FOLLOW-UP OF THE DATA ANALYSIS FOR THE FAST VERTICAL SINGLE-BUNCH INSTABILITY AT SPS INJECTION

E. Métral

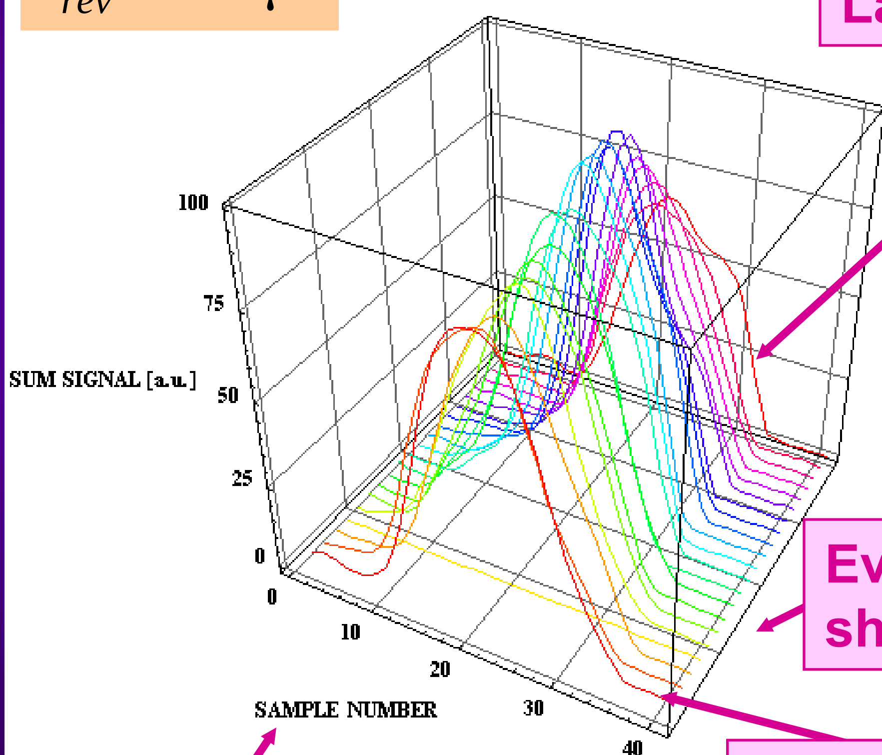
- **2 MDs done on**
  - **Friday 12/09/2003**
  - **Monday 10/11/2003**
- **Until now we looked essentially at the intensity threshold vs. chromaticity, etc… (analytical estimate + simulations with MOSES and HEADTAIL codes)**
- **Now we want to analyze in detail the time evolution (some measurements have be made with the HEADTAIL monitor)**

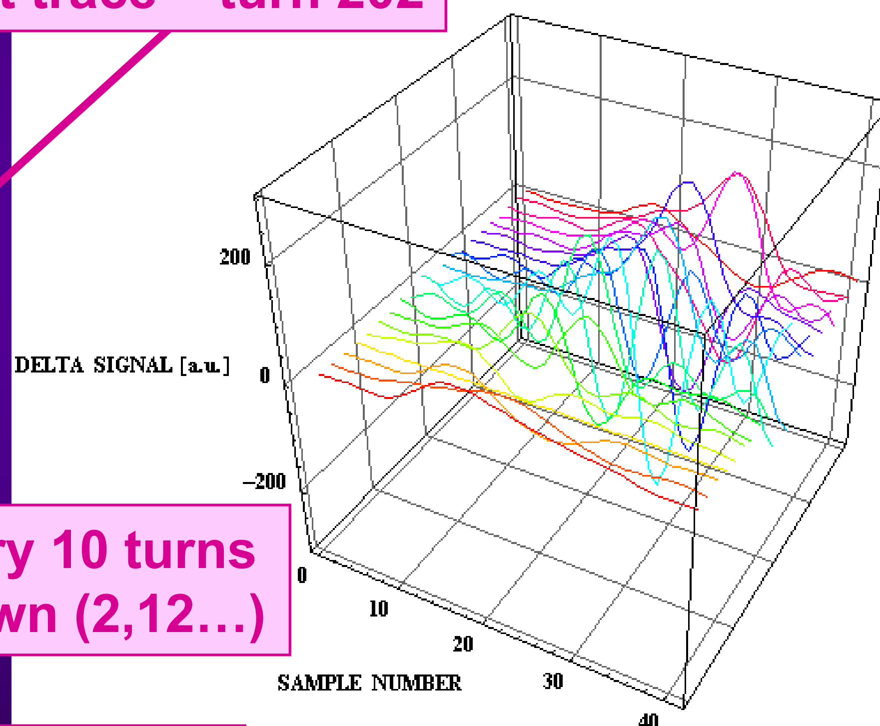
MEASUREMENTS FROM THE HEADTAIL MONITOR (1/6)

⟹ **Data taken and 1st treated by G. Arduini (sent to me on 10/08/05)**

- **Example here (12/09/03): ~ 1.2 10^{11} p, ~ 0.2 eVs, ξ_y ~ 0.05, T_s ~ 300 turns**

$$T_{rev} = 23\ \mu s$$





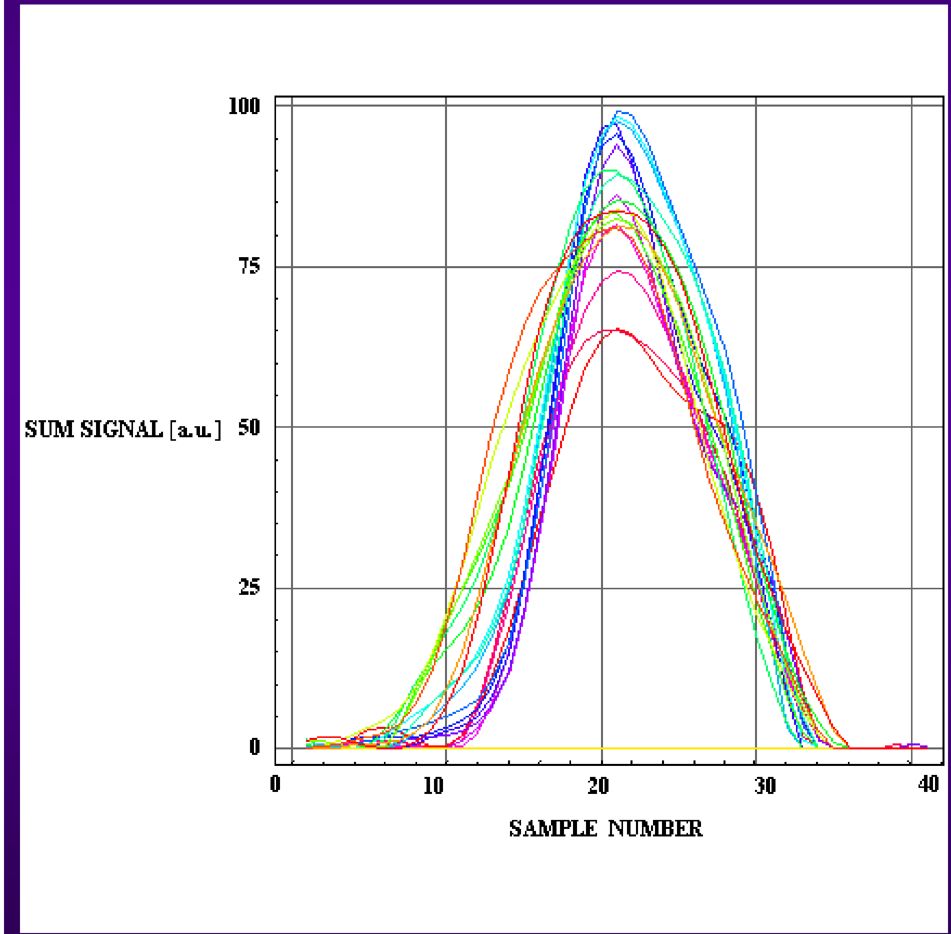
Last trace = turn 202

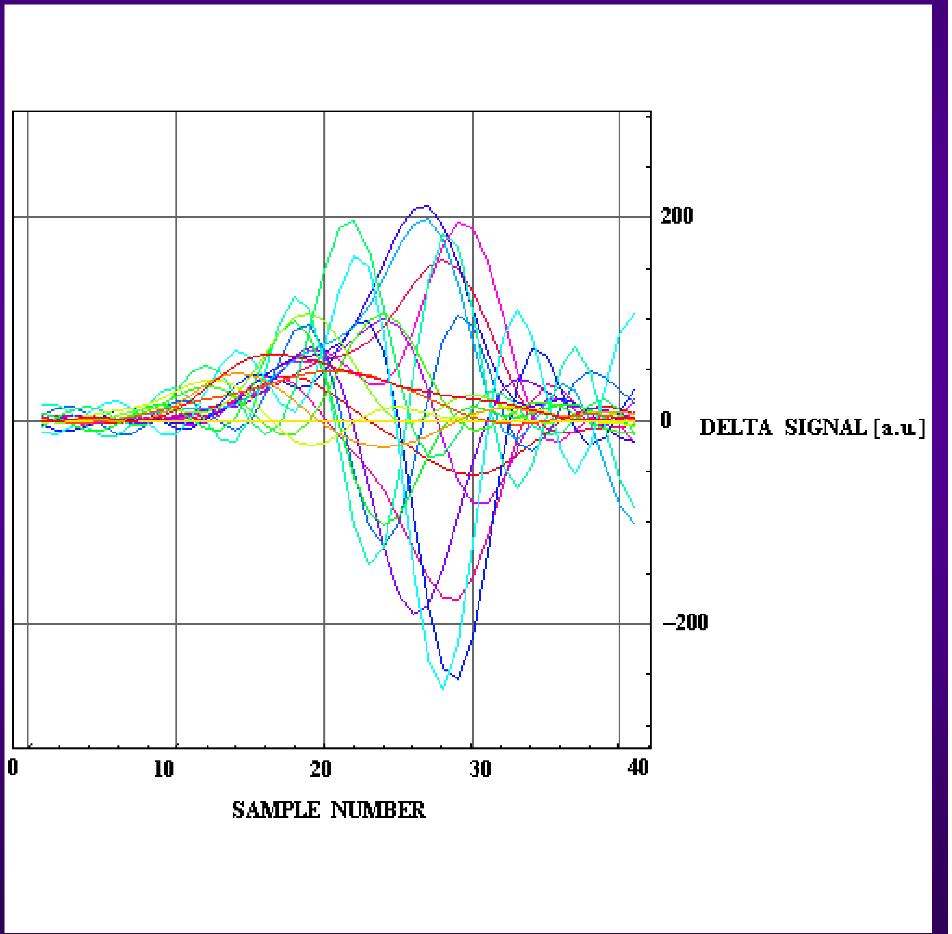
Every 10 turns shown (2,12…)

