advanced theory of collimator wake and application to SPS collimator experiment

nonlinear resistive wall wake field

beam size effect on coherent tune shift

incoherent tune spread

Frank Zimmermann, RLC Meeting, 18.03.05

Potential for nonlinear resistive-wall impedance between two parallel plates was derived by Piwinski (DESY 94-068, Eq. (52)) and re-written by Bane, Irwin, and Raubenheimer (NLC ZDR p. 594).

$$V(x, y, x_0, y_0) = -\kappa f_R(\tau)\left[\frac{-x_- \sinh x_- + y_+ \sin y_+}{\cosh x_- + \cos y_+} + \frac{x_- \sinh x_- + y_- \sin y_-}{\cosh x_- - \cos y_-}\right]$$

$$y_+ = \frac{\pi}{2g}(y + y_0),\ \ y_- = \frac{\pi}{2g}(y - y_0),\ \ x_- = \frac{\pi}{2g}(x - x_0)$$

$2g$: full gap

$$\kappa = \frac{1}{2}\frac{Nr_p}{\gamma\sigma_z}\frac{L}{g}\sqrt{\lambda\sigma_z} \qquad f_R(\tau) = \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{d\tau'}{\sqrt{\tau'}} e^{-\frac{(\tau+\tau')^2}{2}} \qquad \langle f_R \rangle = 0.816$$

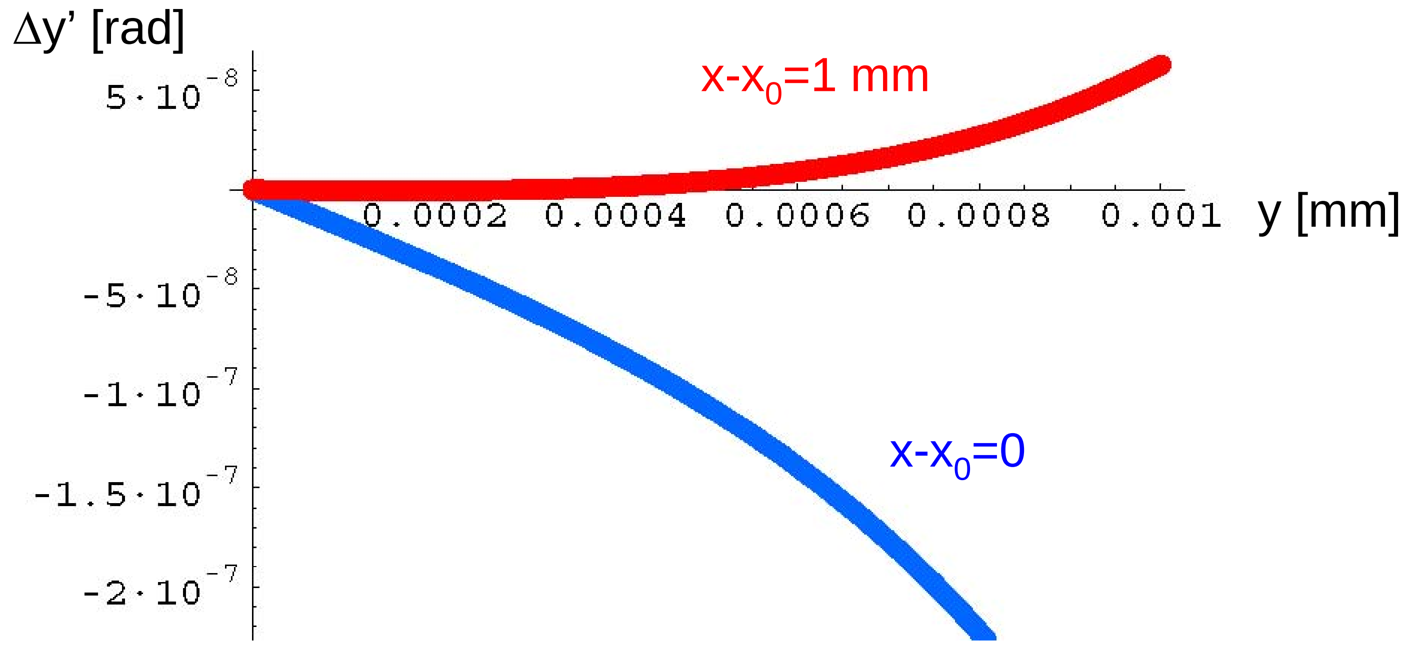
$$\lambda = \rho/(120\pi)$$

nonlinear kick to test particle:

