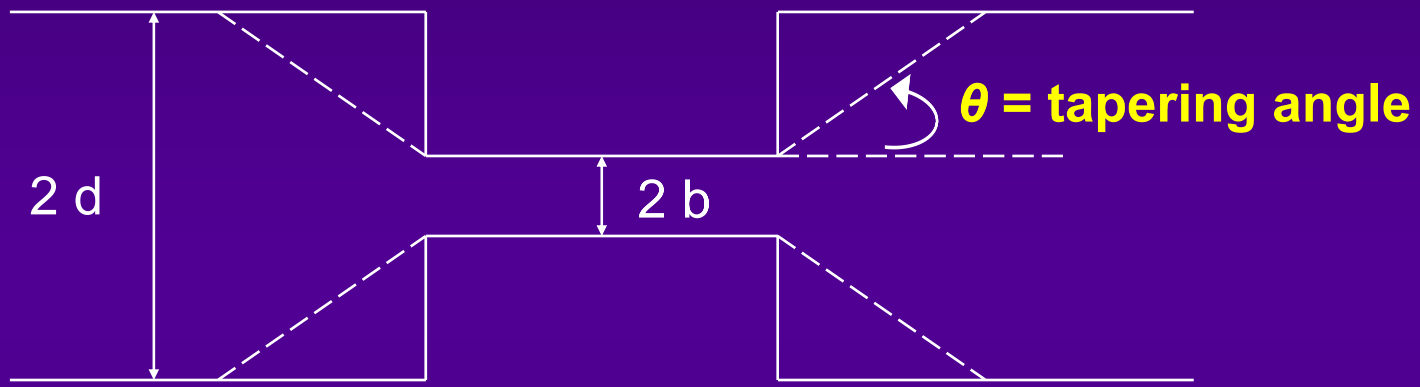
LHC GEOMETRICAL COLLIMATOR IMPEDANCE (from L. Vos' formalism)

E. Metral

⇒ **The trapped modes are not treated here and have to be added**

Geometry of 1 circular collimator



Longitudinal impedance (1/3)

$$Z_l(\omega) = \begin{pmatrix} 0 \ \text{ if } |\omega| < \omega_{c0} \\ A \ \text{ if } |\omega| \geq \omega_{c0} \end{pmatrix} + j \frac{A}{\pi} \operatorname{Log} \left| \frac{1 + \dfrac{\omega}{\omega_{c0}}}{1 - \dfrac{\omega}{\omega_{c0}}} \right|$$

$$A = \frac{Z_0}{4} \operatorname{Log}\left(\frac{d}{b}\right) \sin\theta$$

$$\omega_{c0} = \tau_0 \frac{c}{d}$$ = cut-off frequency of the large pipe

$$Z_0 = 120\,\pi$$

$$\tau_0 = 2.4$$ = 1st root of the 0th order Bessel function

Longitudinal impedance (2/3)

- $\theta = 15^\circ$ $\Rightarrow$ $\sin\theta = 0.26$ $\Rightarrow$ **The impedance is reduced by a factor 4**

