LHC collimator test in the SPS - expected impedance effects & measurements

I assume the following parameters

variable	symbol	value
conductivity	σ	$7 \times 10^4\ \Omega^{-1}\mathrm{m}^{-1}$
collimator length	L_{coll}	1 m
collimator thickness	d	30 cm
collimator gap	b	variable
horizontal beta function	β_x	23.2 m
vertical beta function	β_y	94.8 m
dispersion	D_x	-0.2 m
rms bunch length	σ_z	10.5 cm
bunch population	N_b	8.5×10^{10}
beam energy	E	270 GeV
circumference	C	6900 m
betatron tune	Q_β	26.13

$$Z_\perp(\omega) = -\frac{iZ_0}{\pi b^2} \frac{\left(1+\frac{1}{\lambda_0 b}\right)\tanh(\lambda_0 d)}{\left(2+\frac{1}{\lambda_0 b}+\lambda_0 b\right)\tanh(\lambda_0 d)}$$

Burov-Lebedev impedance

where $$\lambda_0(\omega) = -(1 - i\,\mathrm{sgn}(\omega))\sqrt{\mu_0 \sigma |\omega| / 2}$$

$$\Delta y' = -\frac{N_b r_p y}{2\pi\gamma}\left[\frac{4\pi}{Z_0 c}\right]\int_{-\infty}^{\infty} d\omega\, \mathrm{Im}\, Z_\perp(\omega) \left|\tilde{\rho}(\omega)\right|^2$$

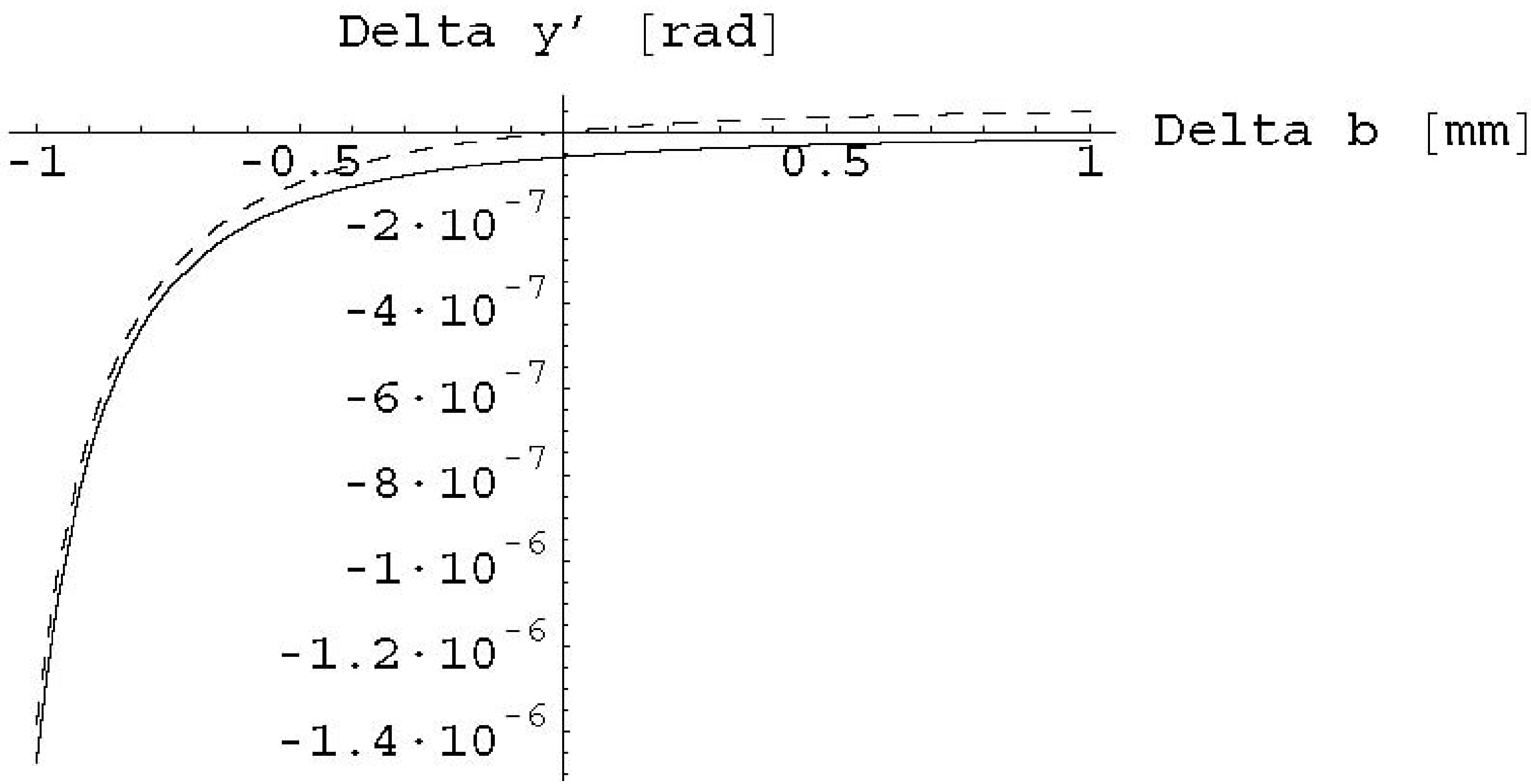
$$= -\frac{N_b e}{2\sqrt{\pi}\sigma_z (E_b / e)} \mathrm{Im}\, Z_{\perp,\mathrm{eff}}(0)\, y \equiv -K_{\mathrm{eff}}\, y$$

transverse deflection of off-center beam

nonlinear deflection at large offsets (Piwinski)

$$y \to \frac{a}{\pi}\left(\frac{\pi y / a + \sin(\pi y / a)}{1 + \cos(\pi y / a)}\right)$$

predicted orbit deflection

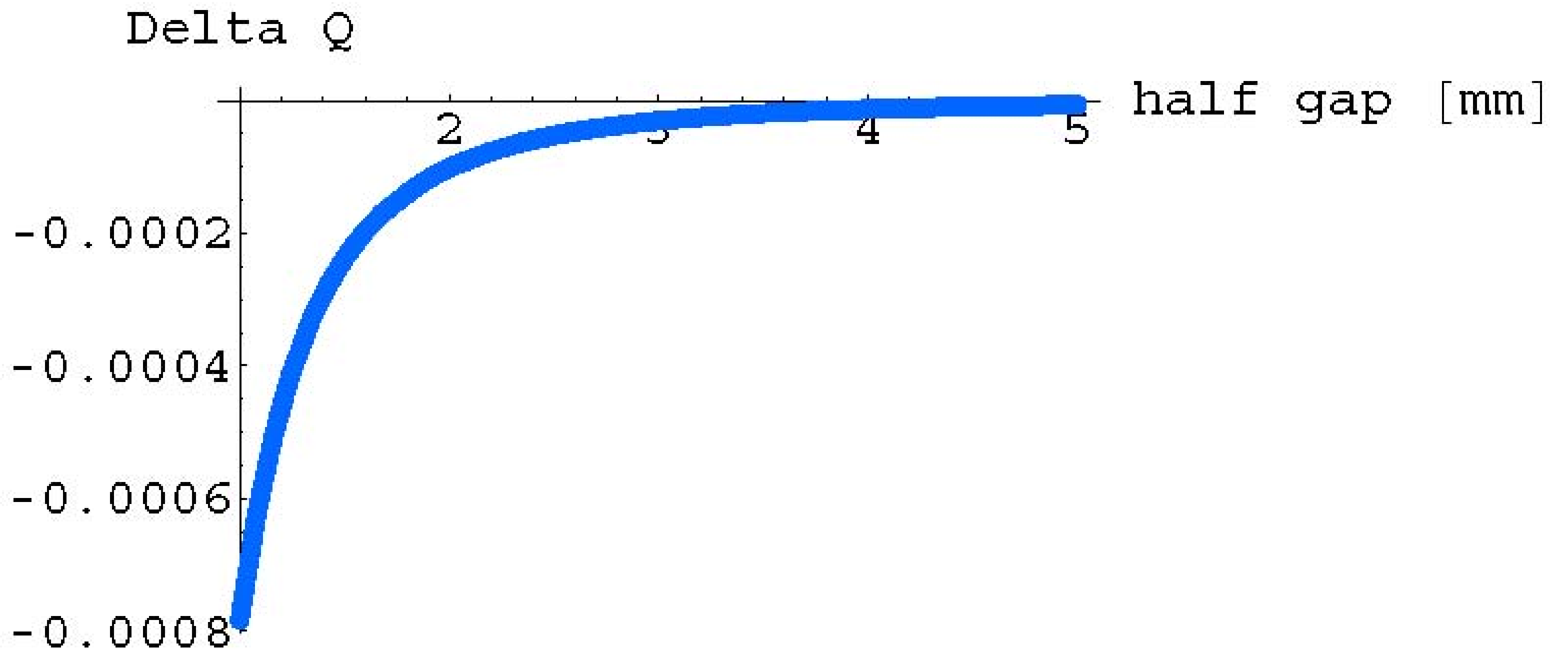


Collimator kick versus jaw position on one side: initial beam-jaw distance is 1.25 mm; the jaw on one side is then moved by 'Delta b'; either the opposite jaw is held fixed at a distance 1.25 mm (dotted), or it is kept at an infinite distance (solid).

Noticable deflection above 1 μrad is seen only when the beam is closer than 0.4 mm to the moving jaw. No deflection was measured in the experiment. OK!