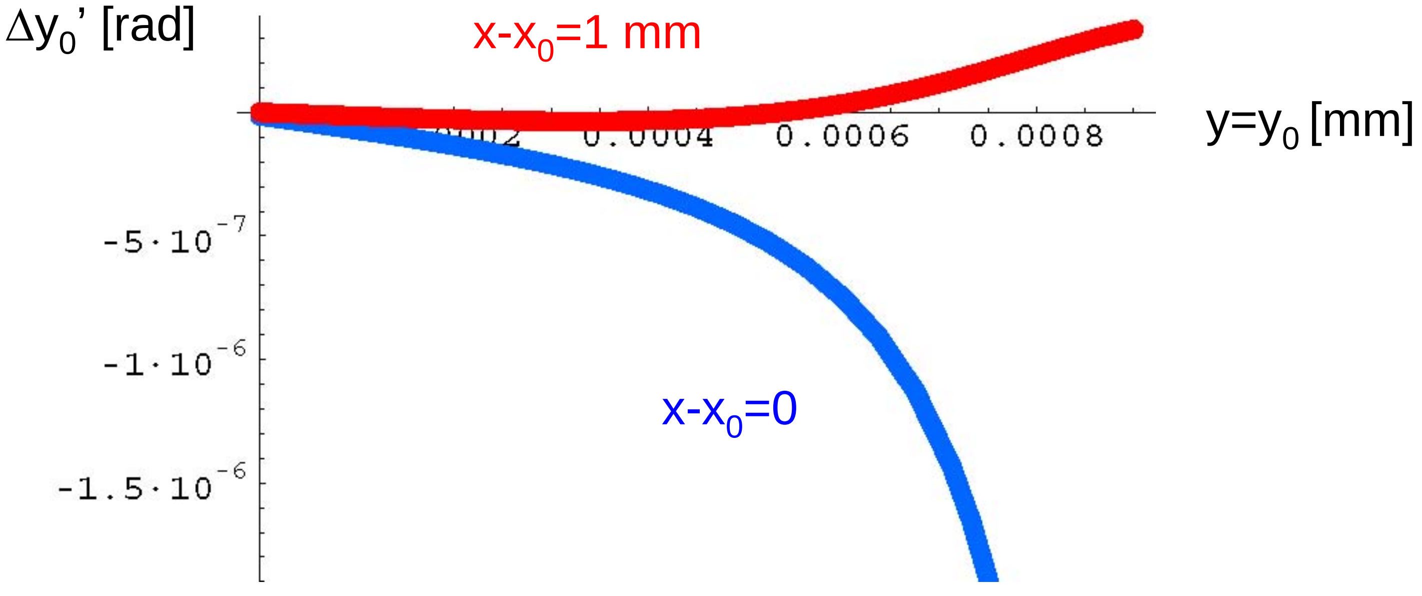
$$\Delta y' = -\left\langle \frac{\partial V}{\partial y} \right\rangle$$

Piwinski formula applies if $\delta_s << \mathrm{Min}\left(d_{wall}, d_{bw}, 1/(bk^2)\right)$

many thanks to Karl Bane who helped understanding ZDR typos & incorrect reference



gap=1 mm,
y_0=0 mm



gap=1 mm,
y_0=y

I calculate the coherent tune shift as follows

$$\Delta Q_{coh} = -\frac{\beta}{4\pi}\kappa\langle f_R\rangle \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-g}^{g} \frac{\partial(\Delta y'(\tilde{x}, y = y_0 + \tilde{y}, x_0, y_0))}{\partial y_0}\Bigg|_{y_0=0} \frac{e^{-\frac{\tilde{y}^2}{2\sigma_y^2}} e^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{x_0^2}{2\sigma_x^2}}}{(2\pi)^{3/2}\sigma_x^2\sigma_y} d\tilde{y}\, dx\, dx_0$$