1st trace = turn 2

Every 125 ps

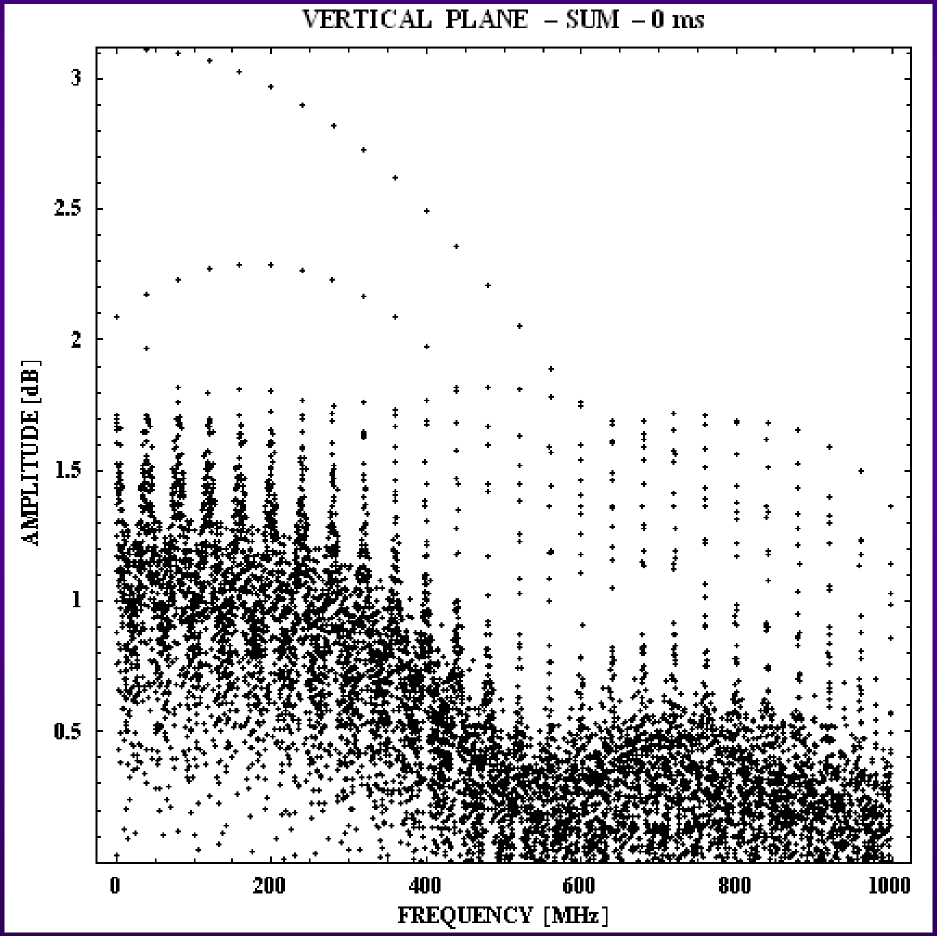
MEASUREMENTS FROM THE HEADTAIL MONITOR (2/6)

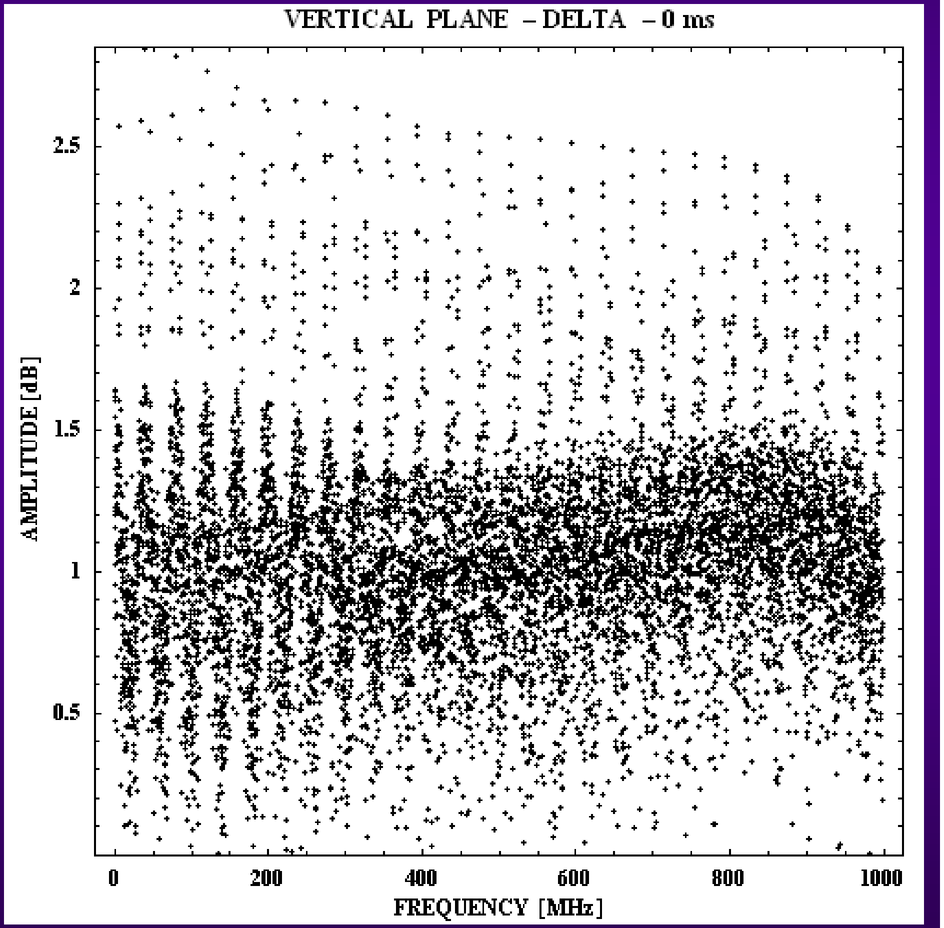




MEASUREMENTS FROM THE HEADTAIL MONITOR (3/6)

FFT applied over the full acquisition depth (372 turns ~ 8.6 ms). The peaks at 40 MHz are due to the fact that at each turn only 25 ns out of the full revolution period of 23.1 μs are acquired

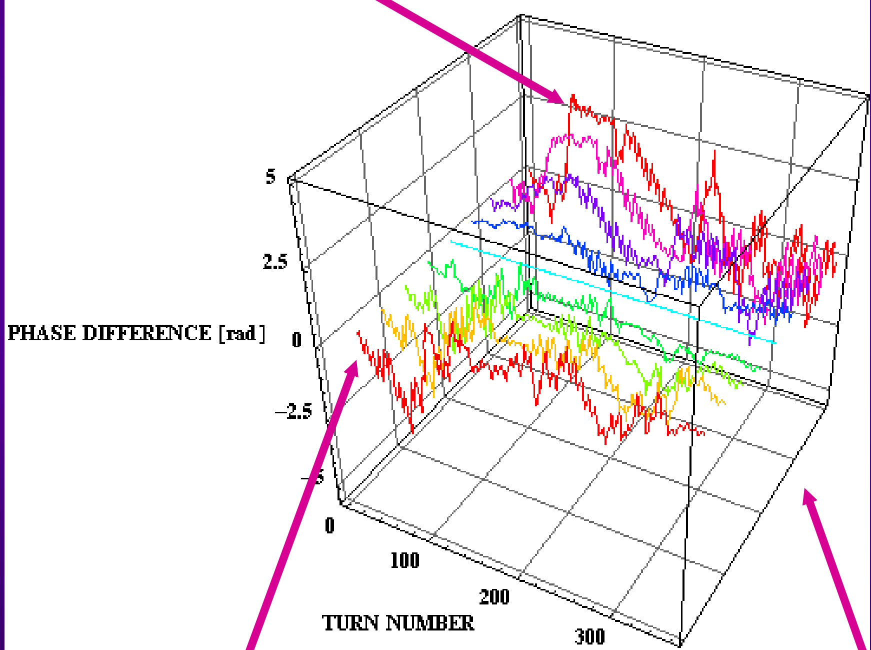


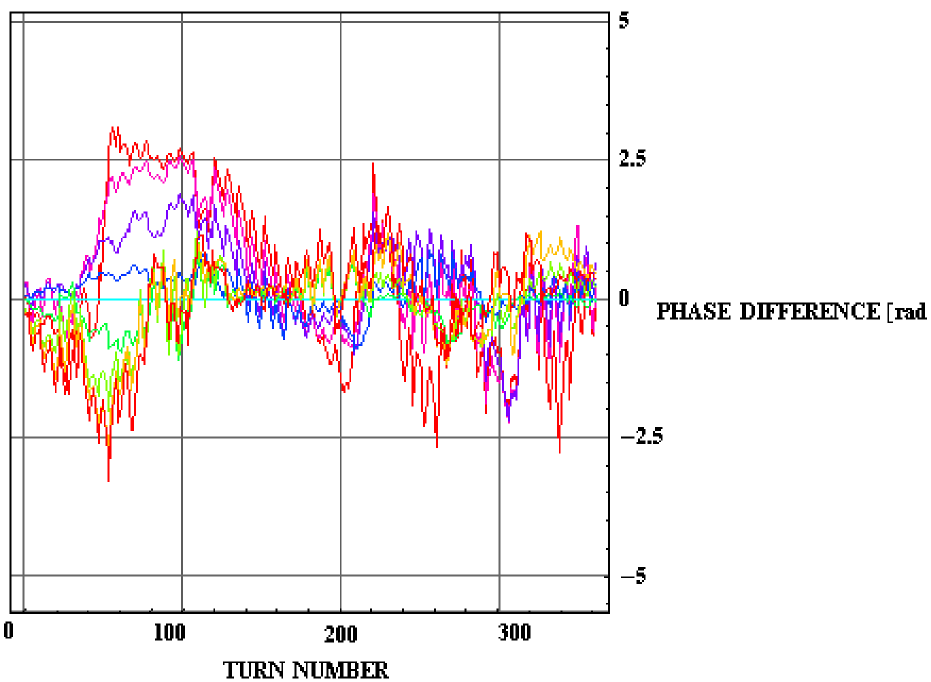


MEASUREMENTS FROM THE HEADTAIL MONITOR (4/6)

Betatron phase difference between different temporal slices with respect to the central slice (=centre of the bunch)