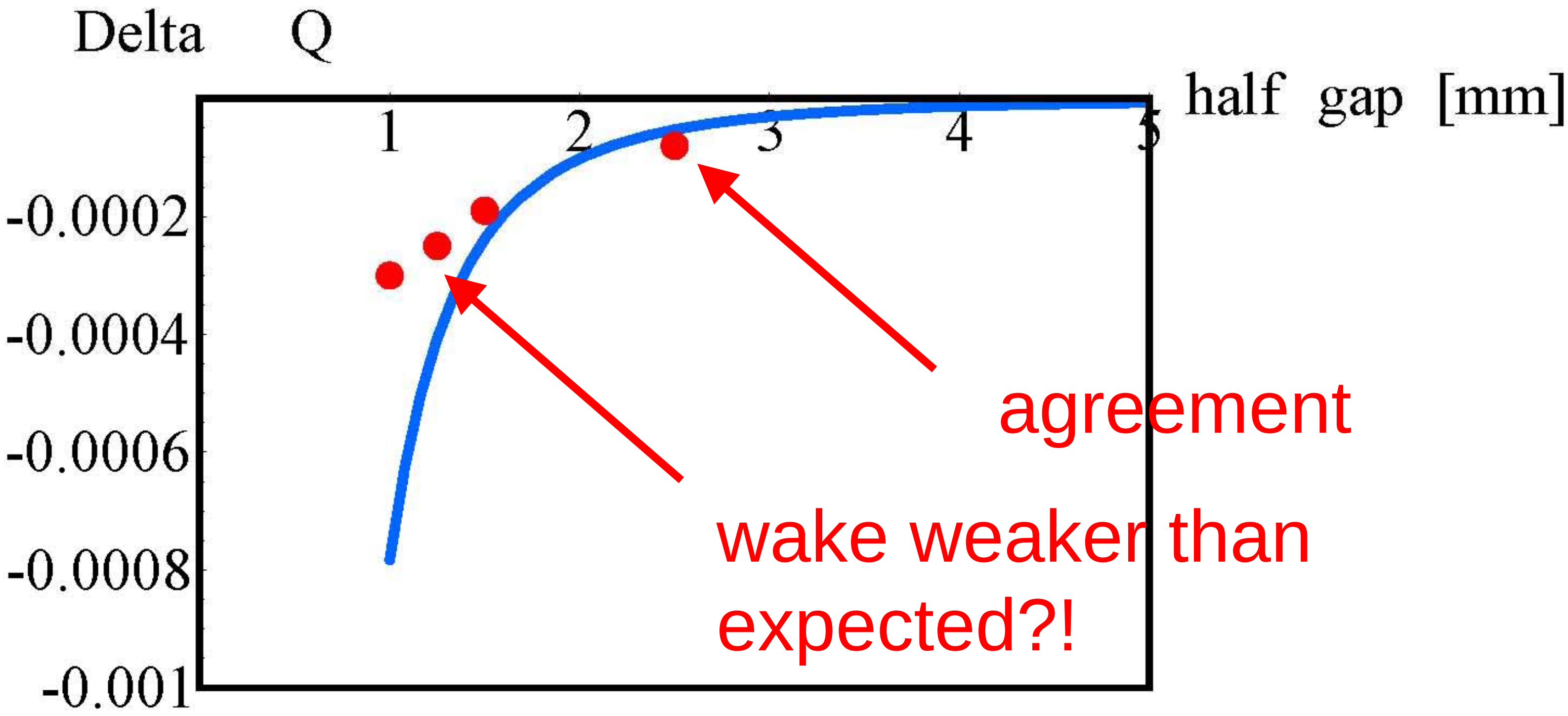
Calculation assumes Burov-Lebedev resistive-wall impedance model.

predicted tune shift



Expected tune shift versus half gap size for a pencil-like beam. Here the two jaws are closed symmetrically.
Calculation assumes Burov-Lebedev resistive-wall impedance model.

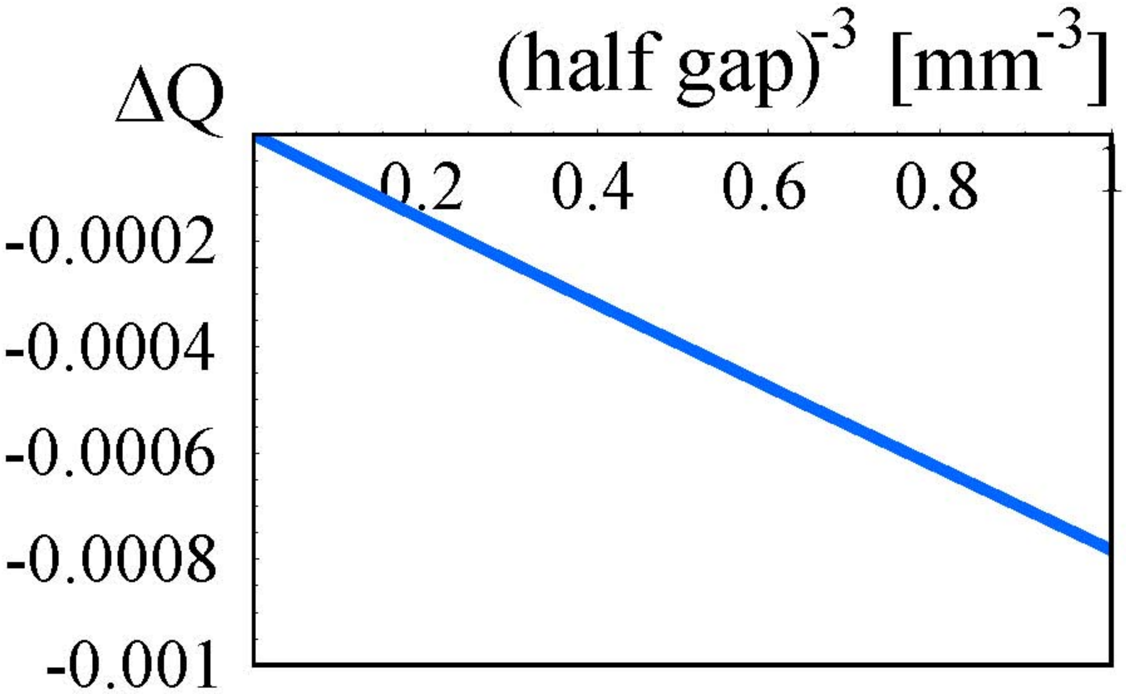
comparison of measured & predicted ΔQ



Expected tune shift of a pencil beam of constant intensity of 8.5×10^{10} protons, on which the measured data (from Marek Glasior's APC talk) are superimposed.

dependence on collimator gap

Ralph remarked that the tune shift should not scale as the inverse cubic aperture. However, in our calculation for the SPS collimator effect, based on the Burov-Lebedev impedance, *we do compute a pure inversely cubic dependence.*



Expected tune shift for the SPS collimator experiment as a function of the inverse third power of the collimator half gap. The dependence ~$1/b^3$ differs from observed dependence ~1/b

form factor for the tune shift at small gaps

$$V(y,y_0) = -\kappa f(\tau)\left[\frac{y_+ \sin y_+}{1+\cos y_+} + \frac{y_- \sin y_-}{1-\cos y_-}\right]$$

wake potential (Piwinski)

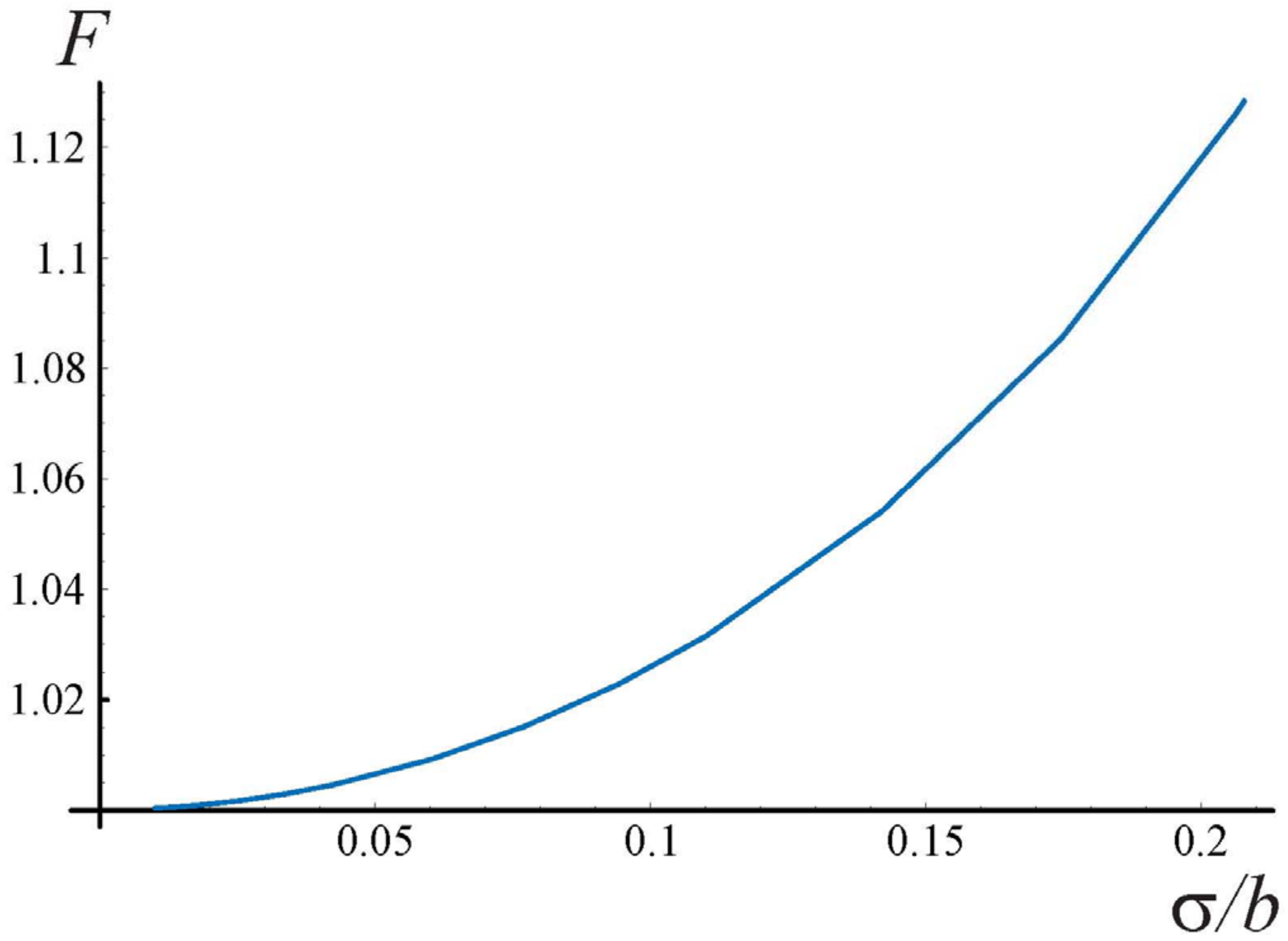
where $$y_- \equiv \frac{\pi}{2b}(y_t + y_0), \qquad y_+ \equiv \frac{\pi}{2g}(y_t - y_0)$$

in expressions for the wake field kick, on the right-hand side should be replaced by

$$y \to \frac{2b^2}{\pi^2} \frac{\frac{1}{2\pi\sigma^2}\int_{-b}^{b}\int_{-b}^{b}\left[\frac{\partial}{\partial y_t}\left[\frac{y_+ \sin y_+}{1+\cos y_+} + \frac{y_- \sin y_-}{1+\cos y_-}\right]\right] e^{-(y_t-y)^2/2\sigma^2} e^{-(y_0-y)^2/2\sigma^2} dy_t dy_0}{\frac{1}{\sqrt{2\pi}\sigma}\int_{-b}^{b} e^{-(y_t-y)^2/2\sigma^2} dy_t}$$

$$= \frac{b}{\pi^3} \frac{\frac{1}{\tilde{\sigma}^2}\int_{-1}^{1}\int_{-1}^{1}\left[\frac{\partial}{\partial \tilde{y}_t}\left[\frac{\tilde{y}_+ \sin \tilde{y}_+}{1+\cos \tilde{y}_+} + \frac{\tilde{y}_- \sin \tilde{y}_-}{1+\cos \tilde{y}_-}\right]\right] e^{-(\tilde{y}_t-\tilde{y})^2/2\tilde{\sigma}^2} e^{-(\tilde{y}_0-\tilde{y})^2/2\tilde{\sigma}^2} d\tilde{y}_t d\tilde{y}_0}{\frac{1}{\sqrt{2\pi}\tilde{\sigma}}\int_{-1}^{1} e^{-(\tilde{y}_t-\tilde{y})^2/2\tilde{\sigma}^2} d\tilde{y}_t} \equiv yF\left(\frac{y}{b},\frac{\sigma}{b}\right)$$