note: for the SPS experiment we need to exchange *x* and *y* planes

and the incoherent tune shift as

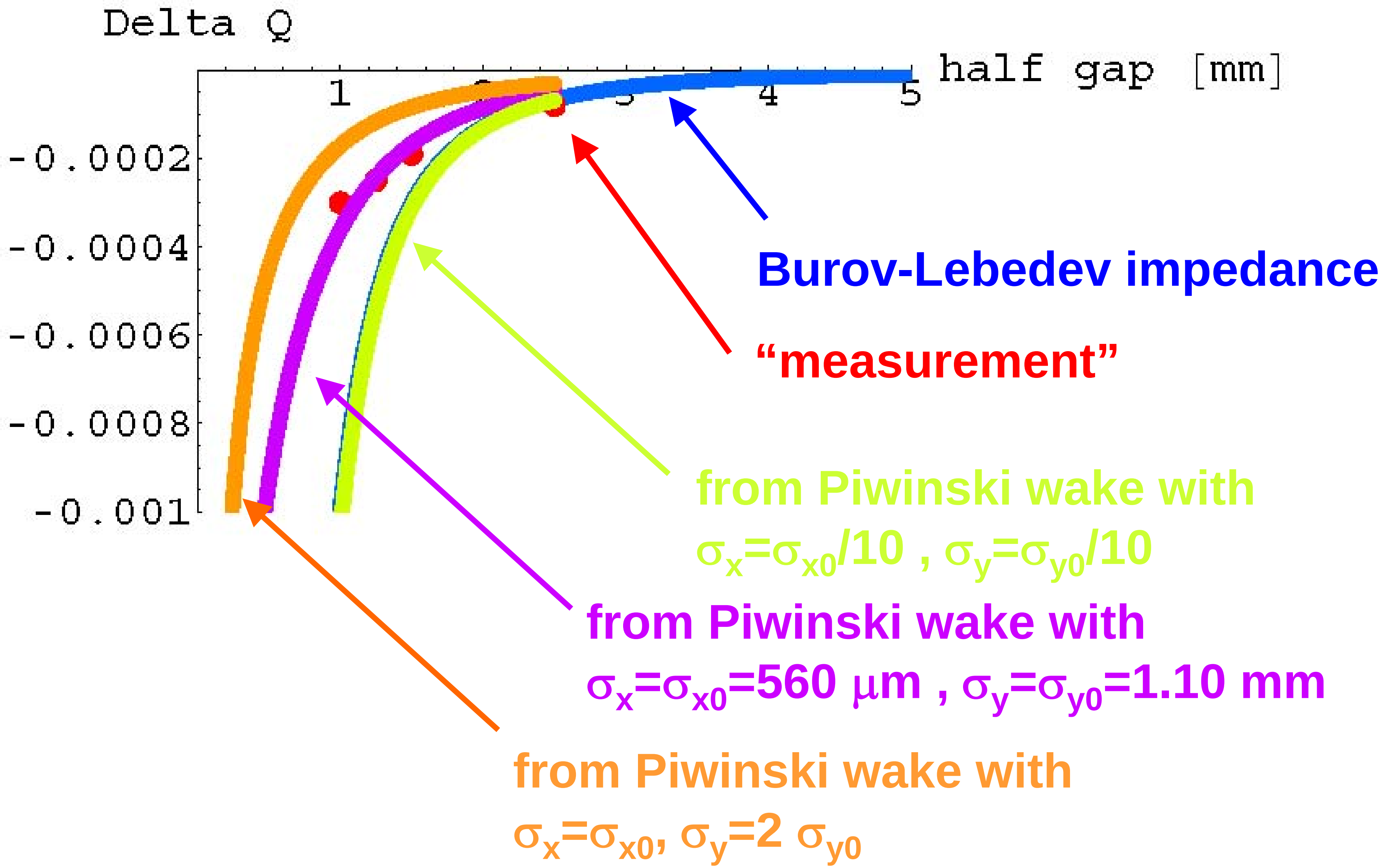
$$\langle \Delta Q_{y,inc}\rangle = -\frac{\beta}{4\pi}\kappa\langle f_R\rangle \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-g}^{g} \frac{\partial(\Delta y'(\tilde{x}, y = \tilde{y}, x_0, y_0))}{\partial \tilde{y}}\Bigg|_{\tilde{y}=0} \frac{e^{-\frac{y_0^2}{2\sigma_y^2}} e^{-\frac{x_0^2}{2\sigma_x^2}}}{2\pi\sigma_x\sigma_y} dy_0\, dx_0\right] \frac{e^{-\frac{x^2}{2\sigma_x^2}}}{(2\pi)^{1/2}\sigma_x} dx$$

$$\langle (\Delta Q_{y,inc})^2\rangle = \int_{-\infty}^{\infty}\left[\frac{\beta}{4\pi}\kappa\langle f_R\rangle \int_{-\infty}^{\infty}\int_{-g}^{g} \frac{\partial(\Delta y'(\tilde{x}, y = \tilde{y}, x_0, y_0))}{\partial \tilde{y}}\Bigg|_{\tilde{y}=0} \frac{e^{-\frac{y_0^2}{2\sigma_y^2}} e^{-\frac{x_0^2}{2\sigma_x^2}}}{2\pi\sigma_x\sigma_y} dy_0\, dx_0\right]^2 \frac{e^{-\frac{x^2}{2\sigma_x^2}}}{(2\pi)^{1/2}\sigma_x} dx$$

$$\Delta Q_{y,rms}^{rms} = \sqrt{\langle (\Delta Q_{y,inc})^2\rangle - \langle \Delta Q_{y,inc}\rangle^2}$$