Slice following the central one by 1 ns

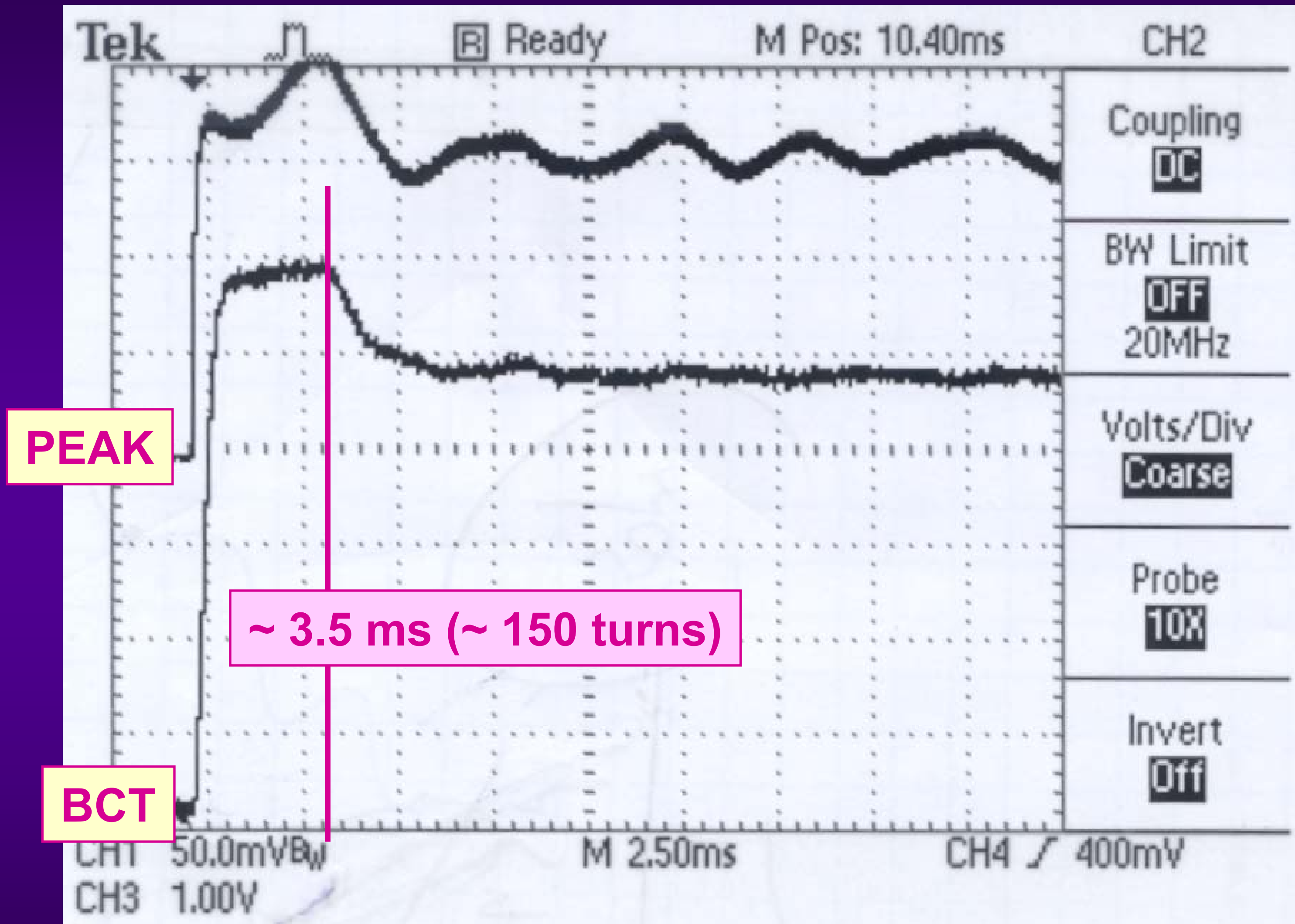




Slice preceding the central one by 1 ns

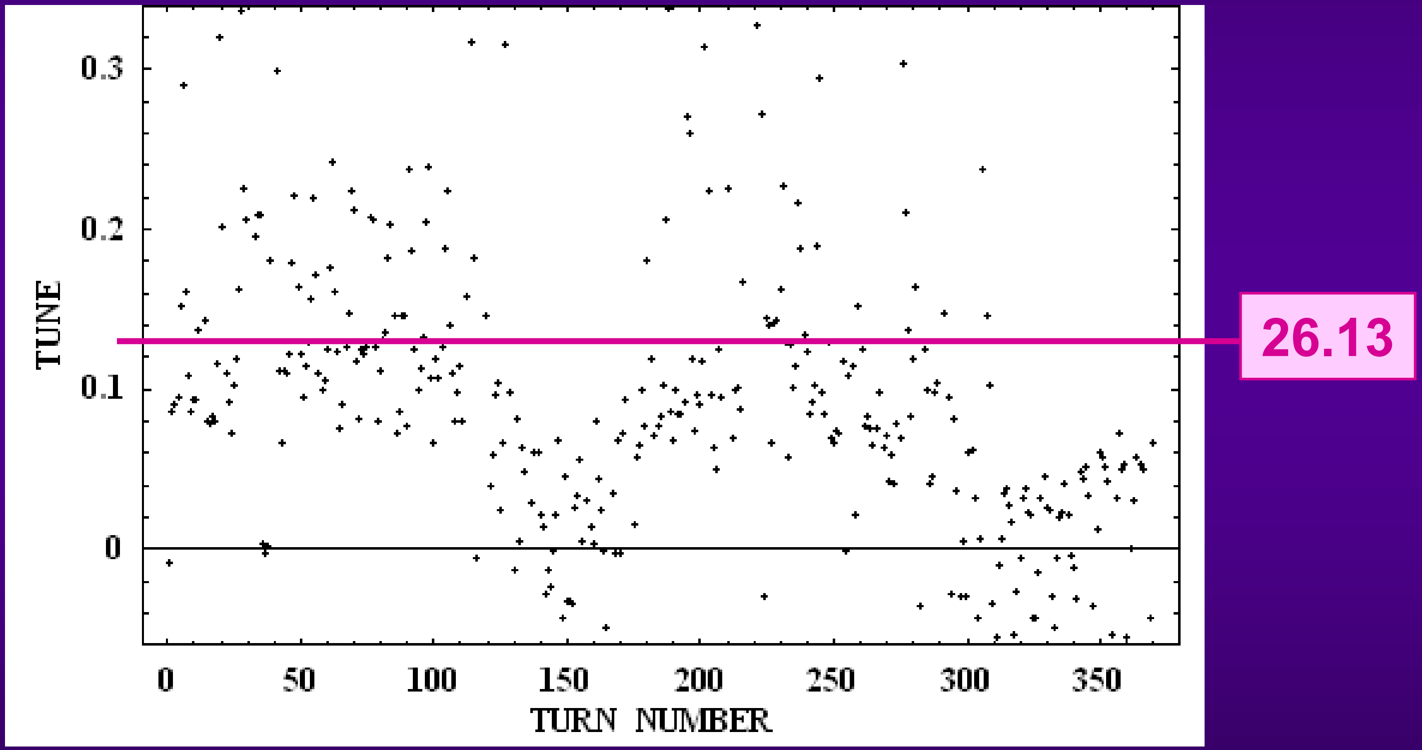
Every 250 ps shown

MEASUREMENTS FROM THE HEADTAIL MONITOR (5/6)



MEASUREMENTS FROM THE HEADTAIL MONITOR (6/6)

Tune deduced at each turn from data analysis (using Hilbert transform techniques)



HEAD-TAIL PHASE SHIFT THEORY (1/5)

Vertical excursion of a slice of a bunch after a vertical kick (linear model) $\Longrightarrow$ Cf. LHC Project Report 602 (Fartoukh&Jones)

$$< y > (\tau , n) = A(\tau , n) \sin\left[2 \pi n Q_y + \phi_\beta (\tau , n)\right]$$

Number of machine turns

Time delay w.r.t central slice $\tau = 0$

with

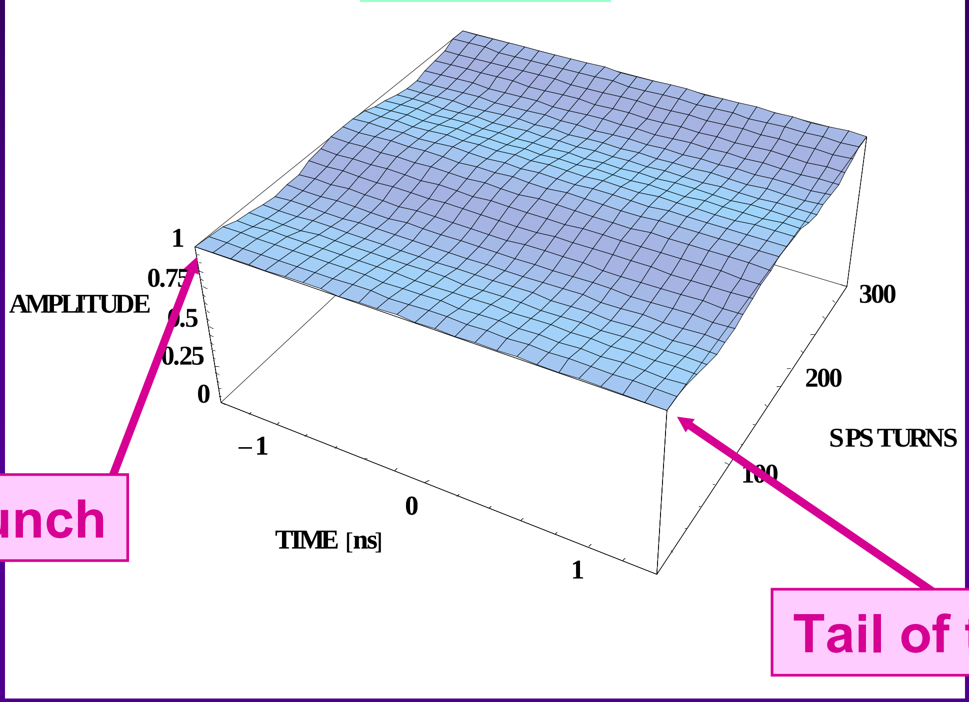
$$A(\tau , n) = A\, e^{-\frac{\left(\frac{\omega_0 Q'_y}{\eta} \sigma_\tau \sin[2 \pi n Q_s]\right)^2}{2}}$$

$$\phi_\beta (\tau , n) = \frac{\omega_0 Q'_y}{\eta} \tau \left(1 - \cos[2 \pi n Q_s]\right)$$

HEAD-TAIL PHASE SHIFT THEORY (2/5)

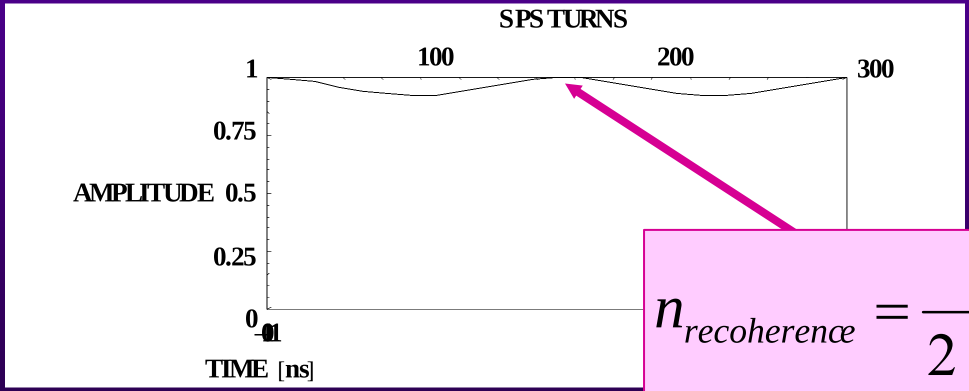
$$\tau_b \left(4\,\sigma \right) = 2.8\ \mathrm{ns}$$