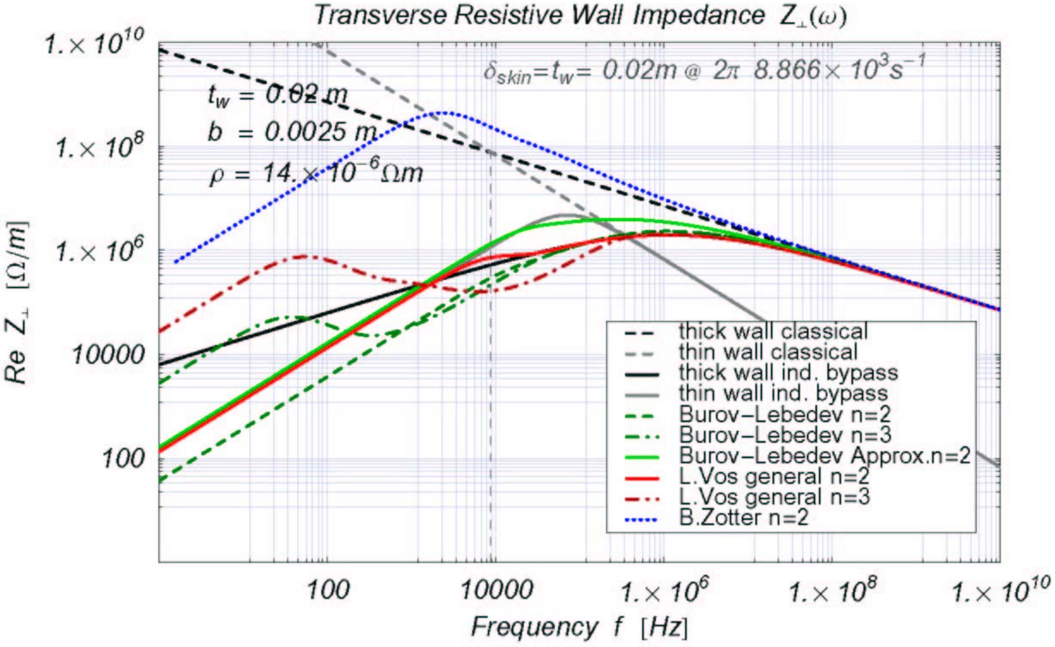
form factor



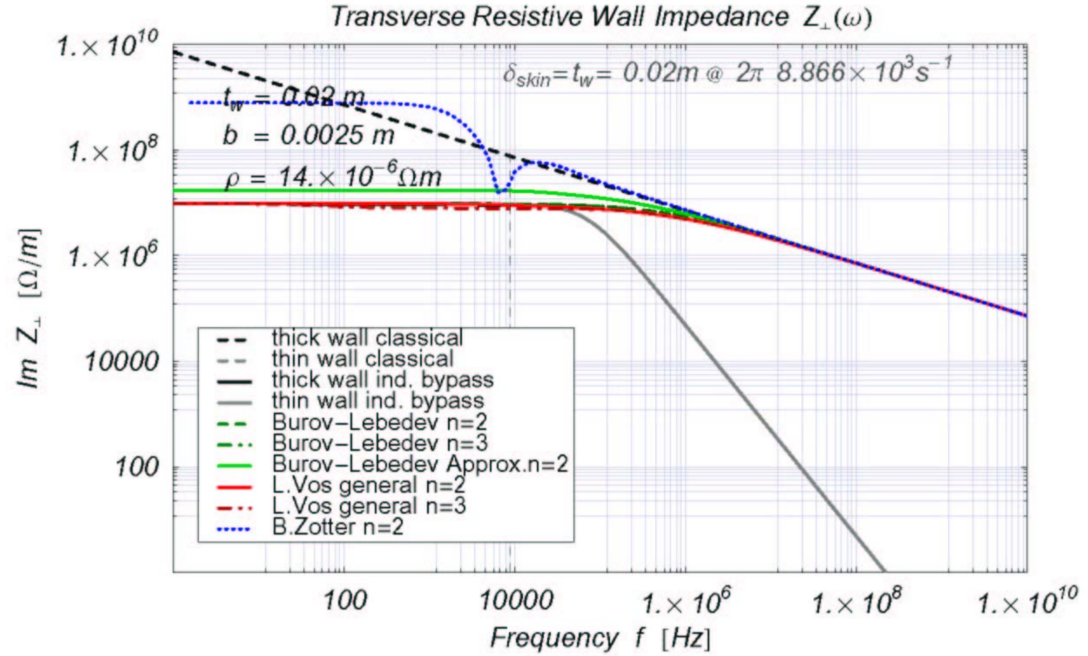
Piwinski form factor *F* for *y/b*<<1 vs. the relative beam size

- despite intensity loss, effect should be slightly enhanced as half gap size approaches rms beam size!

Zotter's impedance model



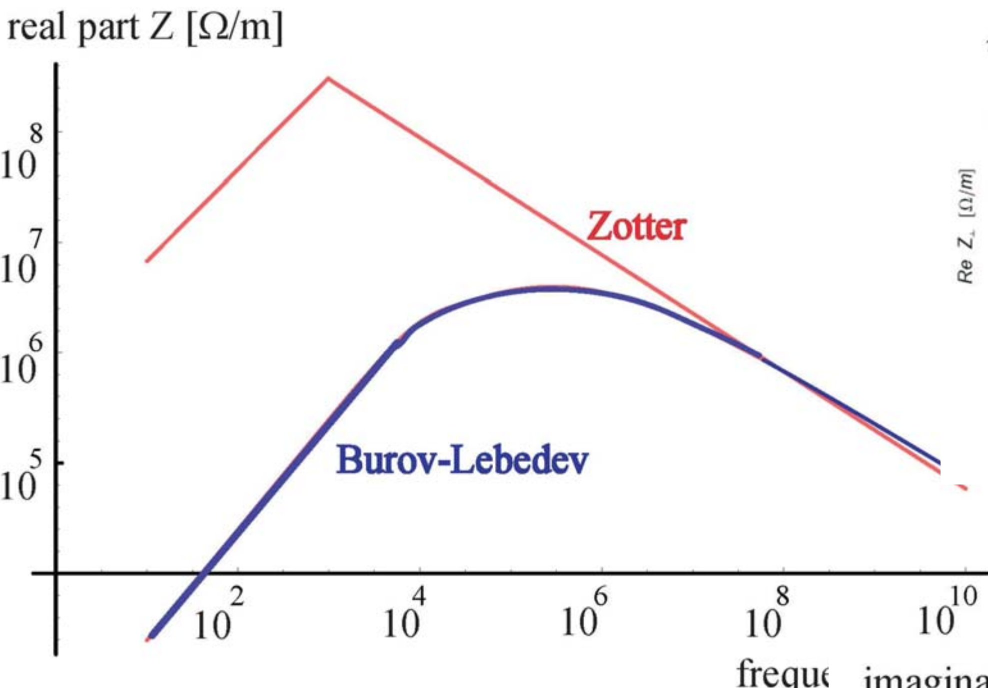
Predictions for the collimator impedance from calculations and formulae by various authors (compiled & plotted by A. Koschik).
Blue curve is B. Zotter's result.

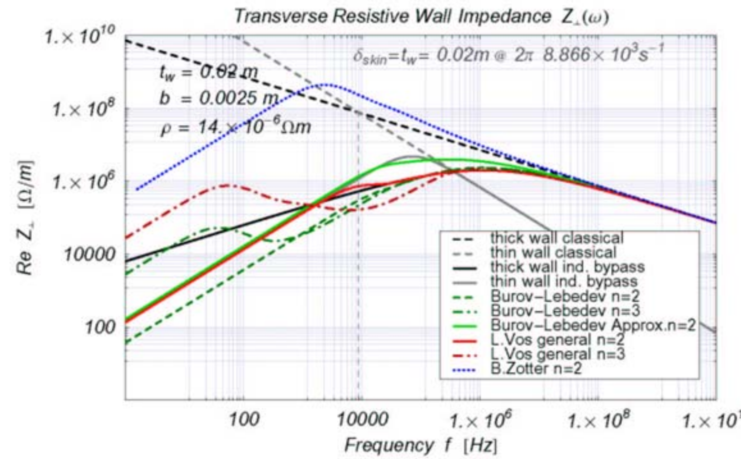


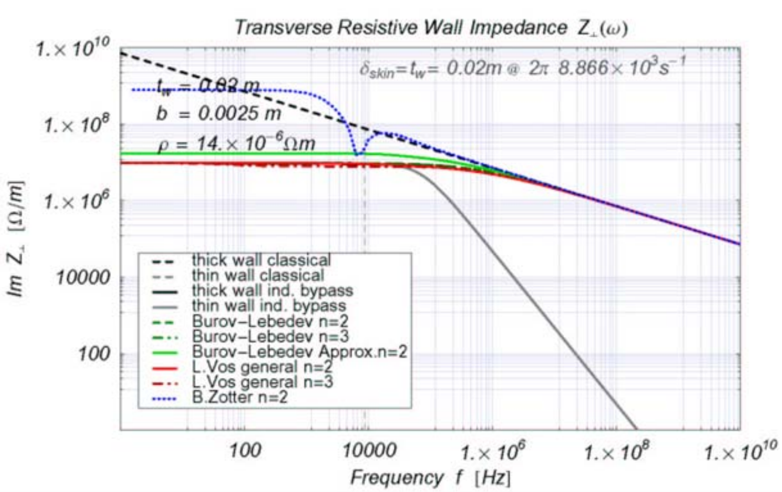
my parametrization of Zotter's impedance

$$\mathrm{Re}\, Z_t = \Omega\mathrm{m} \left(\frac{b_0}{b}\right)^3 \begin{cases} 10^{6^3} f^{0.83} & \text{for} \quad f < 1000\,\mathrm{Hz} \\ 3\times 10^8 \left(\dfrac{f}{1000\,\mathrm{Hz}}\right)^{-0.53} & \text{for} \quad f > 1000\,\mathrm{Hz} \end{cases}$$

$$\mathrm{Im}\, Z_t = \Omega\mathrm{m} \left(\frac{b_0}{b}\right)^3 \begin{cases} 6\times 10^8 & \text{for} \quad f < 10000\,\mathrm{Hz} \\ -3\times 10^8 \left(\dfrac{f}{1000\,\mathrm{Hz}}\right)^{-0.53} & \text{for} \quad f > 10000\,\mathrm{Hz} \end{cases}$$


real part Z [Ω/m]
Zotter
Burov-Lebedev
10^8
10^7
10^6
10^5
10^2
10^4
10^6
10^8
10^10
freque


