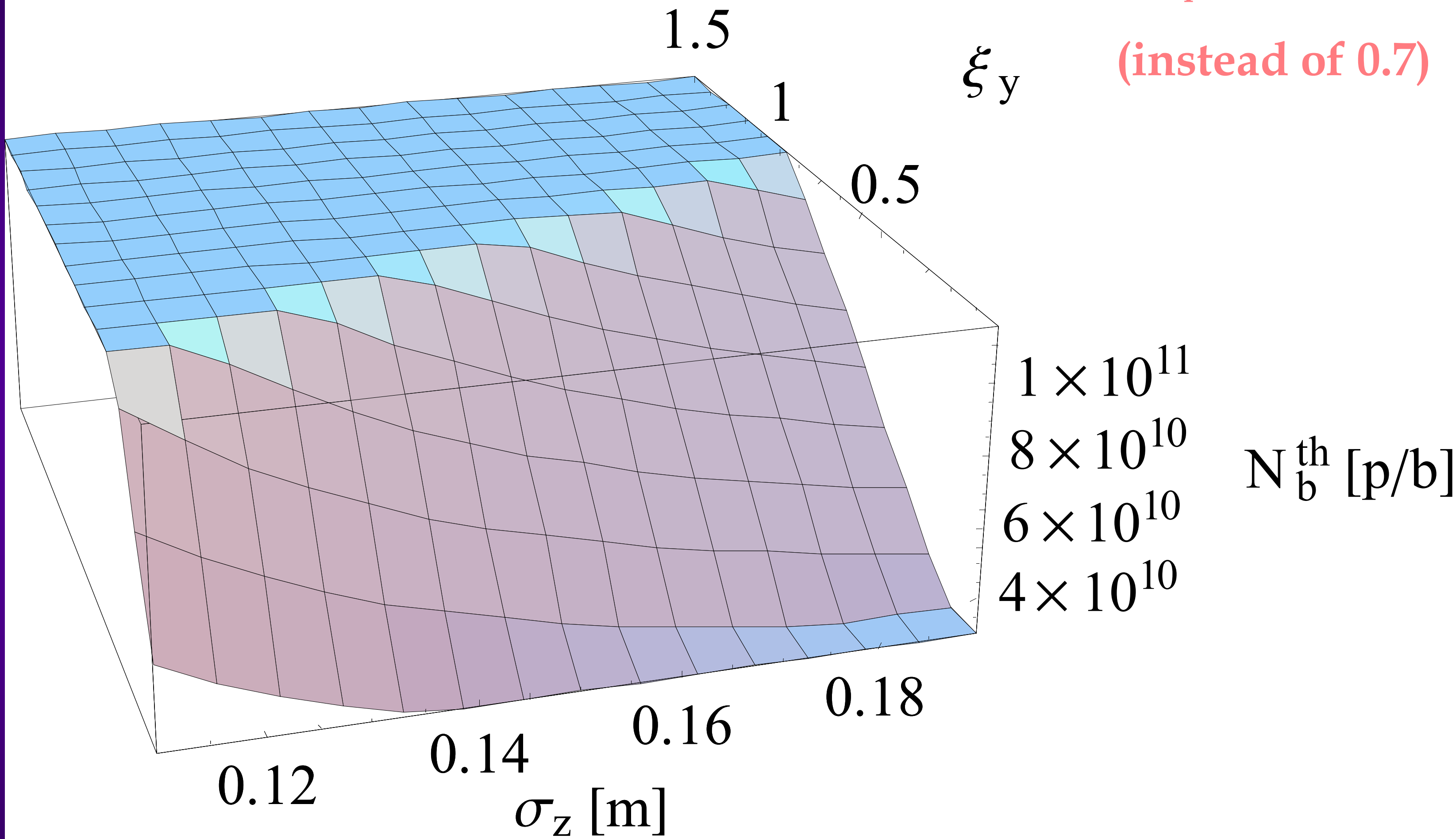
Real beam parameters

Average machine radius	R = 4242.893 m
Slippage factor	$\eta = 3\times10^{-4}$
Beam energy	E = 450 GeV
Bunch population	N_b = 11.5×10^{10} p/b
Longitudinal emittance	ϵ_l = 0.7 eVs
Bunch length	σ_z = 13 cm
Horizontal beam size	σ_x = 1 mm
Vertical beam size	σ_y = 1 mm
Vertical tune	Q_y ~ 59.32

In the LHC at injection (2/3)



In the LHC at injection (3/3)



If $\epsilon_l = 1$ eVs

(instead of 0.7)

In the LHC at top energy (1/2)