$$A\left(\tau\,, n \right)$$



Head of the bunch

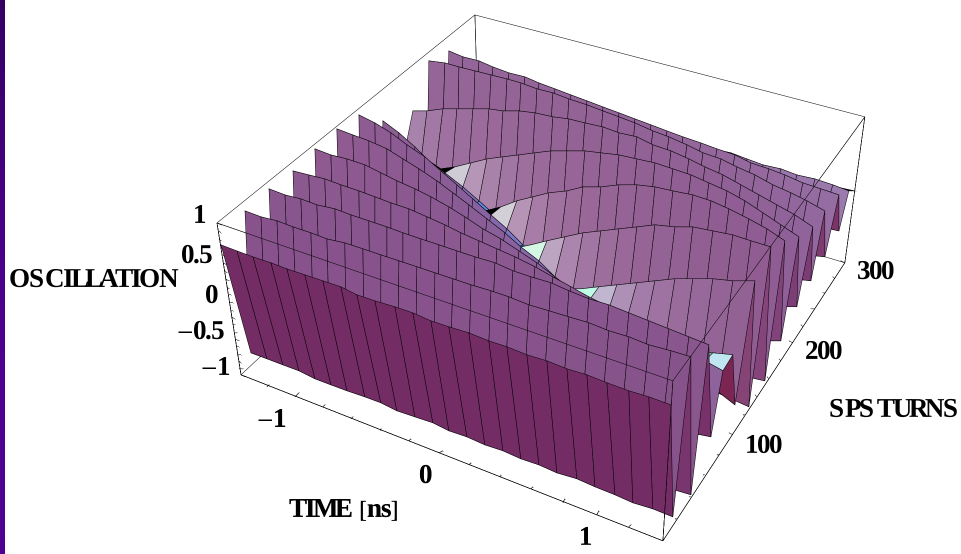
Tail of the bunch



$$n_{recoherence} = \frac{1}{2\,Q_s} \approx 150$$

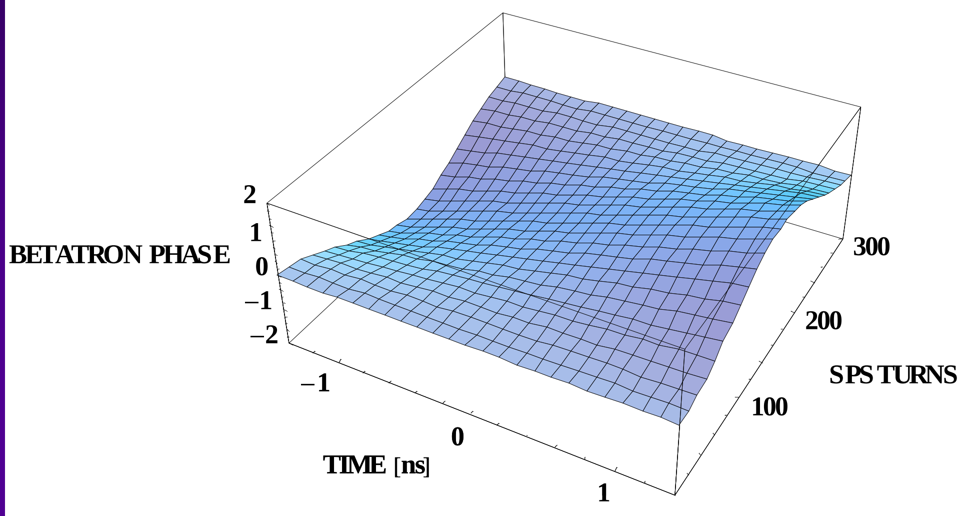
HEAD-TAIL PHASE SHIFT THEORY (3/5)

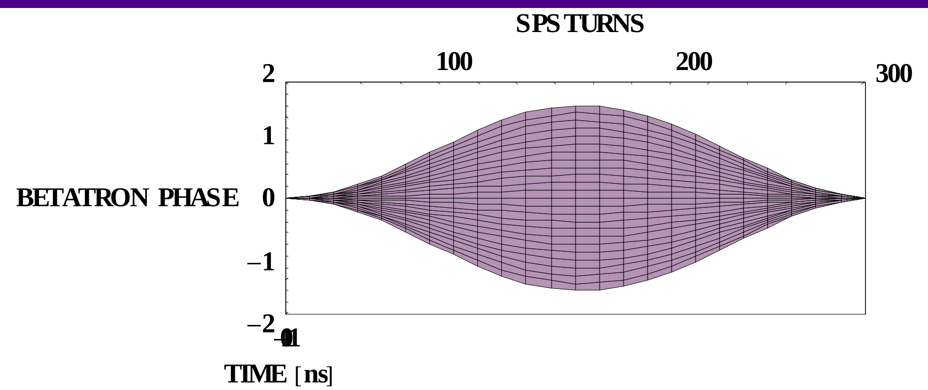
$$< y > (\tau , n)$$



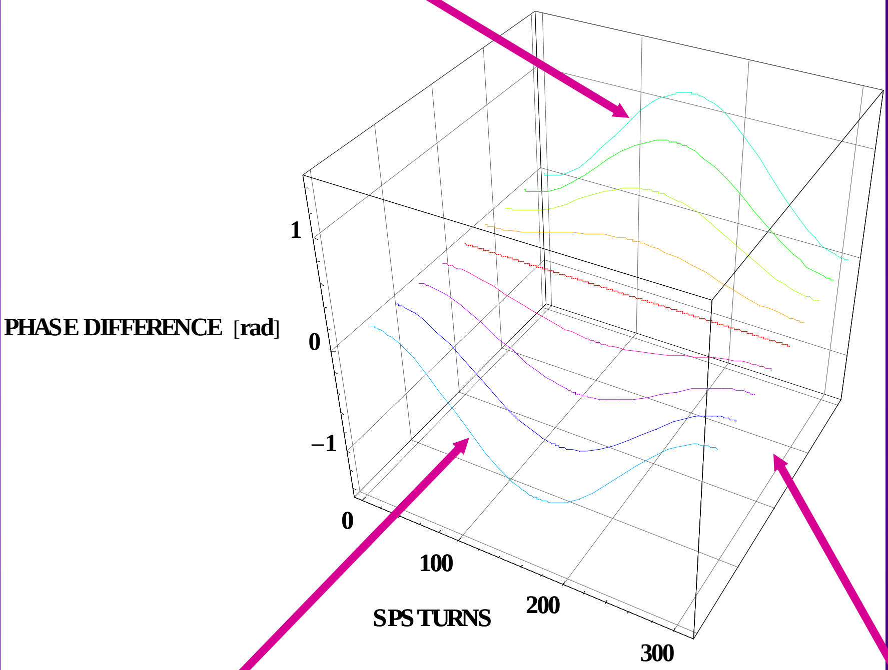
HEAD-TAIL PHASE SHIFT THEORY (4/5)

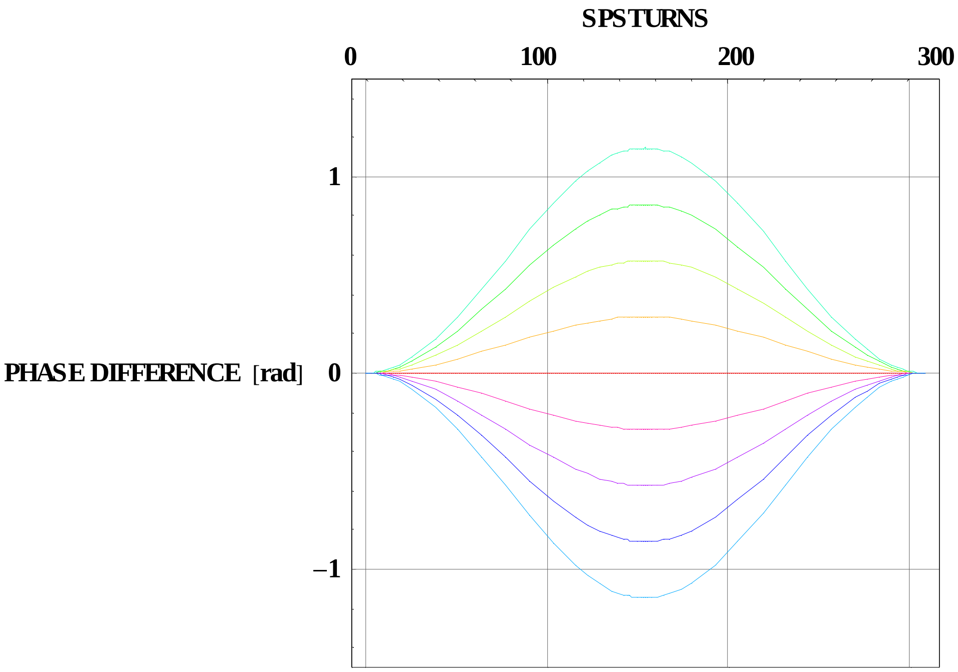
$\phi_{\beta}(\tau, n)$





HEAD-TAIL PHASE SHIFT THEORY (5/5)





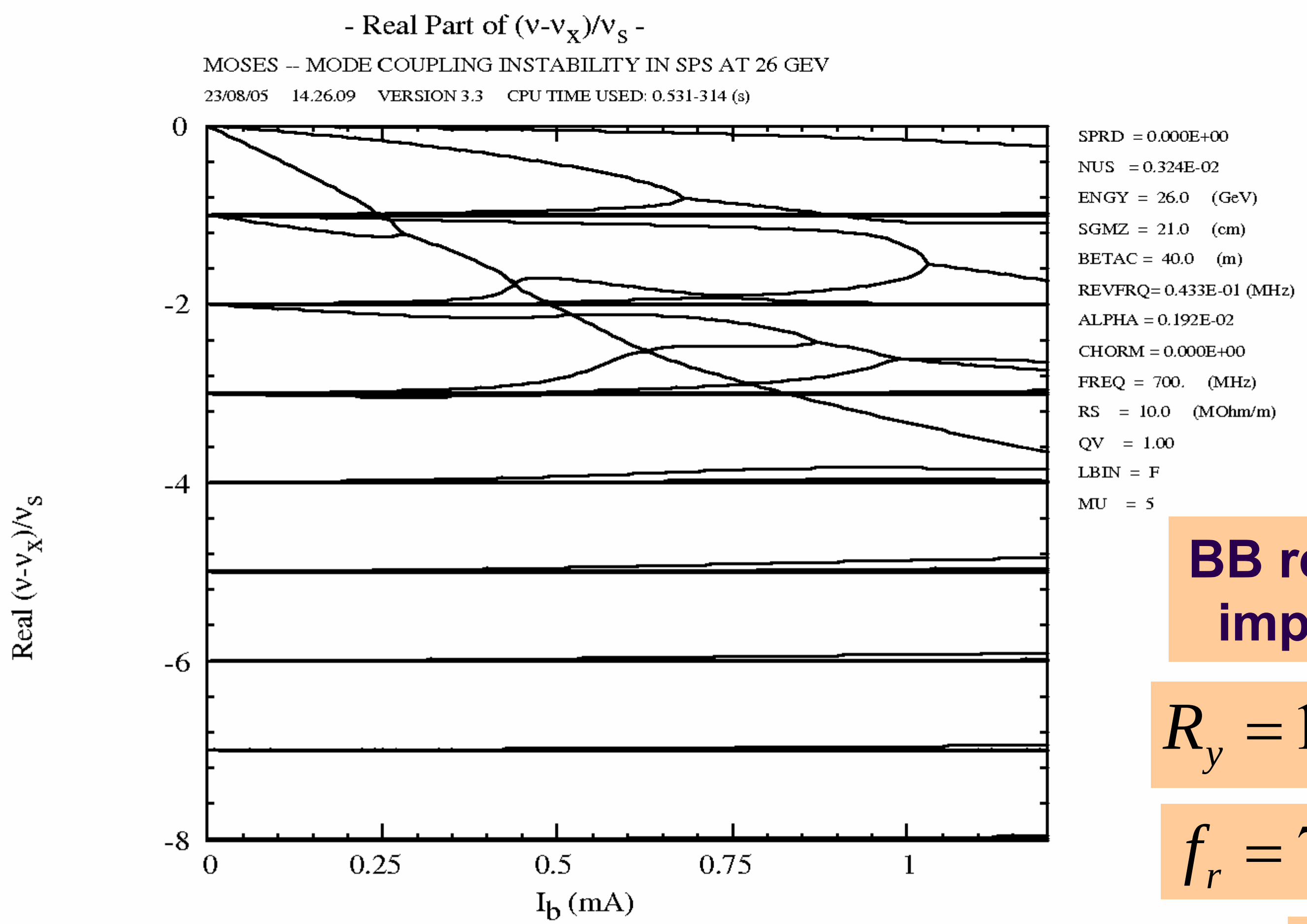
Slice following the central one by 1 ns

Slice preceding the central one by 1 ns

Every 250 ps shown

HEAD-TAIL & TMC INSTABILITY THEORY (1/27)