Transverse Resistive Wall Impedance Z⊥(ω)
δskin=tw= 0.02m @ 2π 8.866× 10^3 s^-1
tw = 0.02 m
b = 0.0025 m
ρ = 14.× 10^-6 Ωm
Re Z⊥ [Ω/m]
Frequency f [Hz]
thick wall classical
thin wall classical
thick wall ind. bypass
thin wall ind. bypass
Burov-Lebedev n=2
Burov-Lebedev n=3
Burov-Lebedev Approx.n=2
L.Vos general n=2
L.Vos general n=3
B.Zotter n=2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thick wall classical
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thick wall ind. bypass
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Burov-Lebedev n=2
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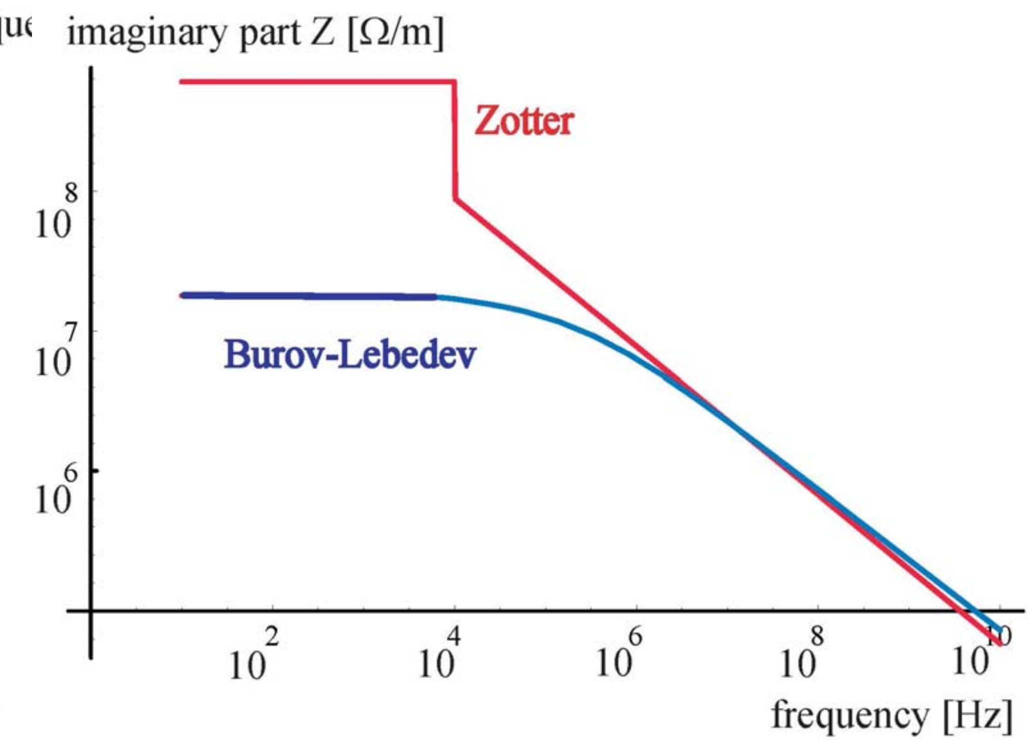
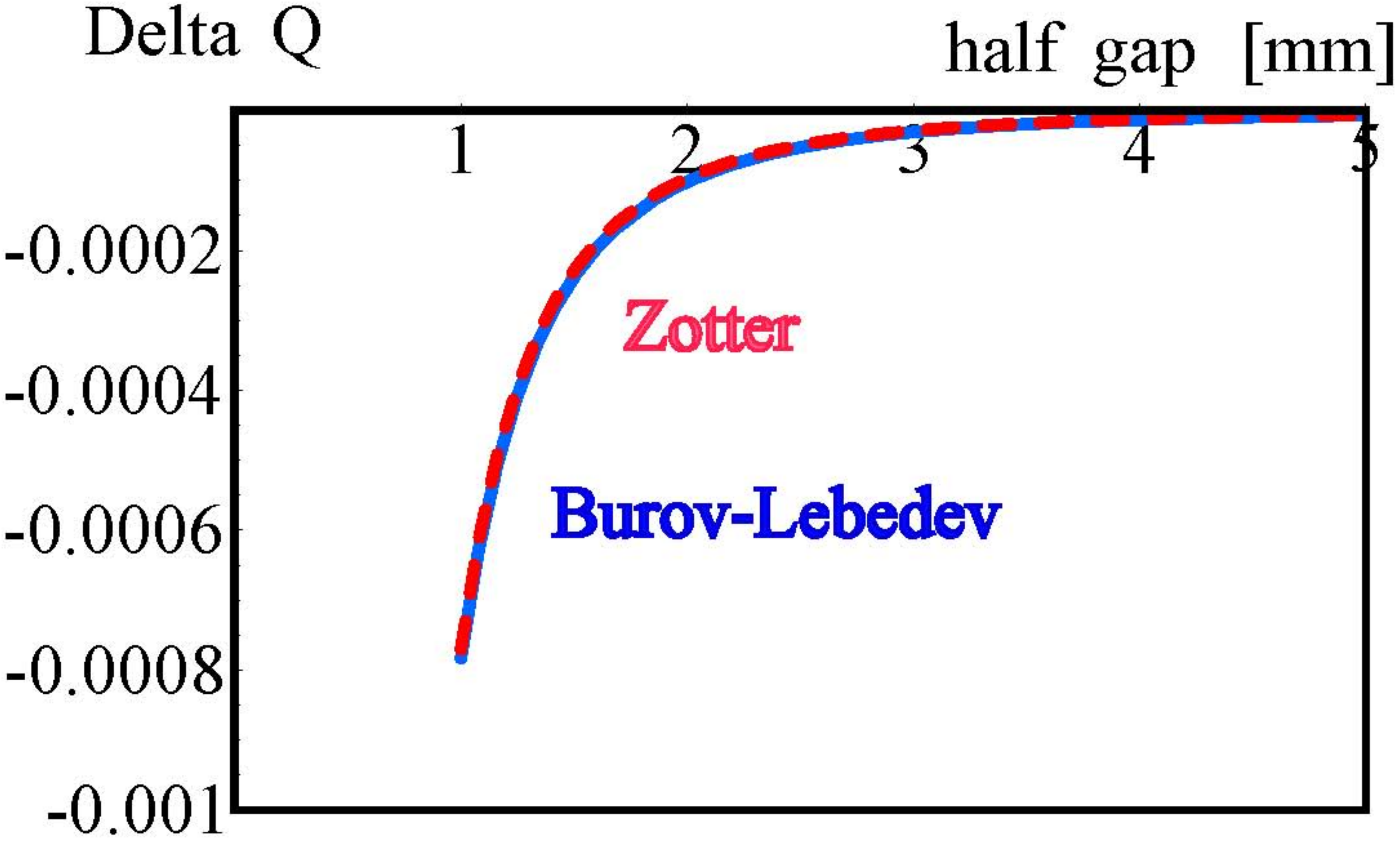
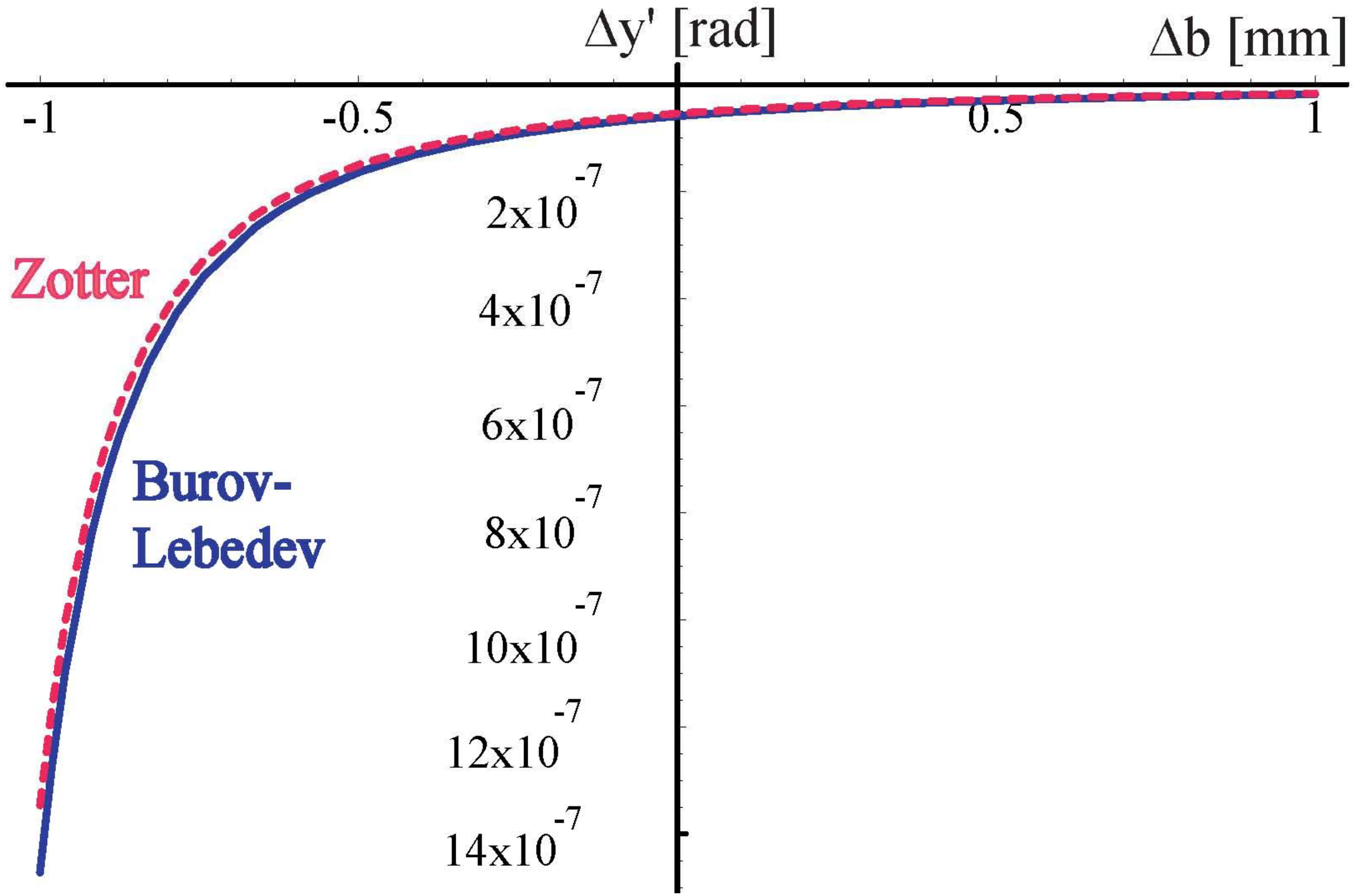

imaginary part Z [Ω/m]
Zotter
Burov-Lebedev
10^8
10^7
10^6
10^2
10^4
10^6
10^8
10^10
frequency [Hz]

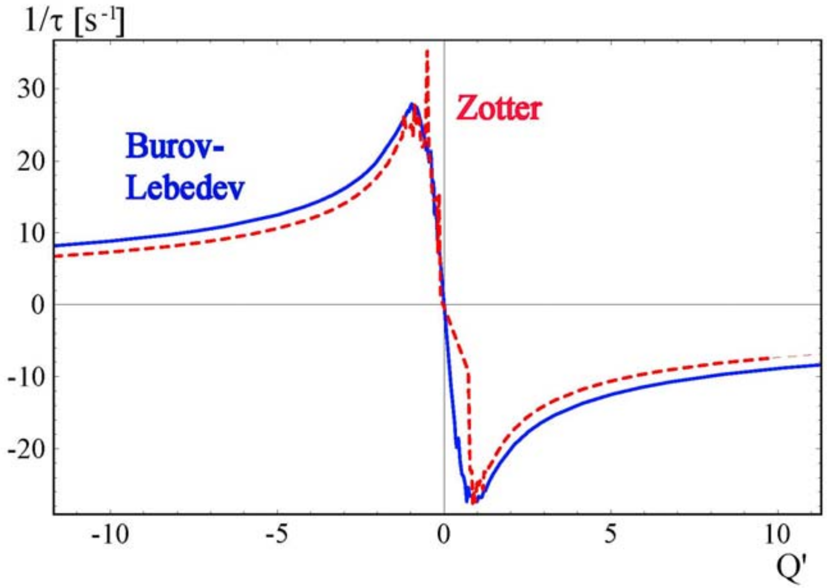


Comparison of *single-bunch tune shifts computed for Burov-Lebedev and Zotter impedance*.