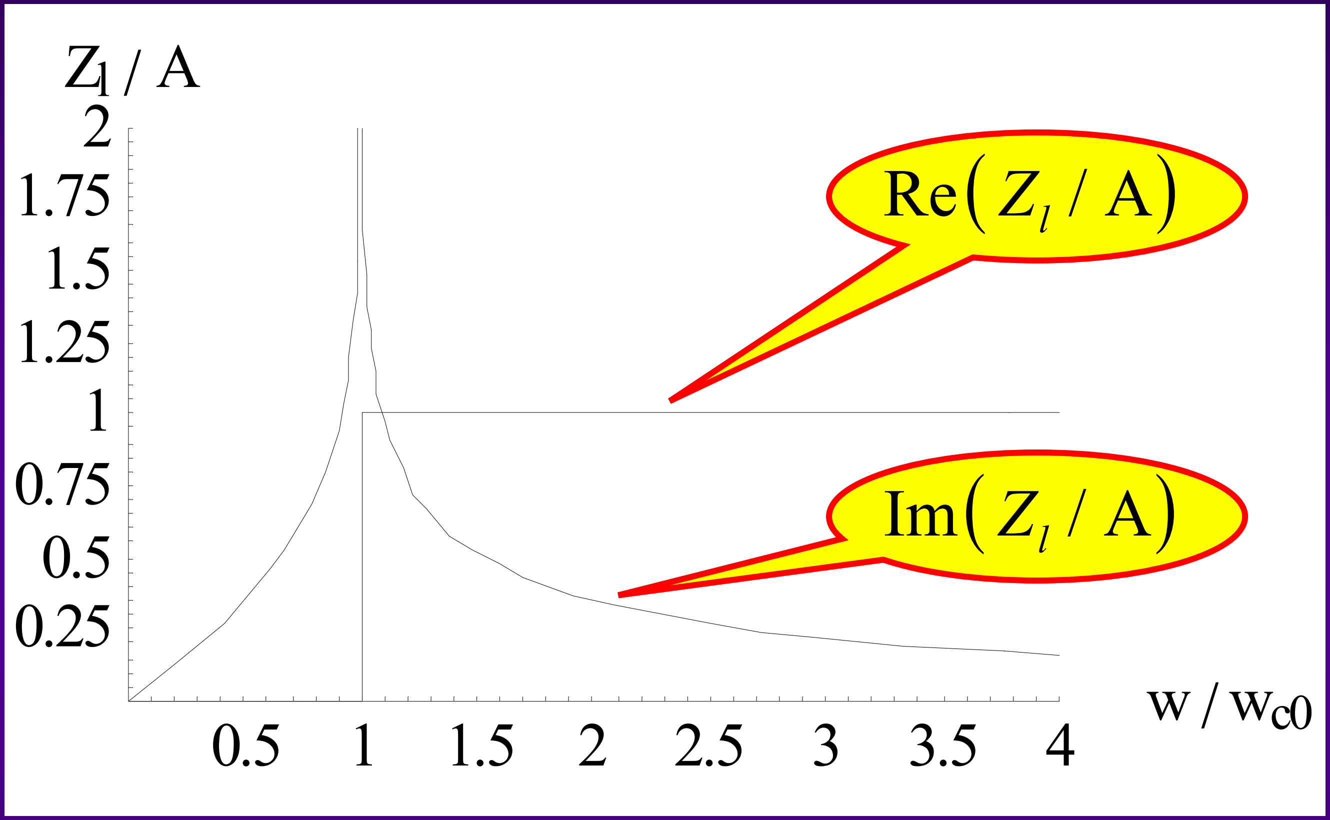
- **For** $\theta = 90^\circ$ $\Rightarrow$ **i.e. no tapering**

  - **L. Vos obtains** $$A = \frac{Z_0}{4} \, \mathrm{Log}\left(\frac{d}{b}\right)$$

  - **In the CAS course (CERN 95-06) "Wake fields and Impedance" by L. Palumbo et al., it is said p. 364, Eq. (150) :**

$$A \approx \frac{Z_0}{\pi} \, \mathrm{Log}\left(\frac{d}{b}\right)$$

Longitudinal impedance (3/3)