Computation with MOSES for 0 chromaticity



BB resonator impedance

$$R_y = 10 \text{ M}\Omega/\text{m}$$

$$f_r = 700 \text{ MHz}$$

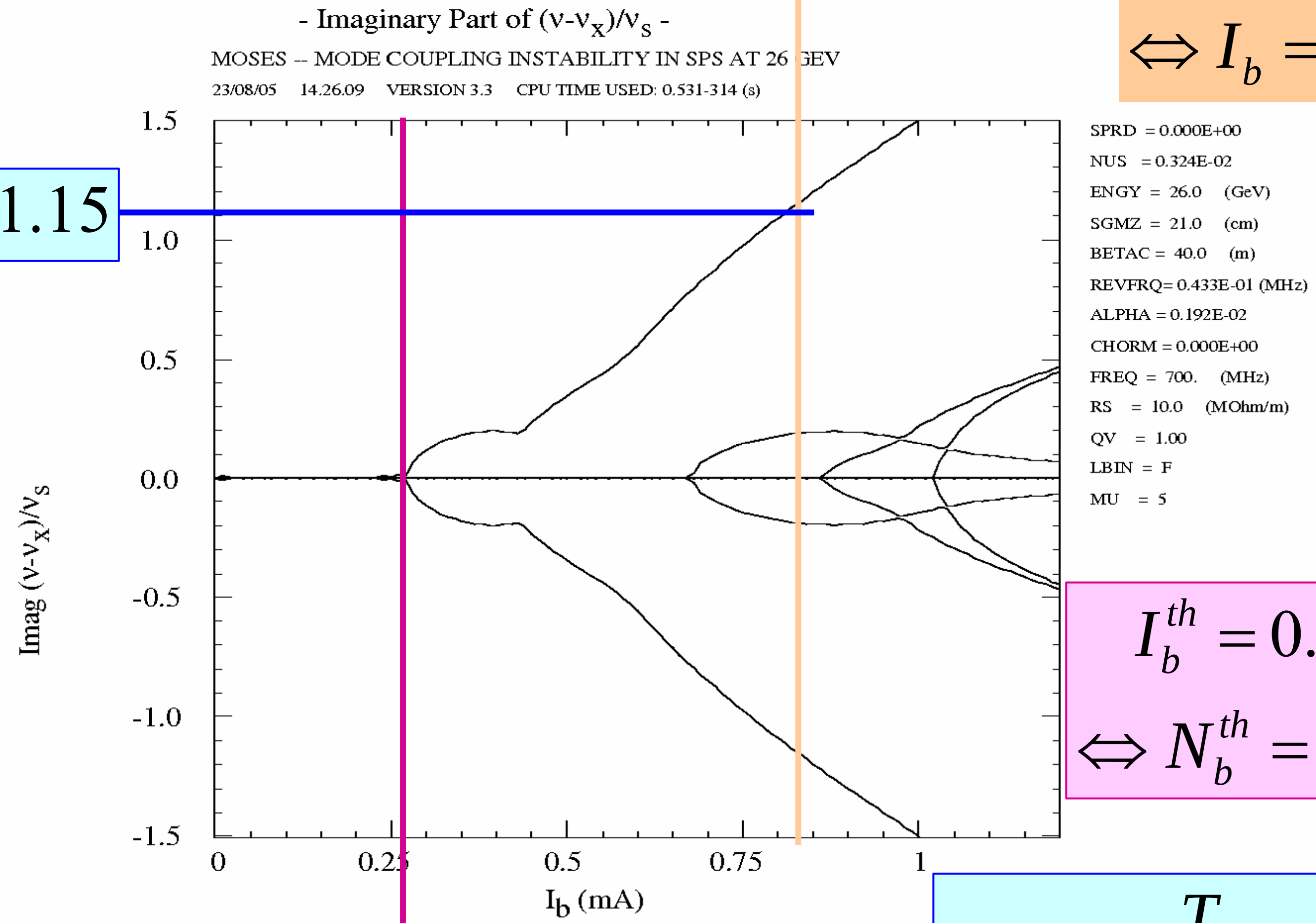
$$Q = 1$$

HEAD-TAIL & TMC INSTABILITY THEORY (2/27)

$$I_b = 0.83\,\mathrm{mA}$$

$$N_b = 1.2\,10^{11}\ \mathrm{p} \Leftrightarrow I_b = 0.83\,\mathrm{mA}$$

$$\alpha \approx 1.15$$



$$I_b^{th} = 0.26\,\mathrm{mA} \Leftrightarrow N_b^{th} = 3.7\,10^{10}\ \mathrm{p}$$

$$I_b^{th} \approx 0.26\,\mathrm{mA}$$

$$\tau_{\mathrm{TMC}} = \frac{T_s}{2\,\pi\,\alpha} = 43\ \ \mathrm{SPS\ turns}$$

HEAD-TAIL & TMC INSTABILITY THEORY (3/27)

If we suppose an initial offset of ~ 1mm and assume that beam losses occur after a growth of a factor ~ 20 (same approximation as for the BBU mechanism discussed later), then beam losses should occur after ~ 3 rise-times, i.e. after ~ 130 turns

HEAD-TAIL & TMC INSTABILITY THEORY (4/27)

Reminder: Simple formula to have a rough estimate of the TMC instability rise-time (for 0 chromaticity)

$$\tau_{TMC} = \frac{T_s}{\pi \sqrt{\left(\frac{N_b}{N_b^{th}} - 1 \right) \left(\frac{N_b}{N_b^{th}} \, q + 1 \right)}}$$

with $q \in [\, 0 \, , 1 \,]$

$$q = 0 \text{ for short bunch, i.e. } 2 \; f_r \; \tau_b \approx 1$$

$$q = 1 \text{ for long bunch, i.e. } 2 \; f_r \; \tau_b >> 1$$

Numerical application

$$2 \; f_r \; \tau_b = 3.9$$

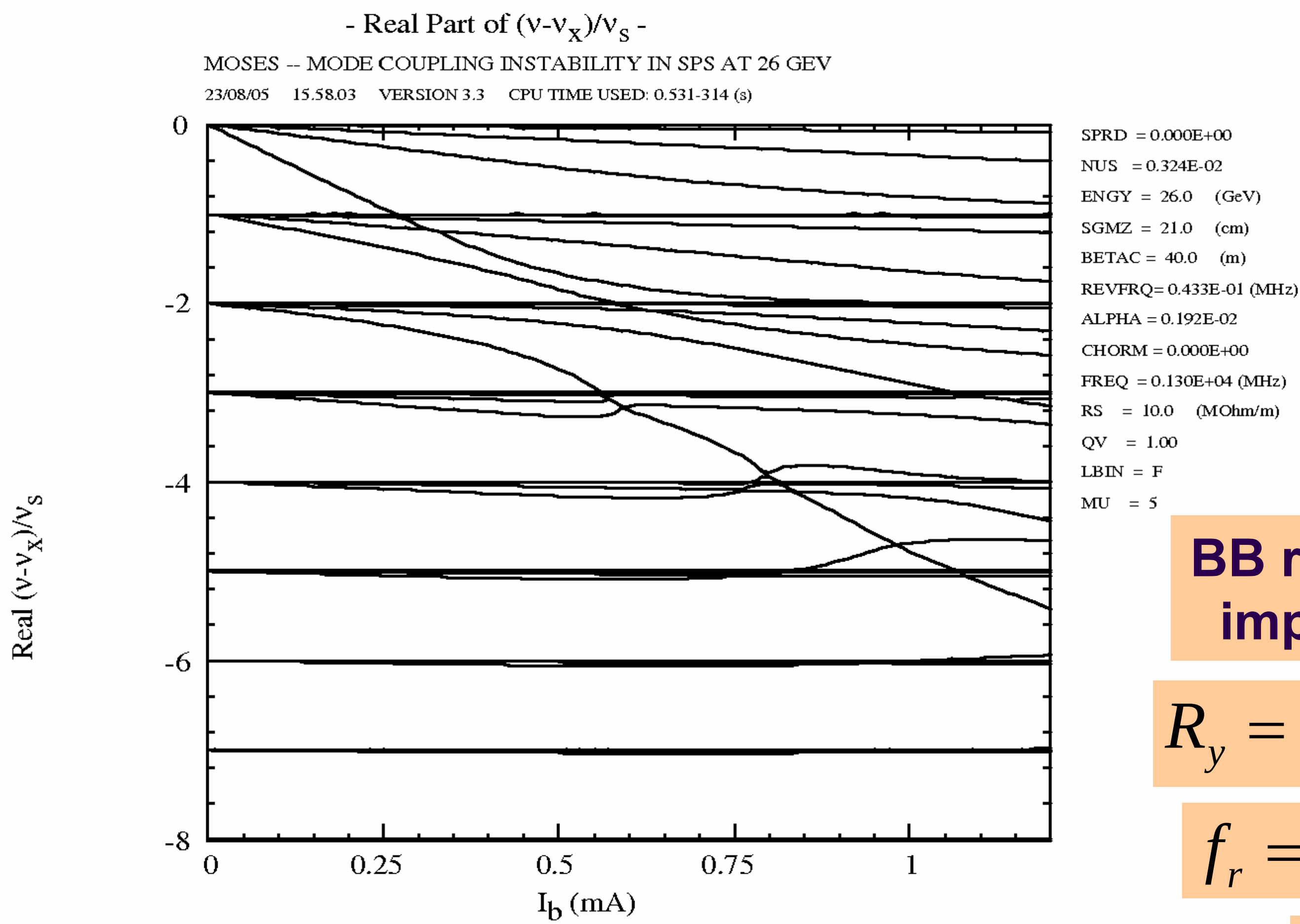
$$\tau_{TMC} \; [\; q = 0 \;] = 66 \;\; \text{SPS turns}$$

$$\boxed{\tau_{TMC} \; [\; q = 1 \;] = 32 \;\; \text{SPS turns}}$$

$$\tau_{TMC} \; [\; q = 0.5 \;] = 41 \;\; \text{SPS turns}$$

Faster by ~25% compared to MOSES

HEAD-TAIL & TMC INSTABILITY THEORY (5/27)