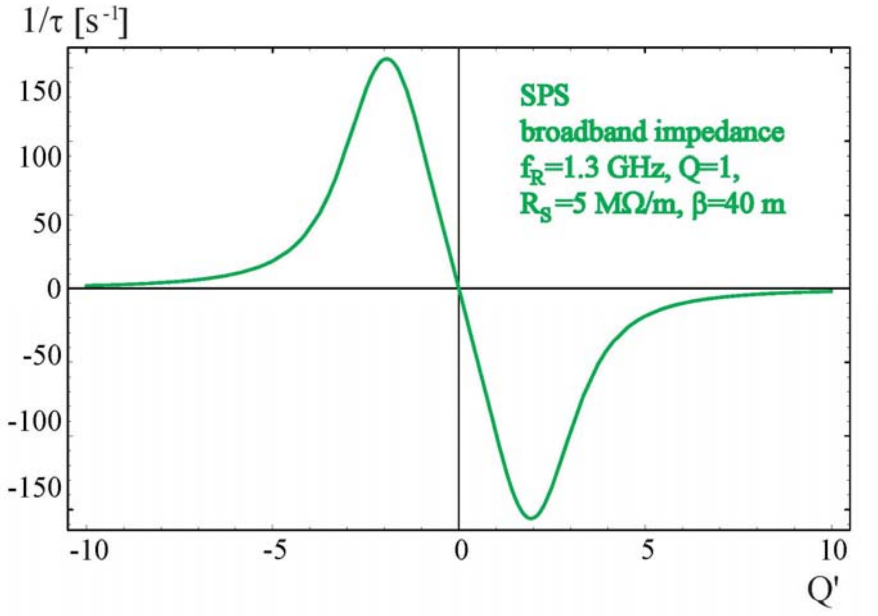


Comparison of *single-bunch deflections computed for Burov-Lebedev and Zotter impedance*. One jaw is at infinity, the other initially at 1.25 mm and then moved by an amount Δ*b*. The resulting deflection of a pencil beam is shown.

horizontal head-tail growth rates

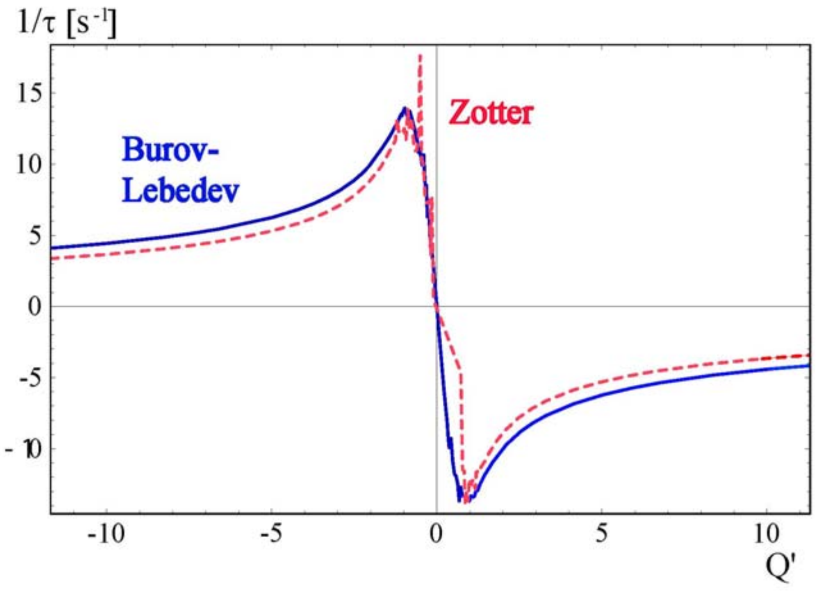


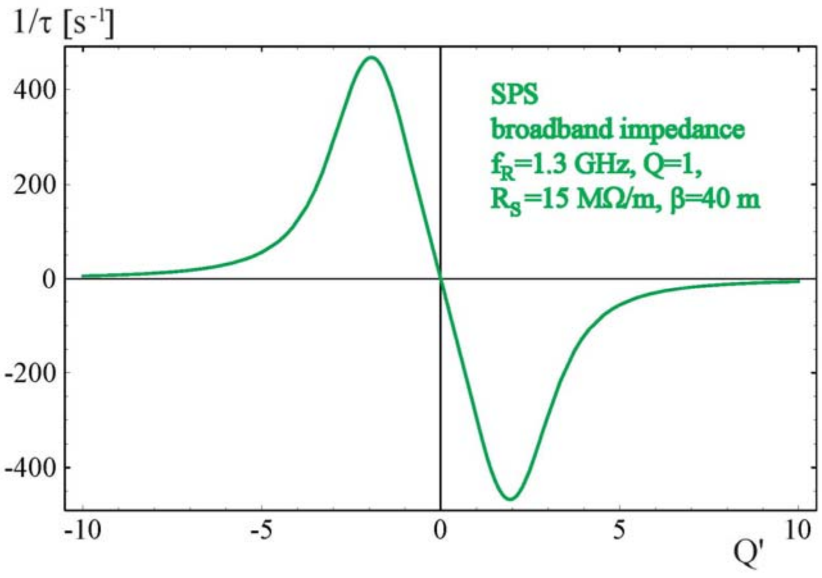
single-bunch horizontal head-tail growth rates due to the collimator



single-bunch horizontal head-tail growth rates due to the SPS horizontal broadband impedance.

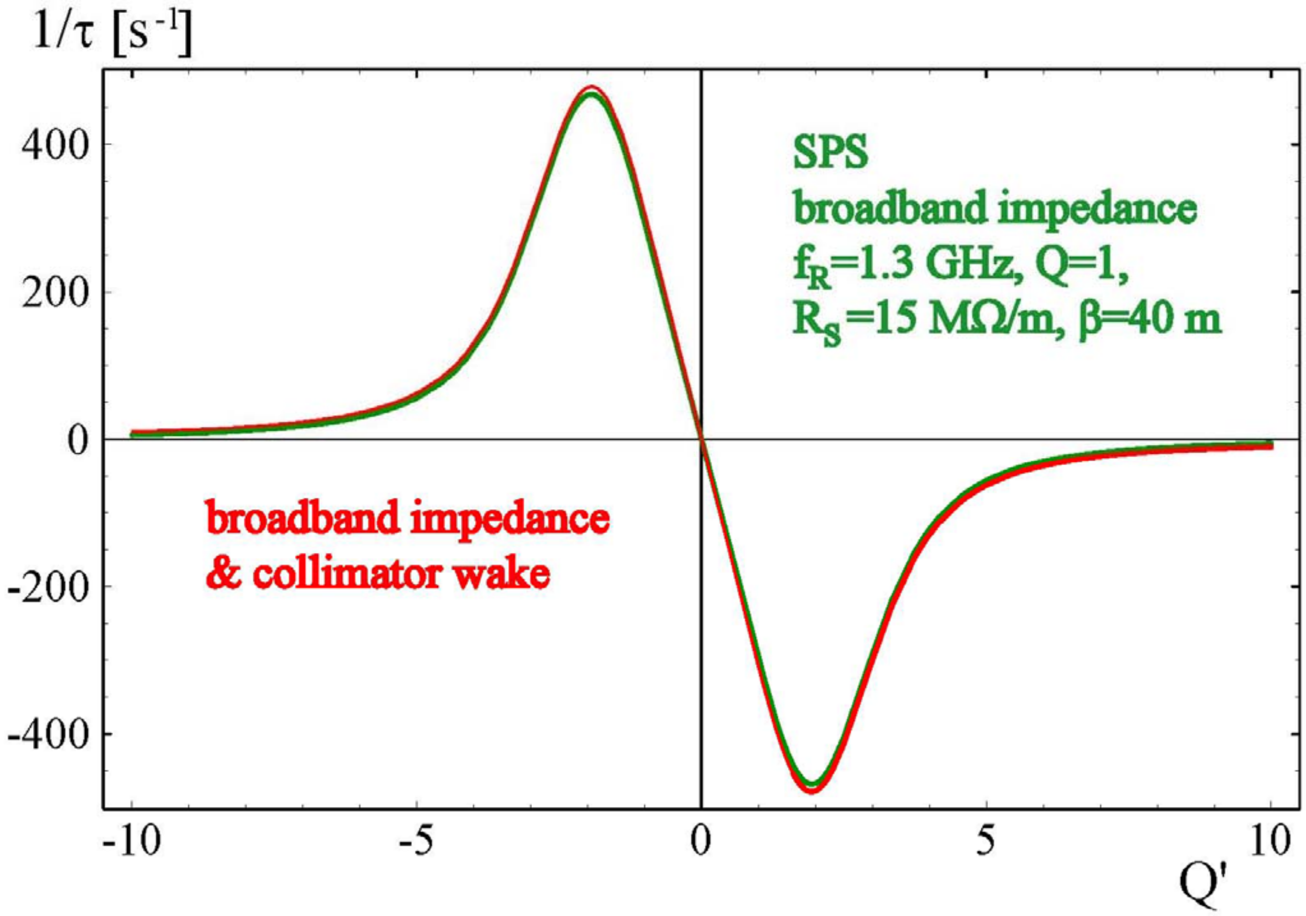
vertical head-tail growth rates





single-bunch vertical head-tail growth rates due to the collimator

single-bunch vertical head-tail growth rates due to the SPS vertical broadband impedance.



single-bunch vertical head-tail growth rates due to the SPS horizontal broadband impedance and with the additional effect of the collimator impedance.