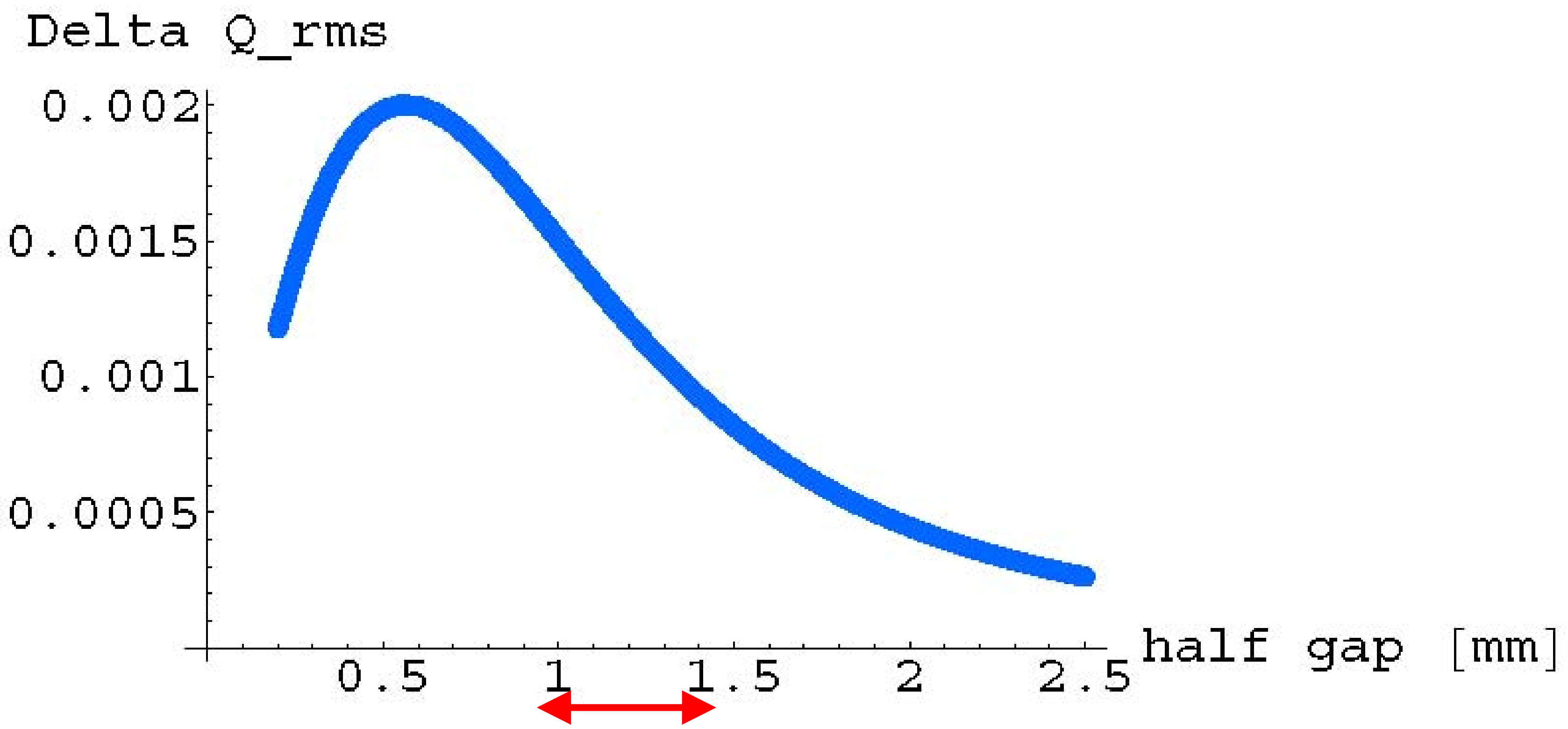
equations take scraping losses into account