BB resonator impedance

$$R_y = 10\ \mathrm{M\Omega/m}$$

$$f_r = 1.3\ \mathrm{GHz}$$

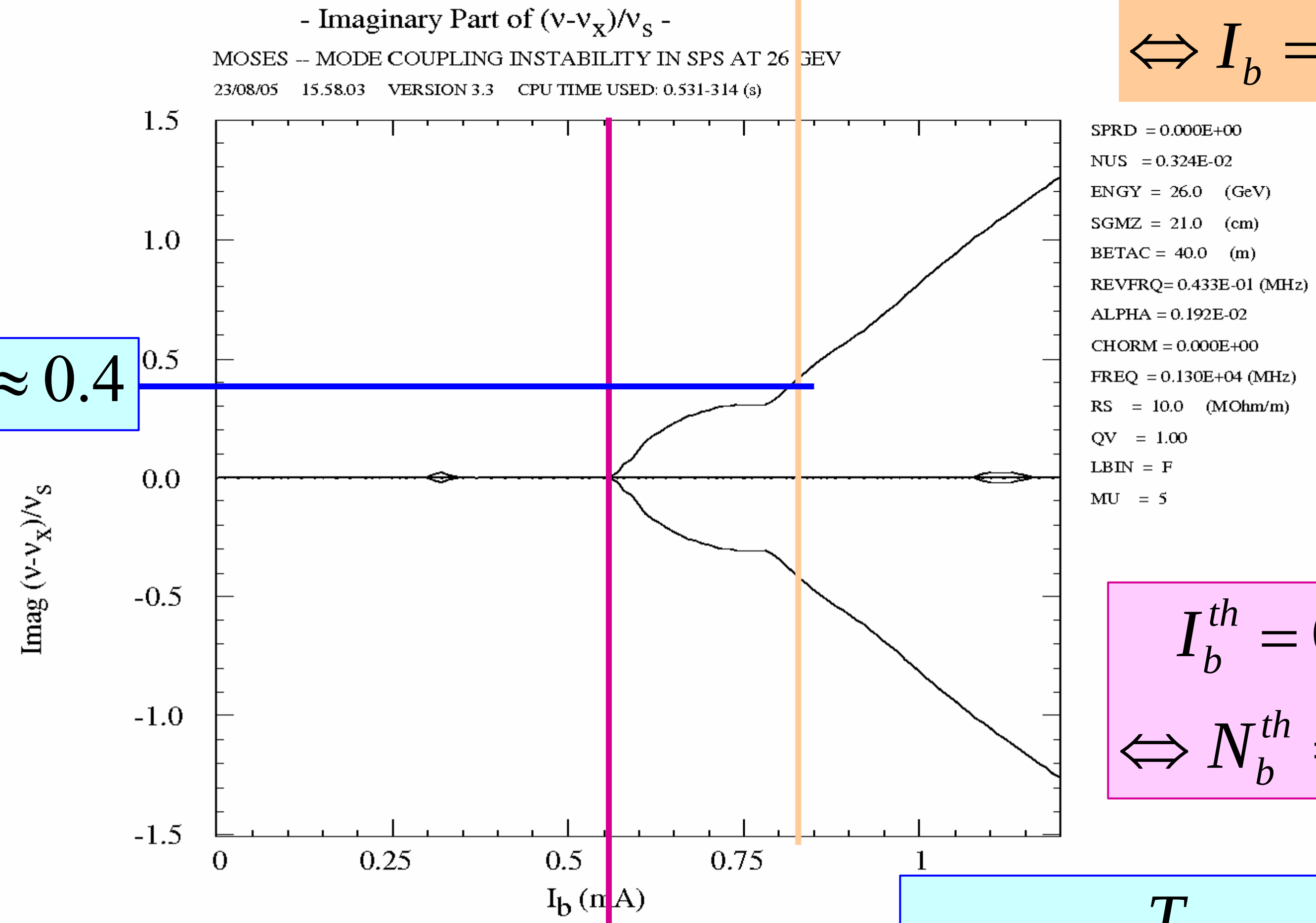
$$Q = 1$$

HEAD-TAIL & TMC INSTABILITY THEORY (6/27)

$I_b = 0.83 \text{ mA}$

$N_b = 1.2\,10^{11}\text{ p}$

$\Leftrightarrow I_b = 0.83 \text{ mA}$



$\alpha \approx 0.4$

$I_b^{th} = 0.6 \text{ mA}$

$\Leftrightarrow N_b^{th} = 8.7\,10^{10}\text{ p}$

$I_b^{th} \approx 0.6 \text{ mA}$

$$\tau_{\text{TMC}} = \frac{T_s}{2\,\pi\,\alpha} = 123 \text{ SPS turns}$$

HEAD-TAIL & TMC INSTABILITY THEORY (7/27)

Using the simple formula one has

$$2\ f_r\ \tau_b = 7.3$$

$$\tau_{TMC}\ [\ q = 0\ ] = 158\ \ \text{SPS turns}$$

$$\boxed{\tau_{TMC}\ [\ q = 1\ ] = 102\ \ \text{SPS turns}}$$

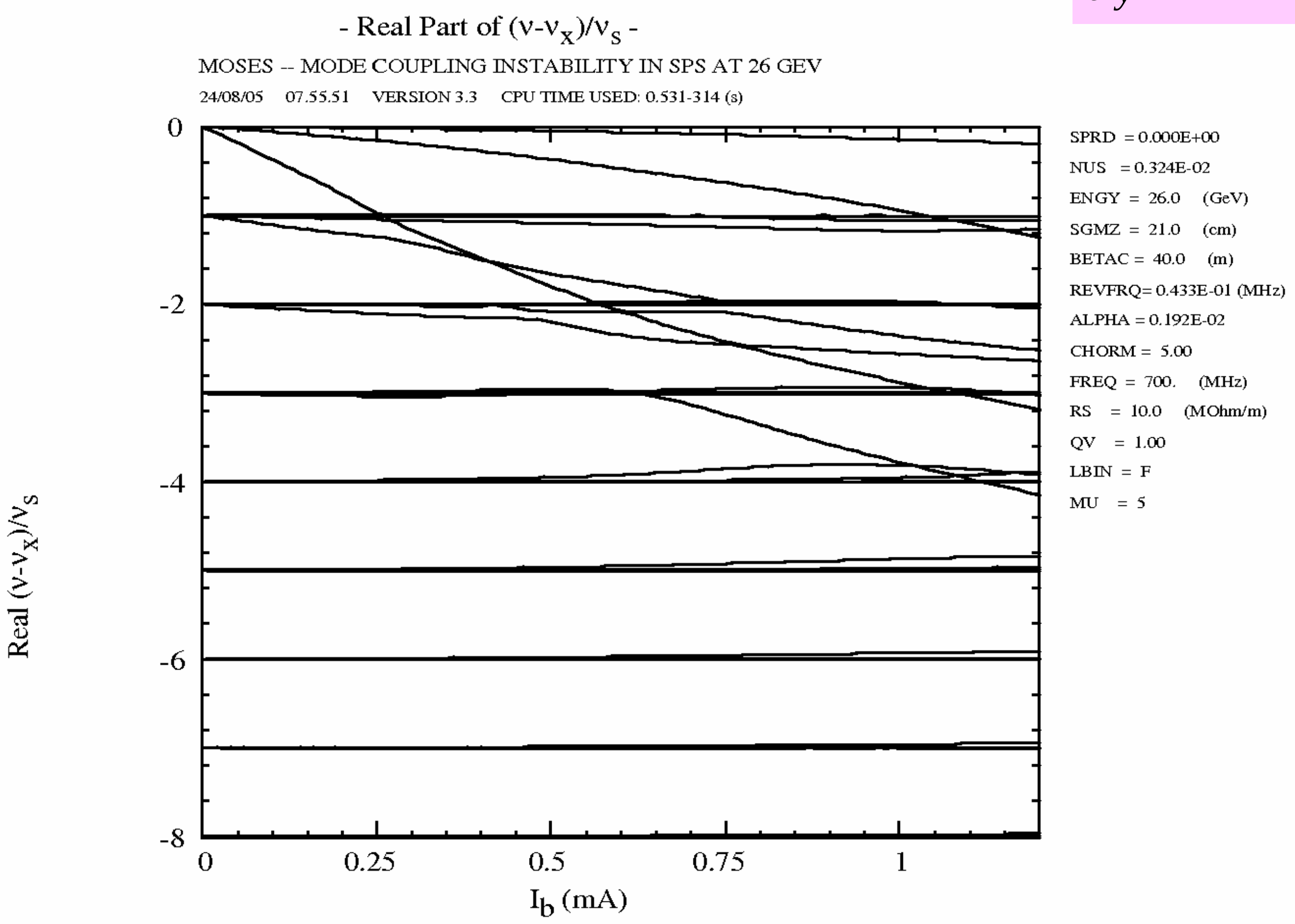
$$\tau_{TMC}\ [\ q = 0.5\ ] = 121\ \ \text{SPS turns}$$

Faster by ~17% compared to MOSES

HEAD-TAIL & TMC INSTABILITY THEORY (8/27)

Computation with MOSES for nonzero chromaticities

$$\xi_y = 5 / Q_y \approx 0.19$$

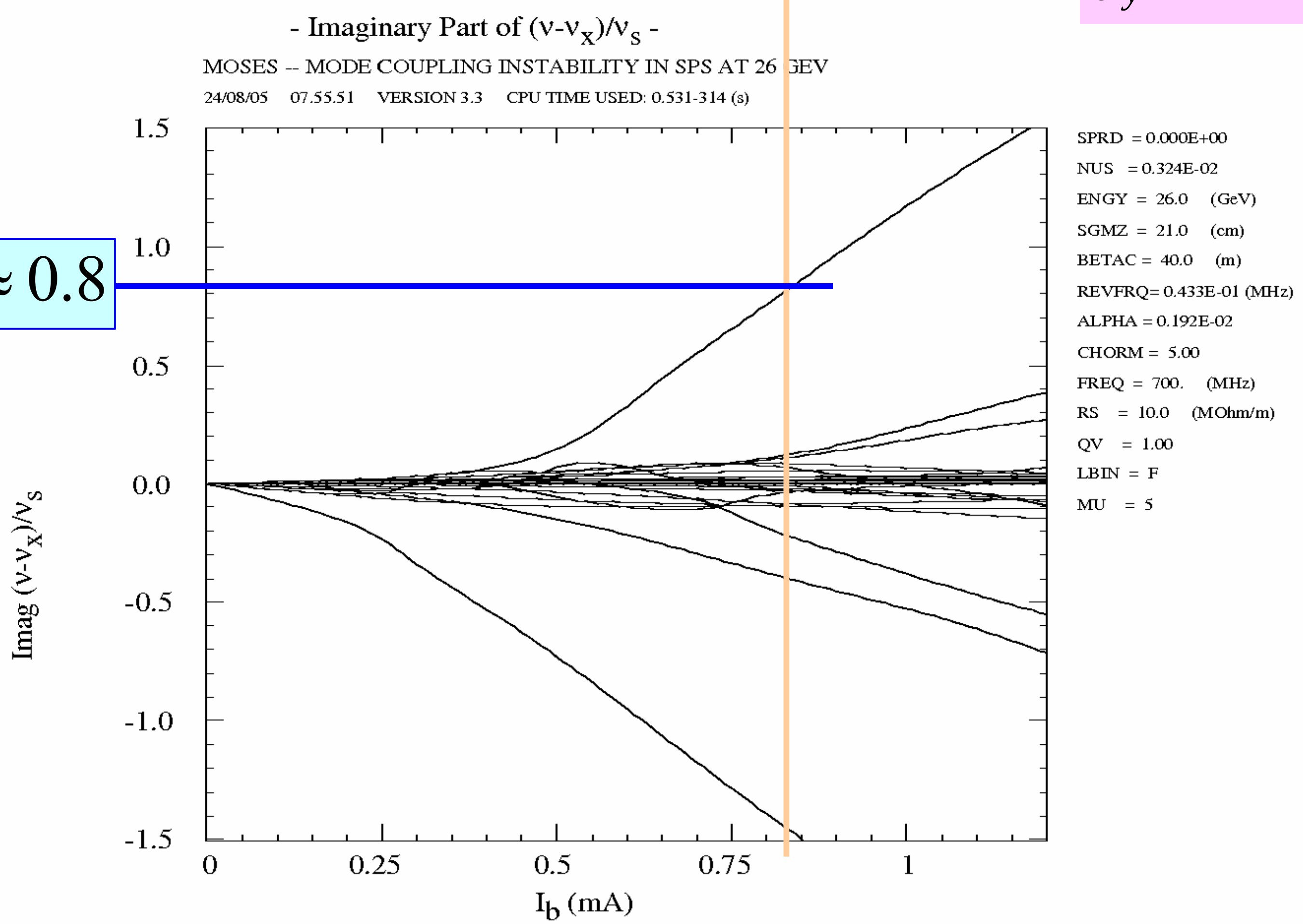


HEAD-TAIL & TMC INSTABILITY THEORY (9/27)