measurements of head-tail growth rates

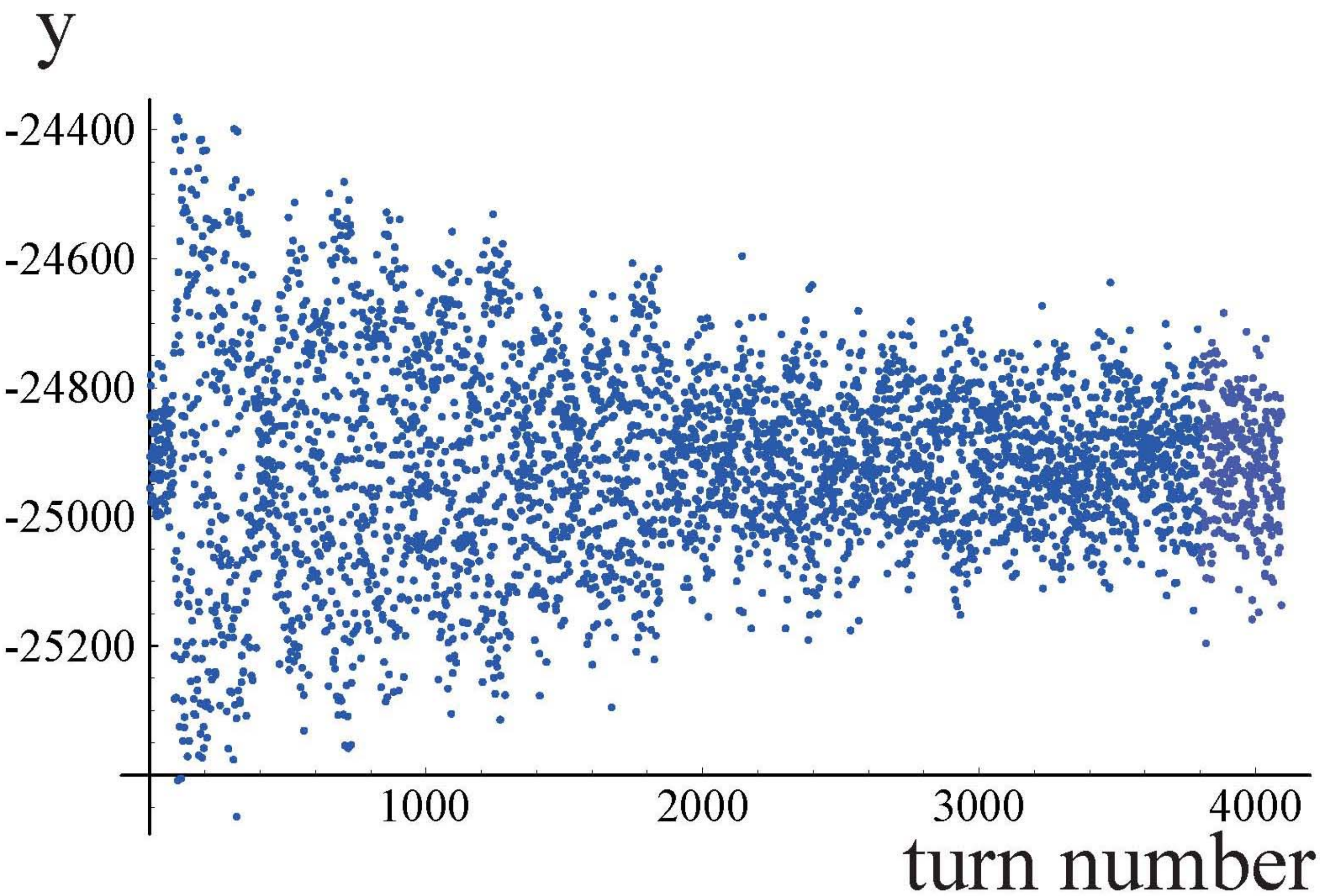
G. Arduini

on October 19, 2004, vertical head-tail growth rates were measured at 270 GeV for a single bunch of about nominal intensity, with horizontal collimators open or closed to a full gap of 4 mm;

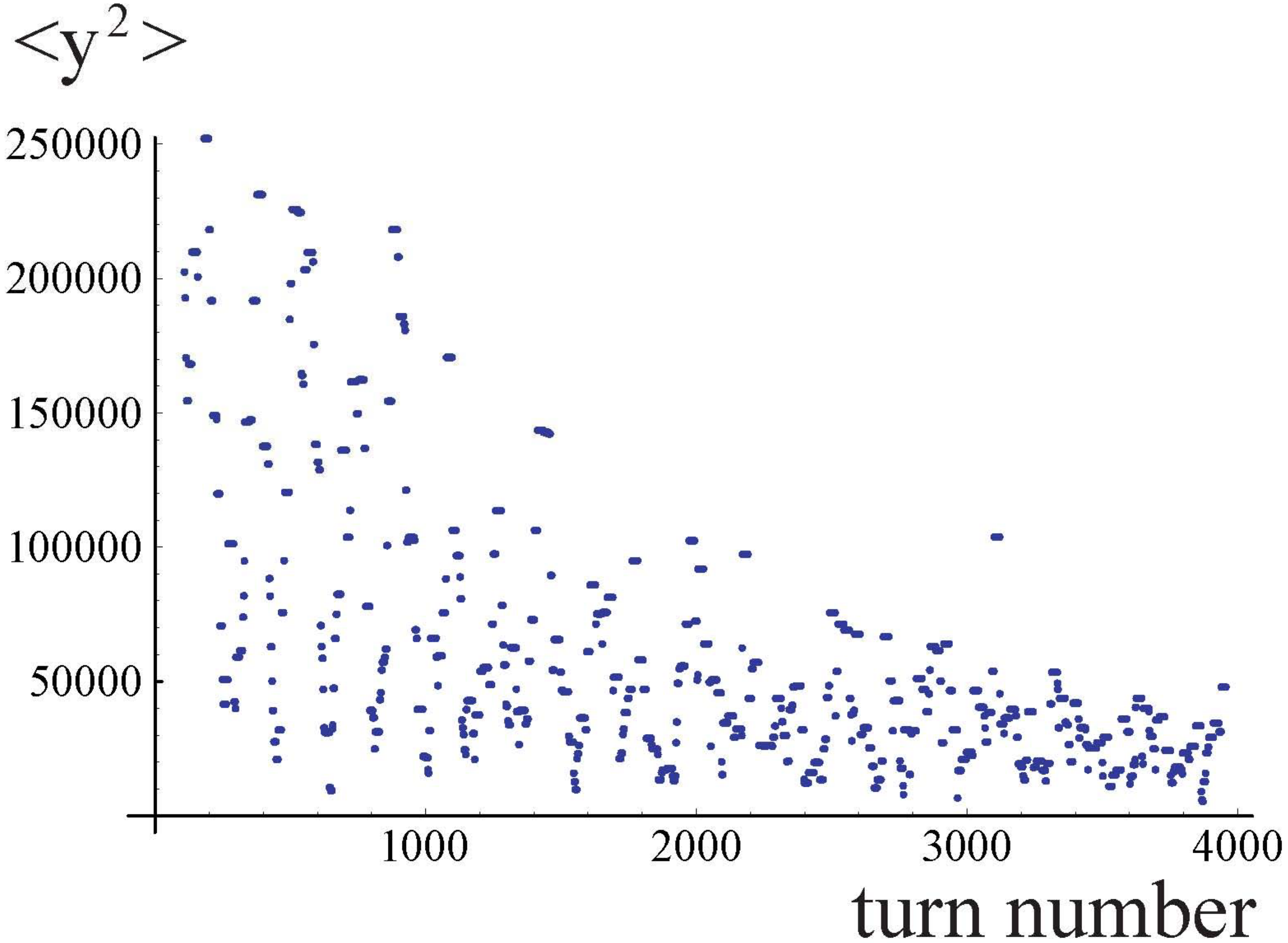
the two collimator positions were alternated from cycle to cycle;

four different settings of the vertical chromaticity ξ_y , namely about 0, 0.05, 0.1 and 0.2;

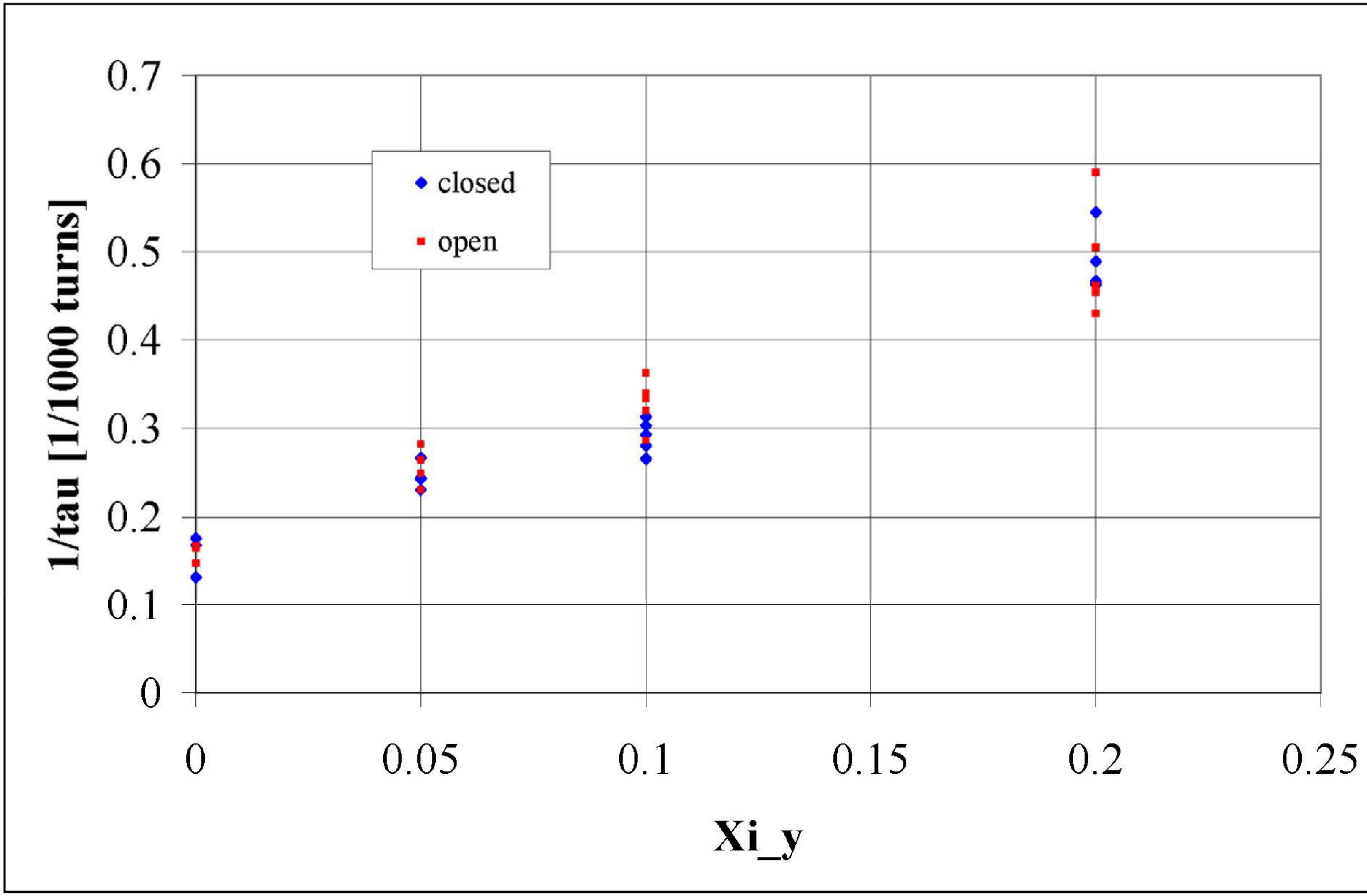
on each cycle a small vertical kick was applied; resulting turn-by-turn oscillation was recorded over 4096 turns.