coherent tune shift



the data are ~best fitted for the actual beam size estimate

rms incoherent tune spread



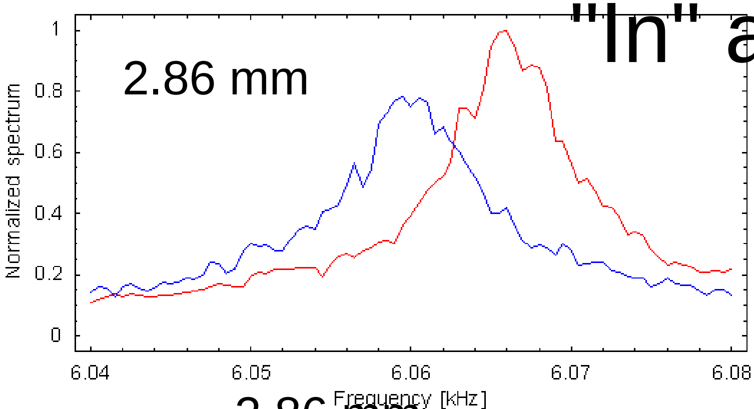
(only from transverse rms, i.e., does not include longitudinal variation of f_R)

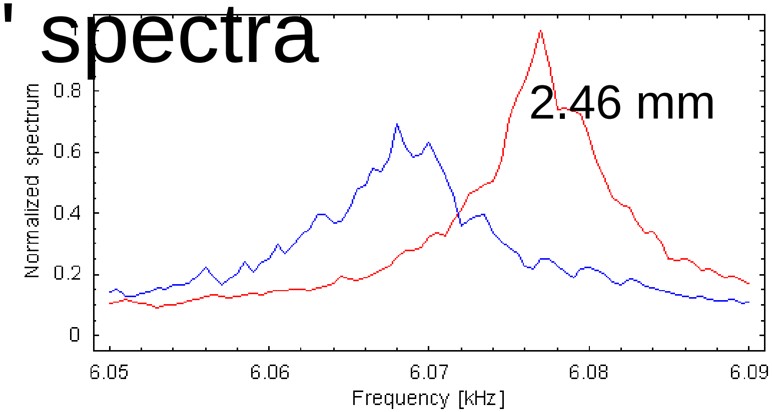
maximum value if gap ~rms beam size

BBQ Measurement by M. Gasior

"In" and "out" spectra

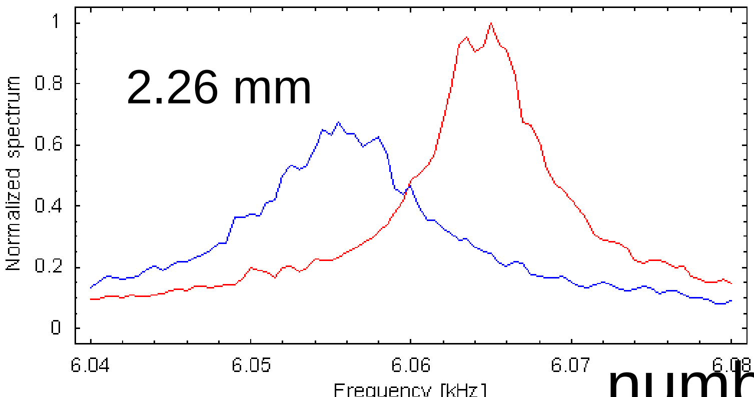
4:01 5:46

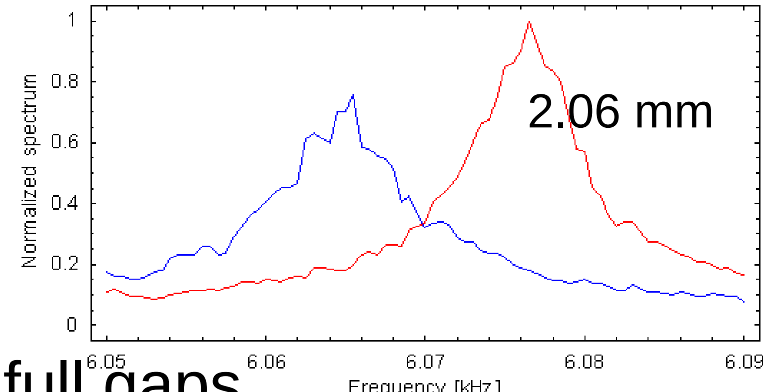




2.86 mm

4:22 5:53





number refer to full gaps

2.26 mm

4:40 6:05