- **Numerical application for an LHC collimator**

$A \approx 220 \sin\theta \ \Omega$

$b = 2 \text{ mm}$ $d = 2 \text{ cm}$ $\Rightarrow$ $f_{c0} = 5.7 \text{ GHz}$

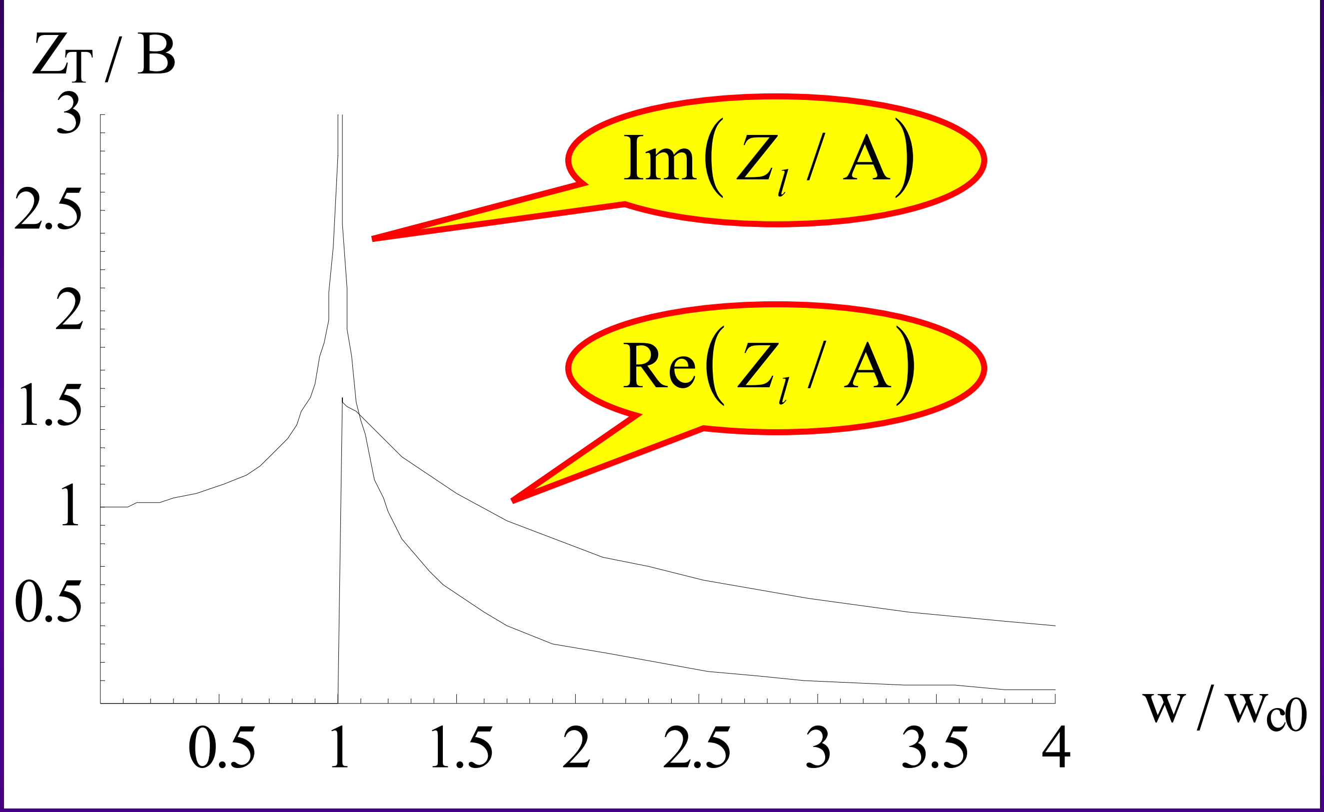
$Z_l / n \approx 0.3 \sin\vartheta \ \text{m}\Omega$

Transverse impedance (1/2)

$$Z_T(\omega) = \begin{pmatrix} 0 \ \text{ if } |\omega| < \omega_{c0} \\ \dfrac{\pi B \omega_{c0}}{2\omega} \ \text{ if } |\omega| \geq \omega_{c0} \end{pmatrix} + j \frac{B \omega_{c0}}{2\omega} \mathrm{Log} \left| \frac{1 + \dfrac{\omega}{\omega_{c0}}}{1 - \dfrac{\omega}{\omega_{c0}}} \right|$$

$$B = \frac{c Z_0}{\pi b^2 \omega_{c0}} \mathrm{Log}\left(\frac{d}{b}\right) \sin\theta$$

Transverse impedance (2/2)



- **Numerical application for an LHC collimator**

$$B \approx 0.6 \sin\theta \ \ \mathrm{M\Omega/m}$$