$I_b = 0.83\,\mathrm{mA}$

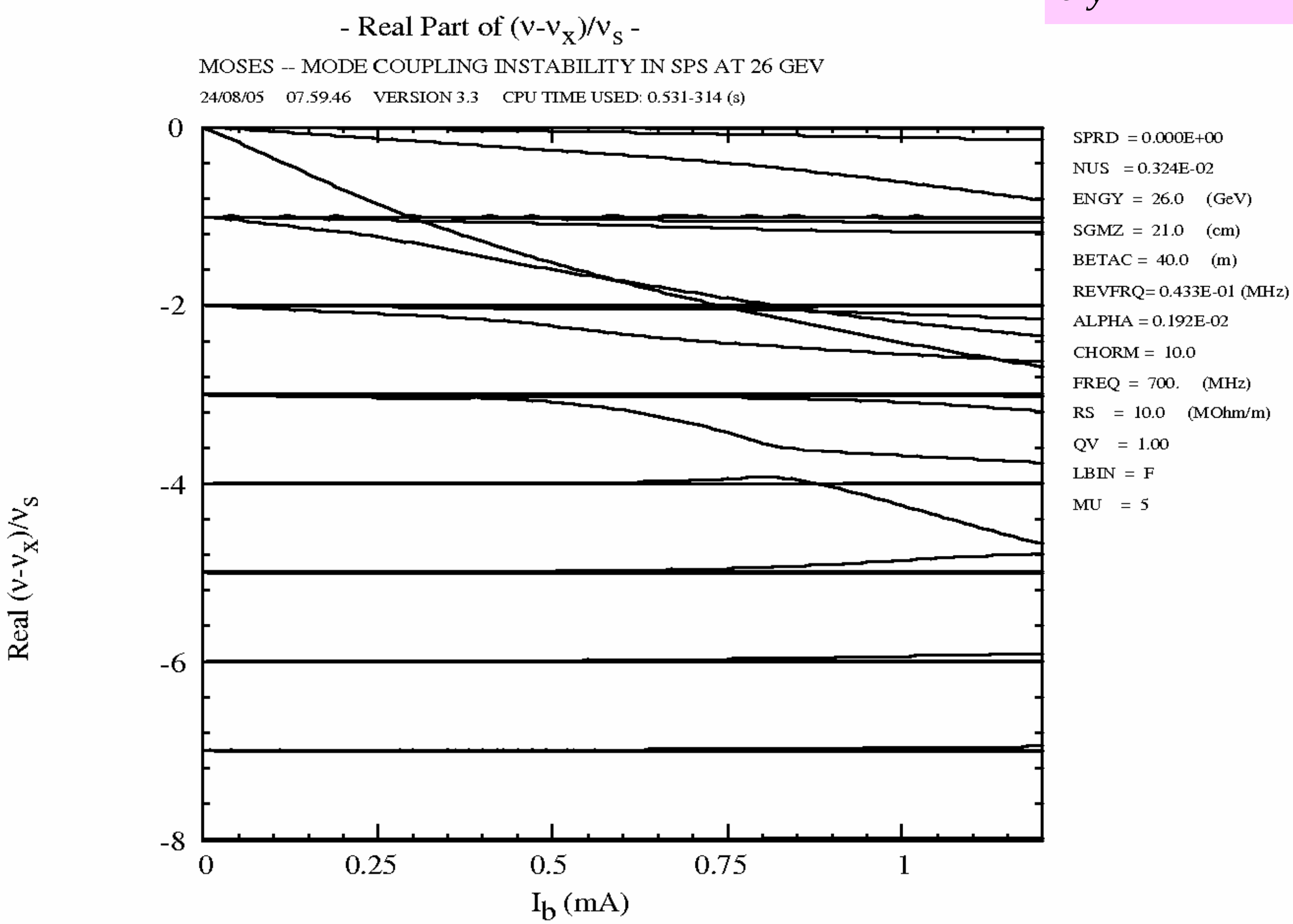
$\xi_y = 5 / Q_y \approx 0.19$

$\alpha \approx 0.8$

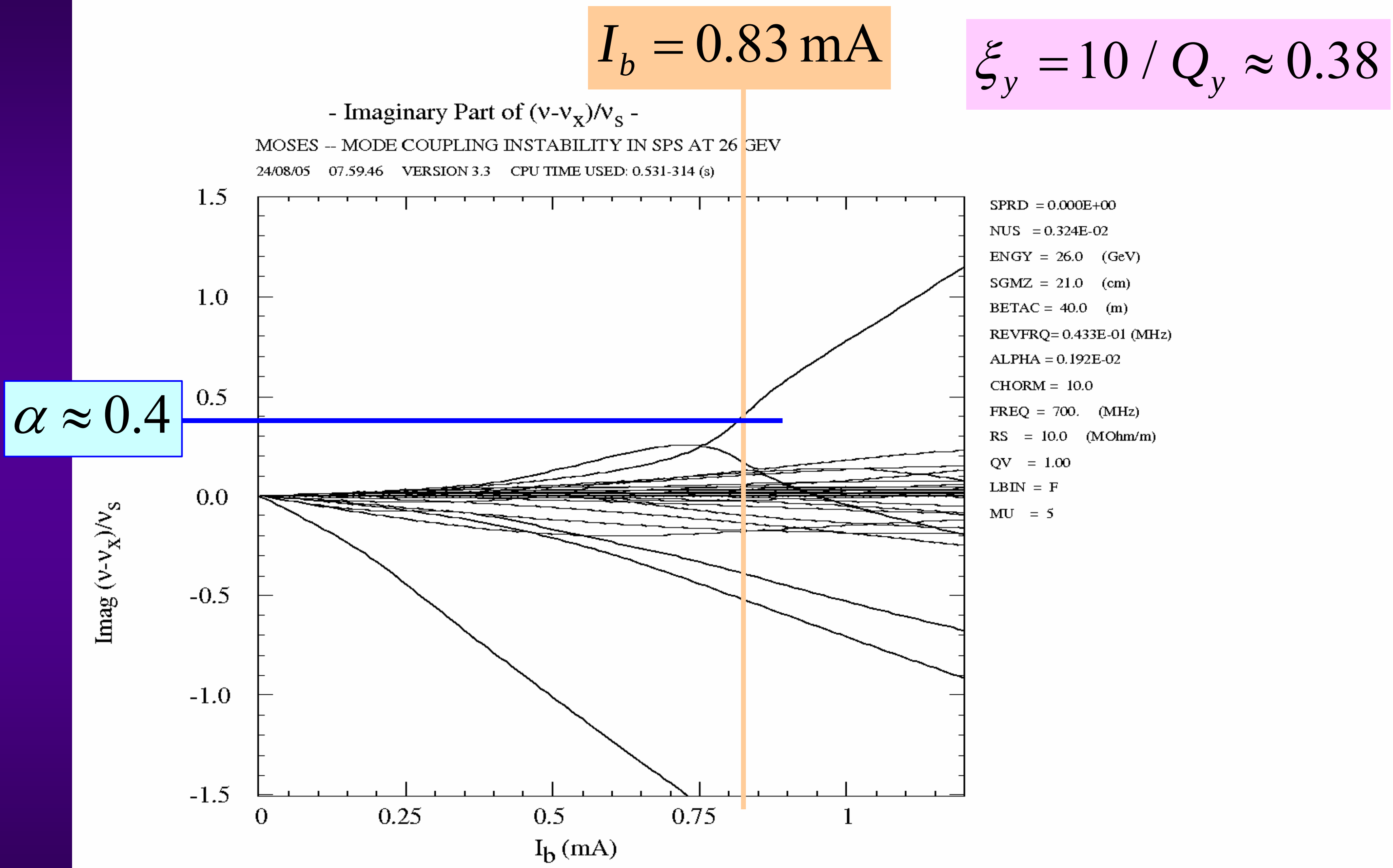


HEAD-TAIL & TMC INSTABILITY THEORY (10/27)

$$\xi_y = 10 / Q_y \approx 0.38$$

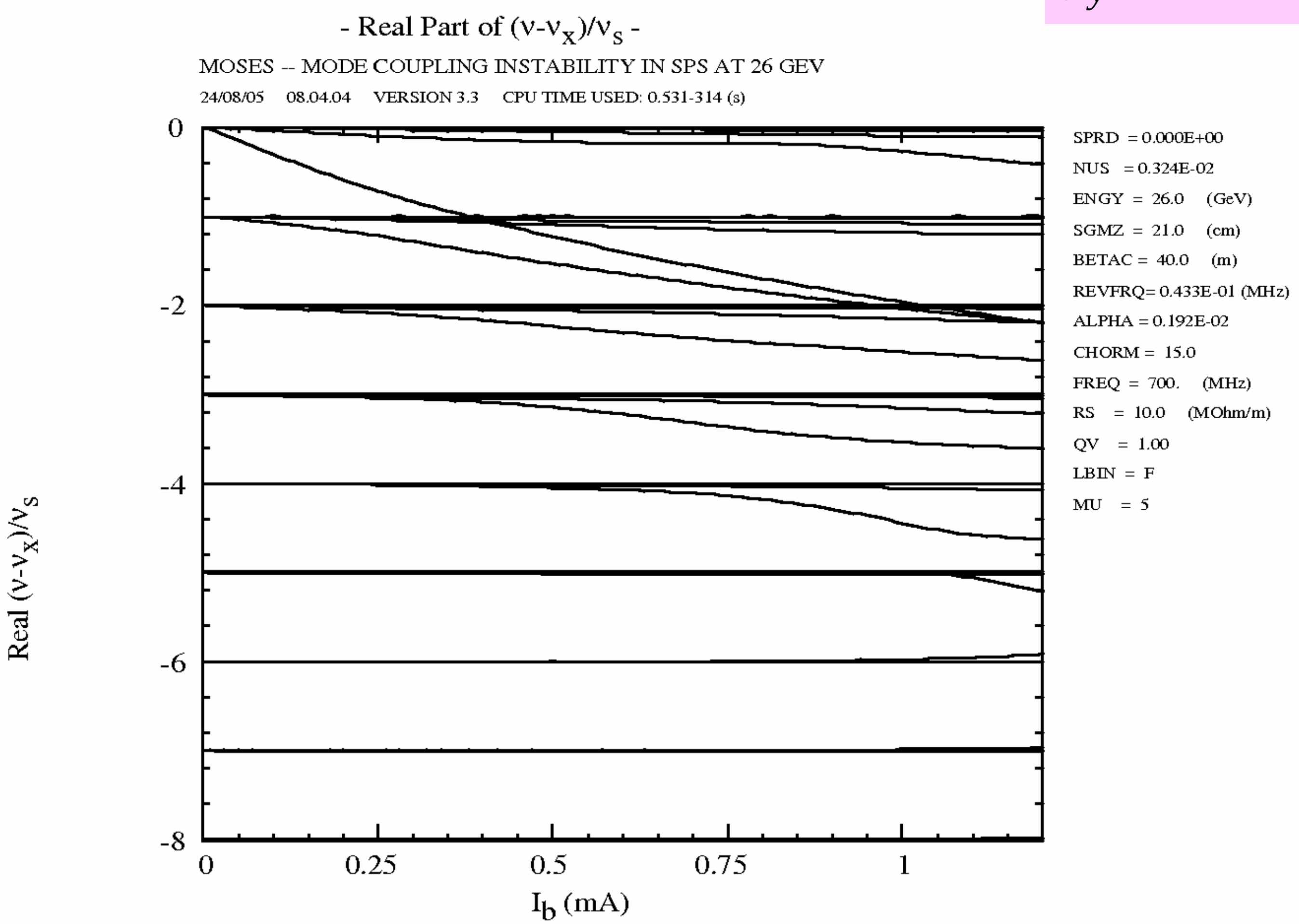


HEAD-TAIL & TMC INSTABILITY THEORY (11/27)



HEAD-TAIL & TMC INSTABILITY THEORY (12/27)