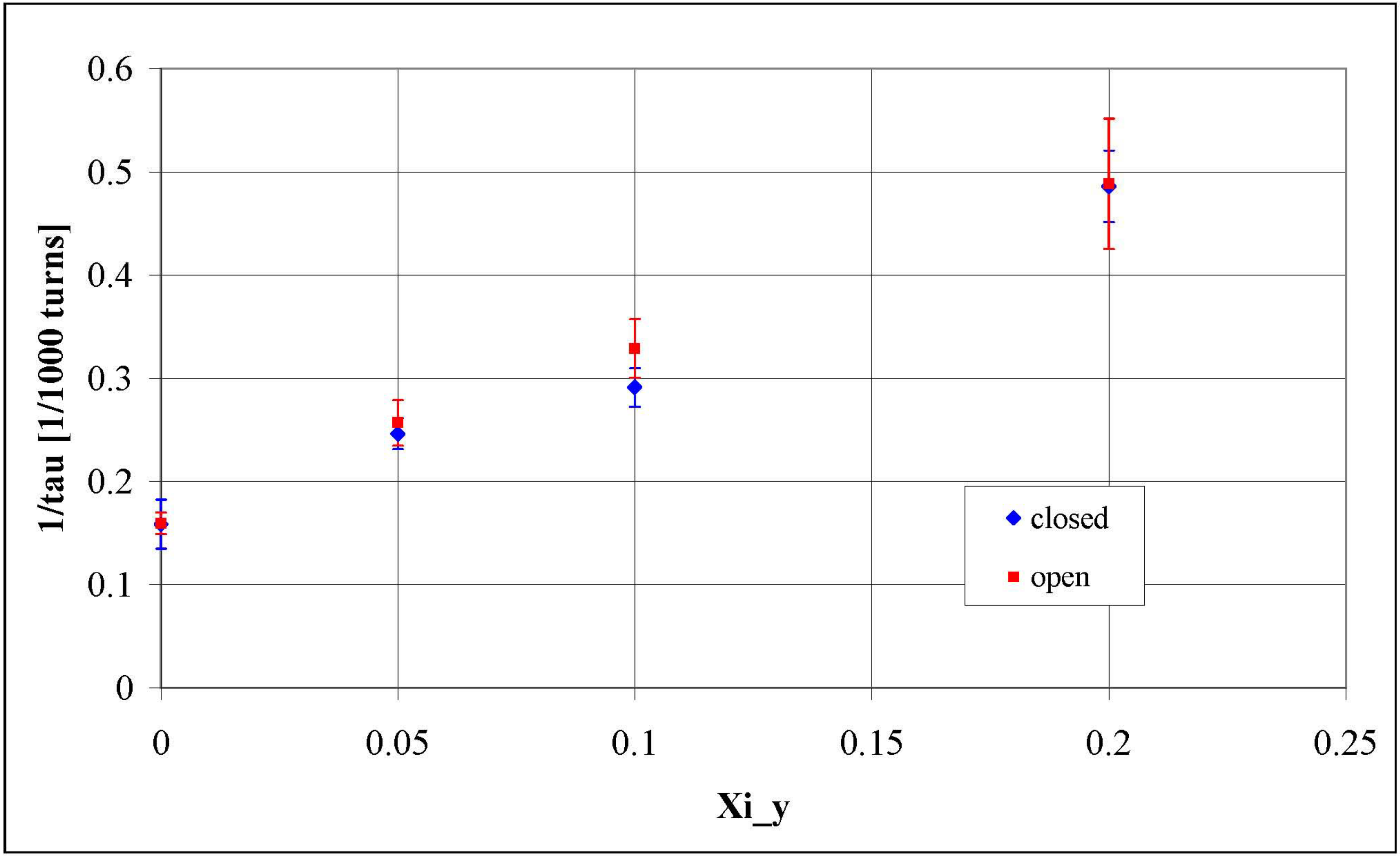
Example raw data for vertical growth-rate measurement. Multiple frequencies in the spectrum cause beating.



Running maximum of the squared amplitude as a function of turn number; to this smoother curve an exponential fit was applied.



Fitted amplitude decay time for all data sets as a function of chromaticity.



Mean and rms value of the fitted decay time vs. ξ_y.

There is a faint sign for a decrease in the damping rate, when the collimator is closed, opposite to expectation.

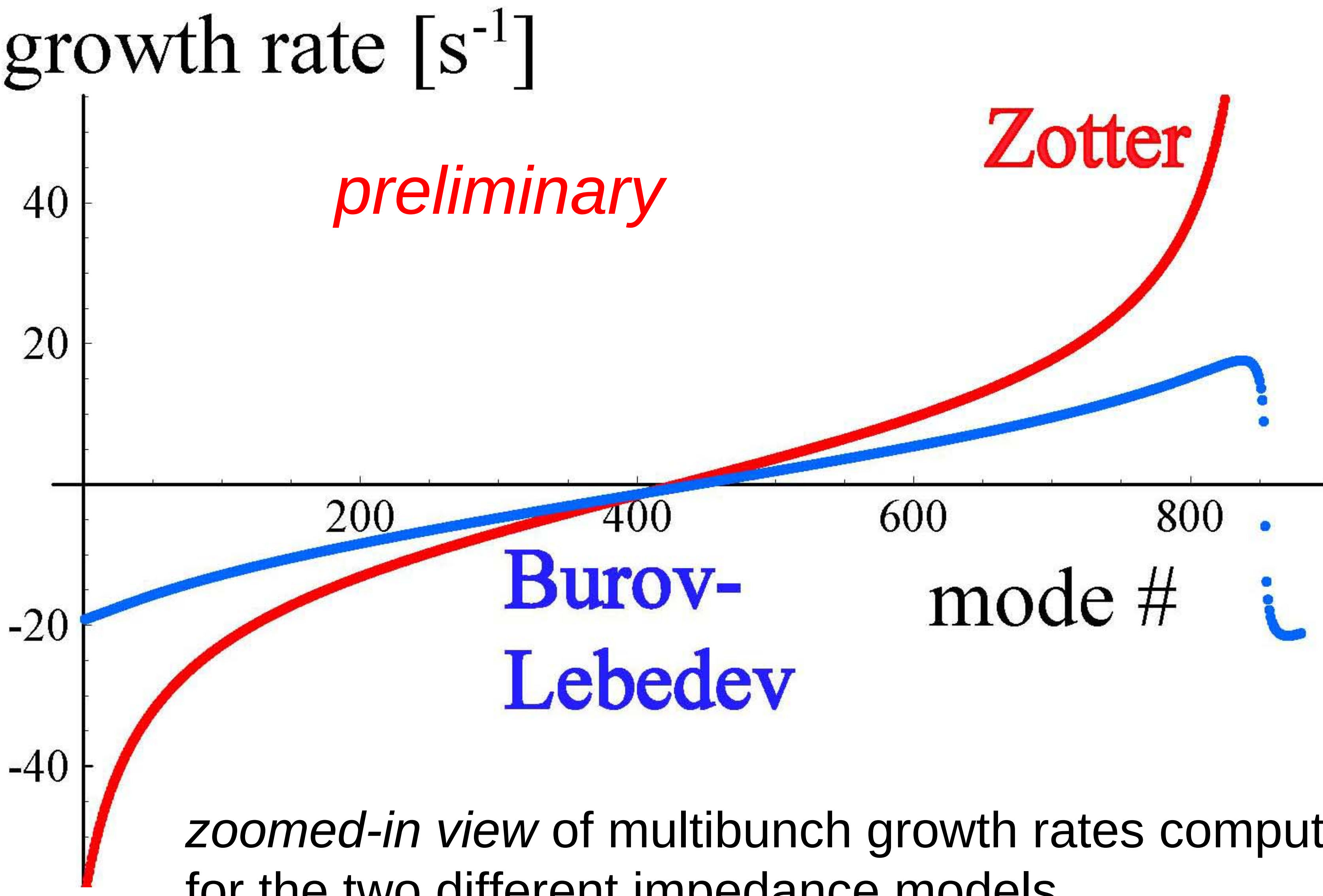
multibunch effects

multibunch growth rates from the formula (A. Chao, (4.114))

$$\Omega^{(\mu)} - \omega_\beta = -i \frac{n_b N_b r_p c}{2\gamma T_0^2 \omega_\beta} \sum_{p=-\infty}^{\infty} Z_1^{\perp}\left[\omega_\beta + \left(p n_b + \mu\right)\omega_0\right]$$

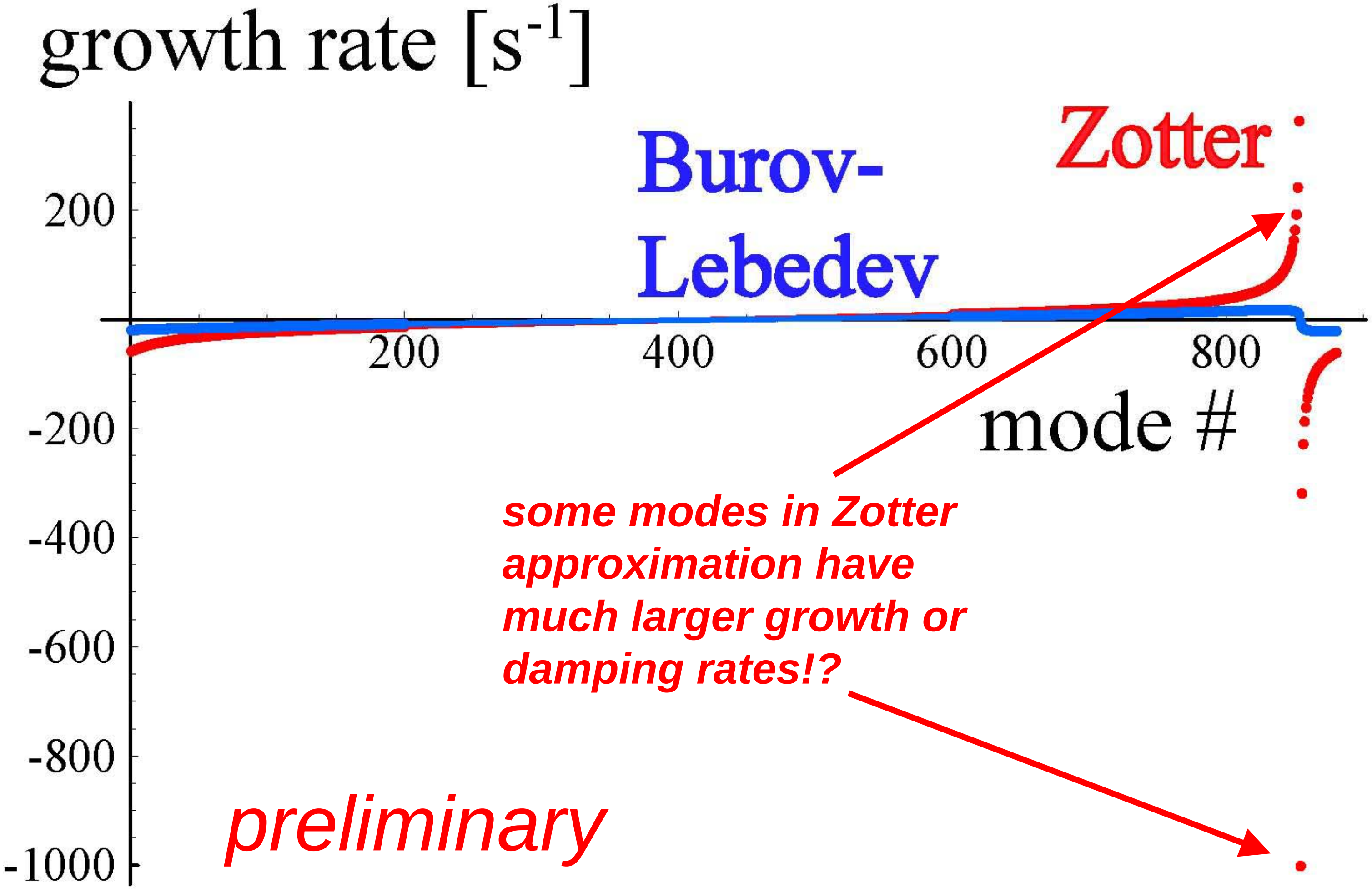
where I took n_b=880 as the total number of bunches, assuming entire SPS ring is filled at 25-ns bunch spacing