Beam parameters used to evaluate the e⁻cloud induced impedance

Bunch population	$N_{b0} = 11.5 \times 10^{10}$ p/b
Bunch length	$\sigma_{z0} = 7.5$ cm
Horizontal beam size	$\sigma_{x0} = 0.3$ mm
Vertical beam size	$\sigma_{y0} = 0.3$ mm

$$\Rightarrow \left| Z_{y0} \right| = 487 \text{ M}\Omega/\text{m}$$

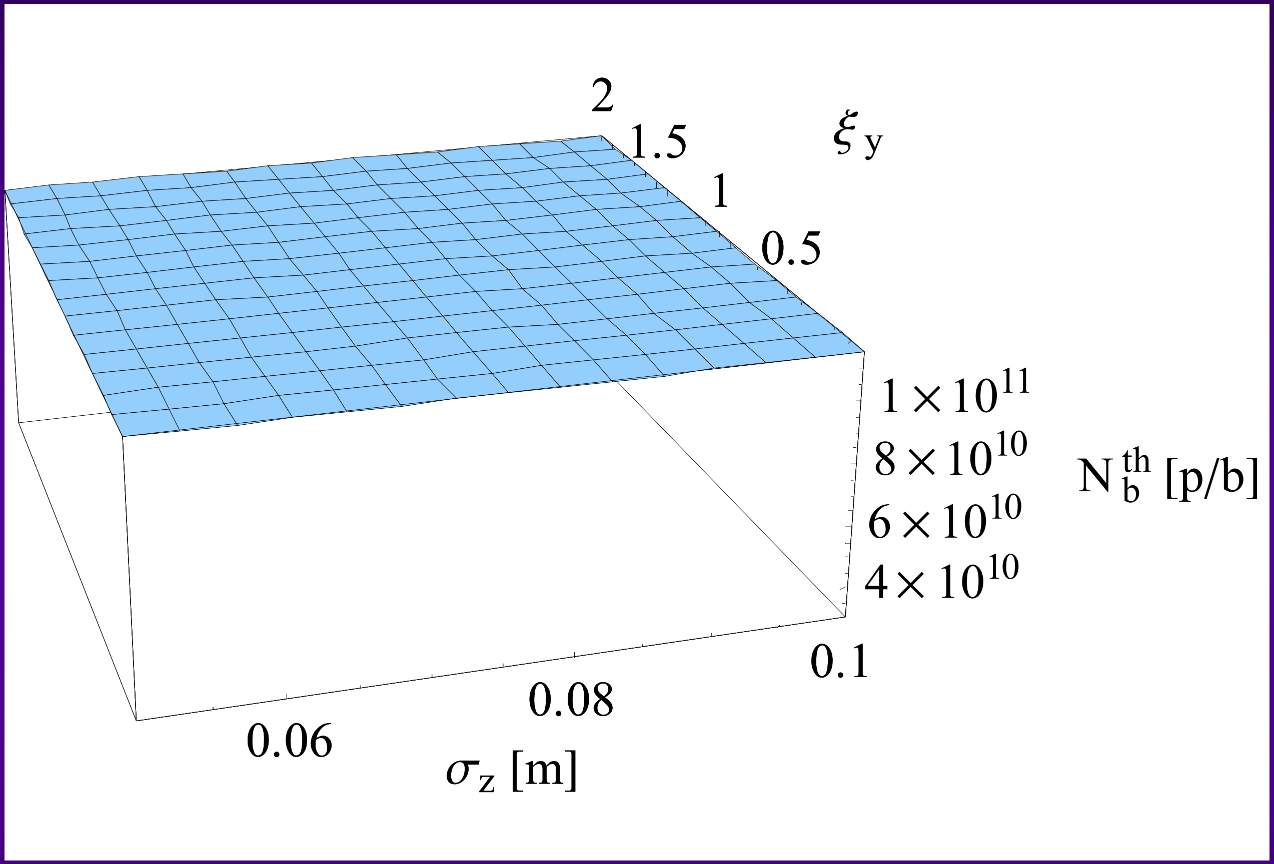
$$f_{r0} = 4.67 \text{ GHz}$$

$$Q \approx 1$$

Real beam parameters

Average machine radius	R = 4242.893 m
Slippage factor	$\eta = 3 \times 10^{-4}$
Beam energy	E = 7000 GeV
Bunch population	$N_b = 11.5 \times 10^{10}$ p/b
Longitudinal emittance	$\epsilon_l = 2.5$ eVs
Bunch length	$\sigma_z = 7.5$ cm
Horizontal beam size	$\sigma_x = 0.3$ mm
Vertical beam size	$\sigma_y = 0.3$ mm
Vertical tune	$Q_y \sim 59.32$

In the LHC at top energy (2/2)



$\Rightarrow$ **No instability even for 0 chromaticity. The intensity threshold for 0 chromaticity is ~4×10^{11} p/b**

Conclusion

- **Can we push the chromaticity to ~1 at injection in the LHC, or to ~0.5 if one can have a longitudinal emittance of 1 eVs ?**

- **If yes, the beam should be stable (concerning the fast vertical single-bunch instability induced by the e-cloud) according to this model, which is in good agreement with the SPS observations !**