(no concise formula of growth rate seems to be available for partially filled rings; calculation for uniform filling gives upper bound on the real growth rate, according to a theorem by Kohaupt)



zoomed-in view of multibunch growth rates computed for the two different impedance models

- *here is a clear difference!*



full-scale view of multibunch growth rates computed for the two different impedance models, considering a completely filled SPS ring

A few *measurements* of multi-bunch instability growth rates were performed on October 18, 2004 (G. Arduini).

Four batches of 48 bunches each were stored with reduced damper gain and chromaticity. The 48 bunches were chosen to keep electron-cloud effects small. The beam was kicked vertically with horizontal collimators open or closed.

The fluctuation for either collimator position, between stable and unstable responses (persistent oscillations at large amplitude) more than shadowed any difference between the two collimator positions.

This condition was far from a completely filled ring (192 bunches instead of ~880).

wake functions

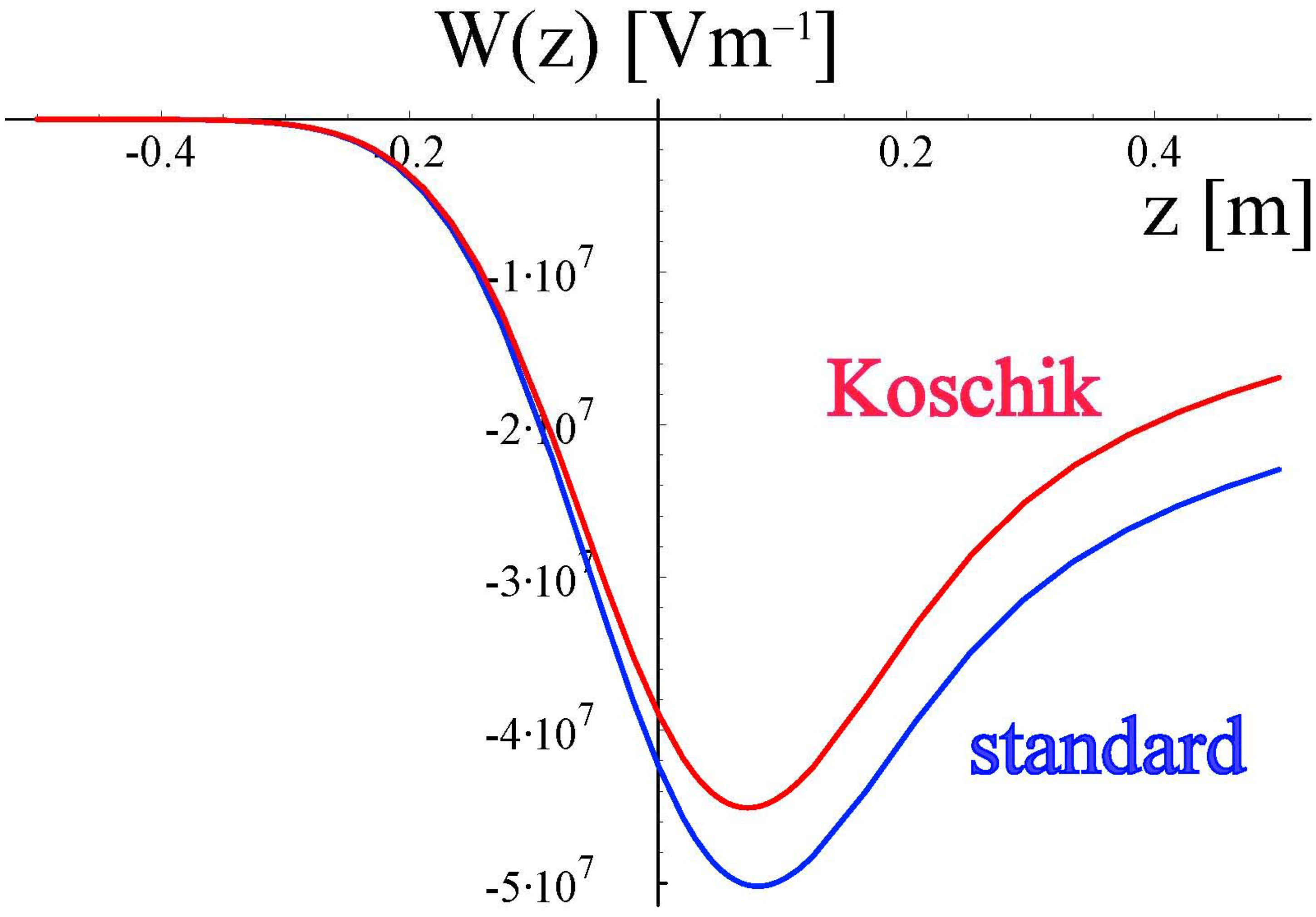
$$W_{\perp 0}(z) = -\frac{2c}{\pi b^3}\sqrt{\frac{\rho Z_0}{4\pi}}\frac{1}{\sqrt{|z|}}$$

standard expression

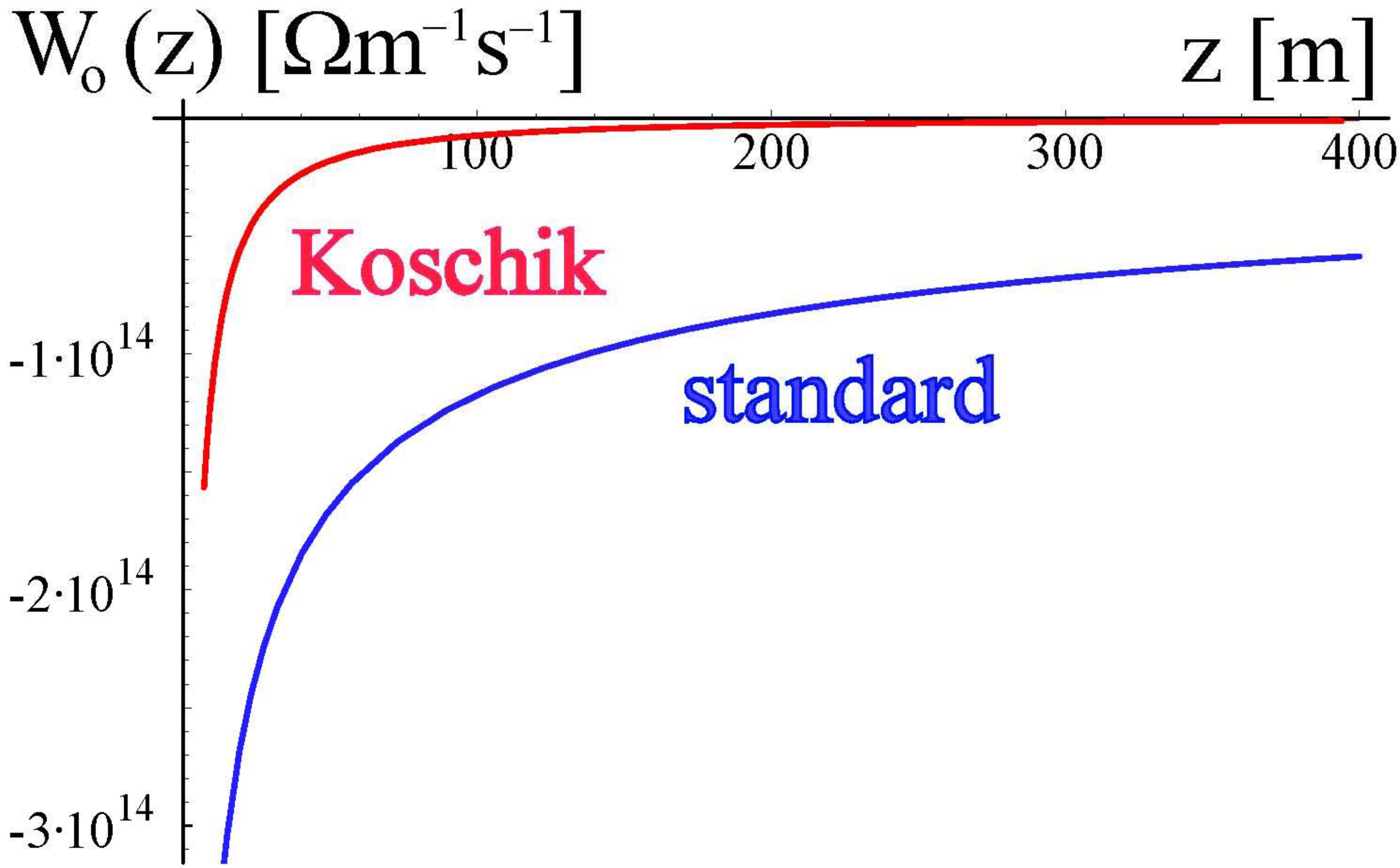
$$W_{\perp 0}^{Koschik}(z) = -\frac{2c}{\pi b^3}\sqrt{\frac{\rho Z_0}{4\pi}}\frac{1}{\sqrt{|z|}}$$

$$+\exp\left[\frac{4\rho}{Z_0 b^2}|z|\right]\frac{2c\rho}{b^4\pi}\left(1-\mathrm{erf}\left[\frac{4\rho}{Z_0 b^2}|z|\right]\right)$$

A. Koschik's wake with inductive bypass



short-range resistive-wall wake field, obtained by folding the Green function wake with the bunch profile, for the SPS collimator computed from standard expression and using Koschik's formula



Green-function resistive-wall wake field, shown over an extended distance behind a point source, for the SPS collimator computed either from standard expression and using the Koschik formula.