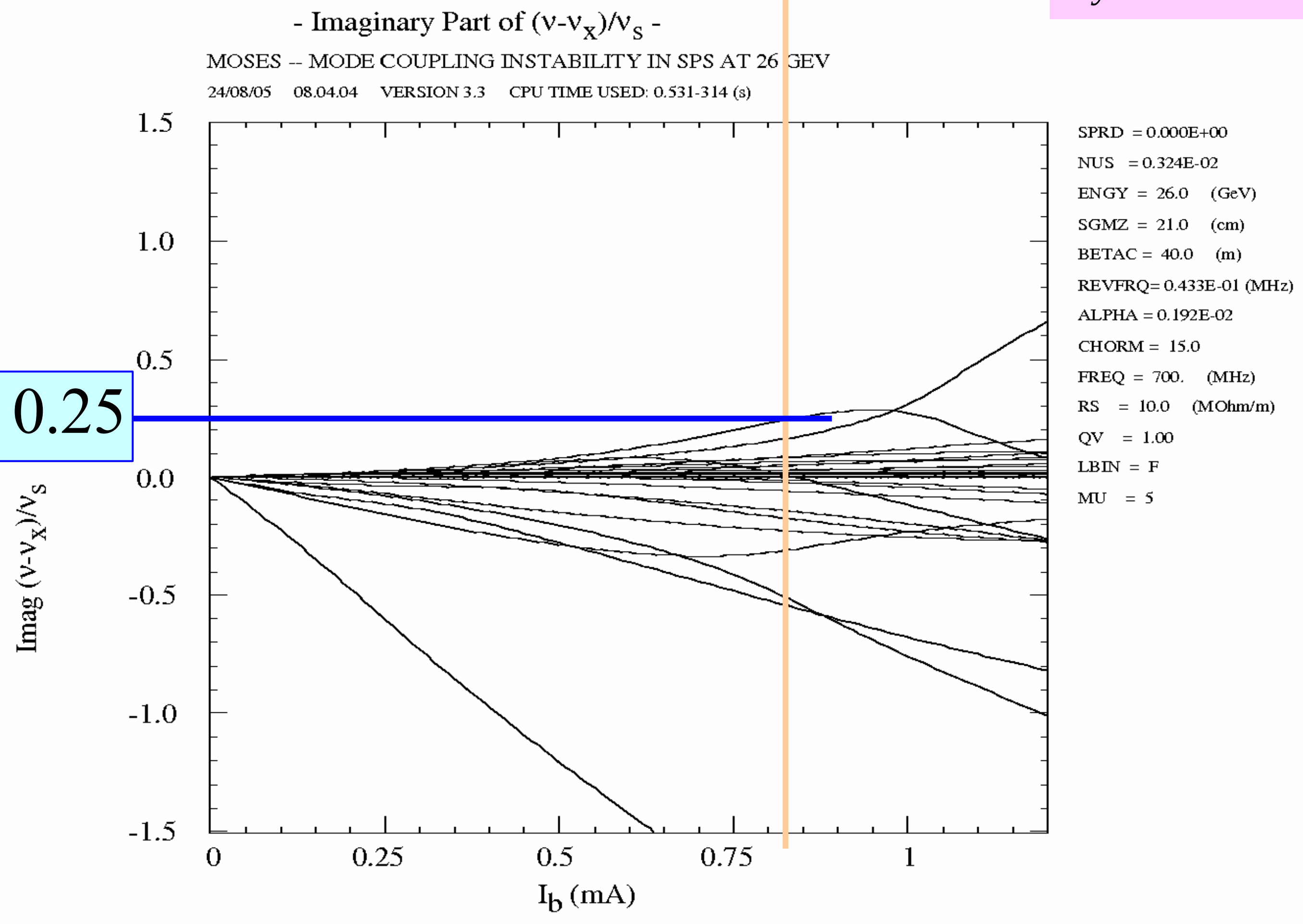
$$\xi_y = 15 / Q_y \approx 0.57$$



HEAD-TAIL & TMC INSTABILITY THEORY (13/27)

$I_b = 0.83\,\mathrm{mA}$

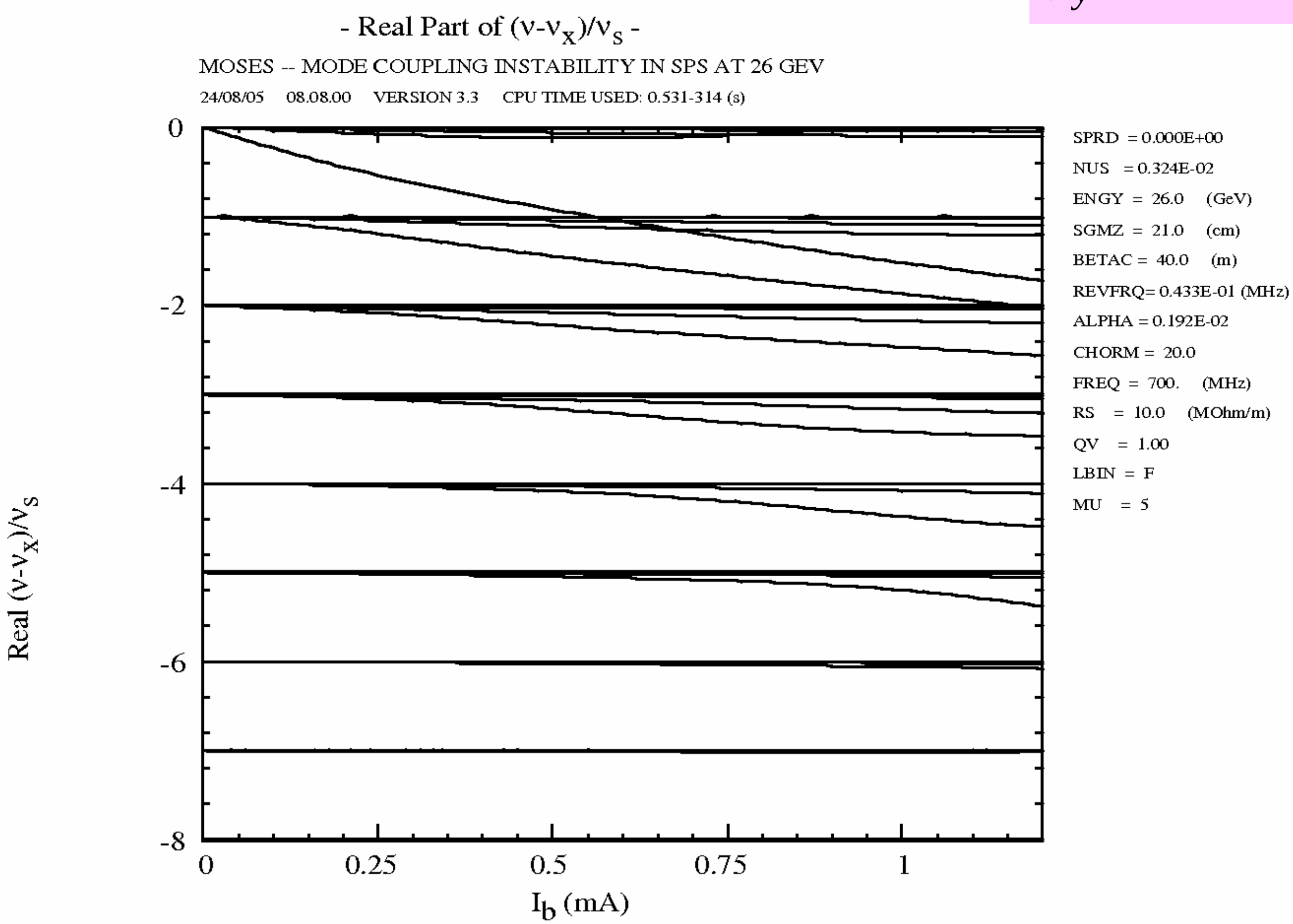
$\xi_y = 15 / Q_y \approx 0.57$



$\alpha \approx 0.25$

HEAD-TAIL & TMC INSTABILITY THEORY (14/27)

$$\xi_y = 20 \,/\, Q_y \approx 0.77$$

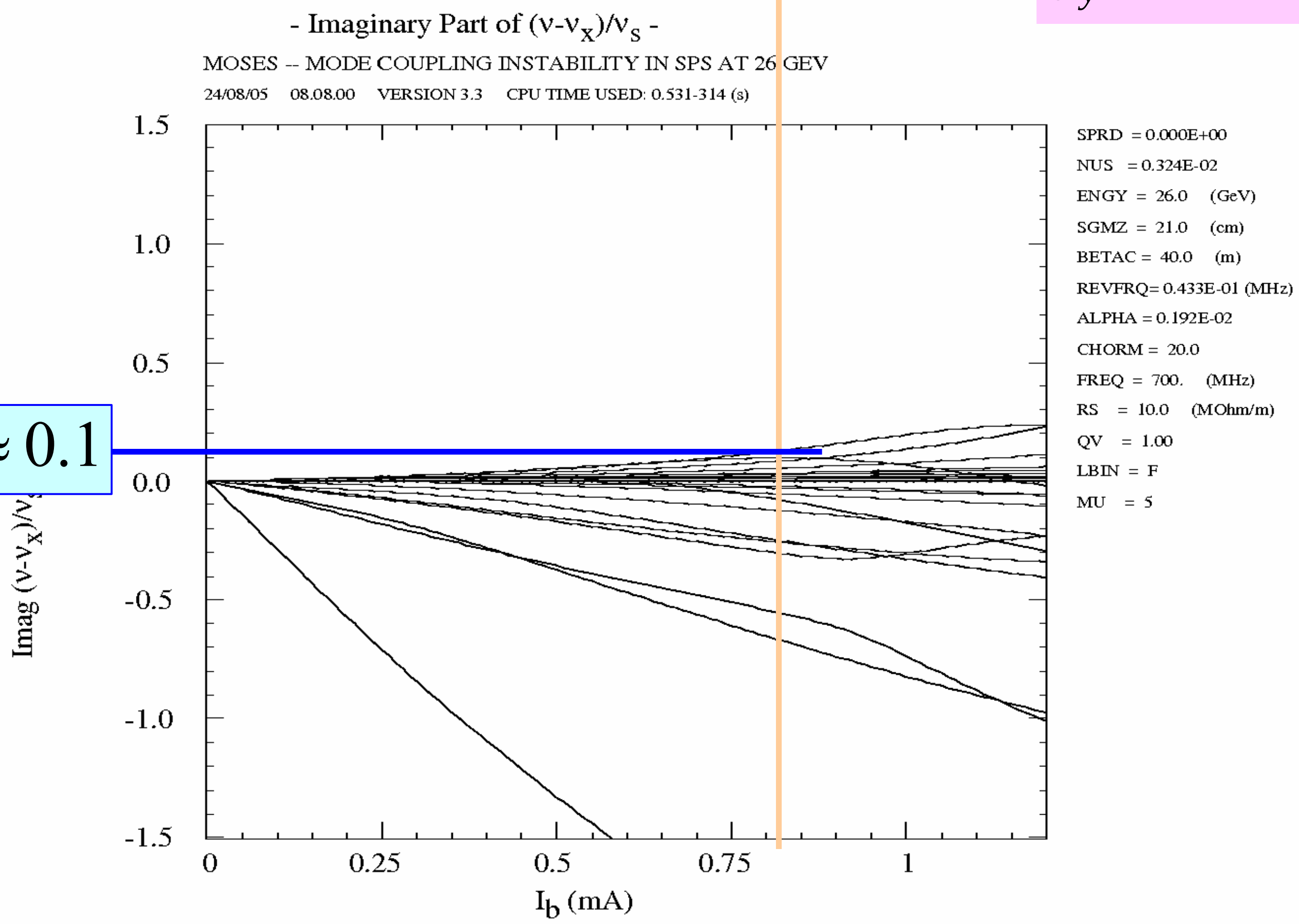


HEAD-TAIL & TMC INSTABILITY THEORY (15/27)

$I_b = 0.83\,\mathrm{mA}$

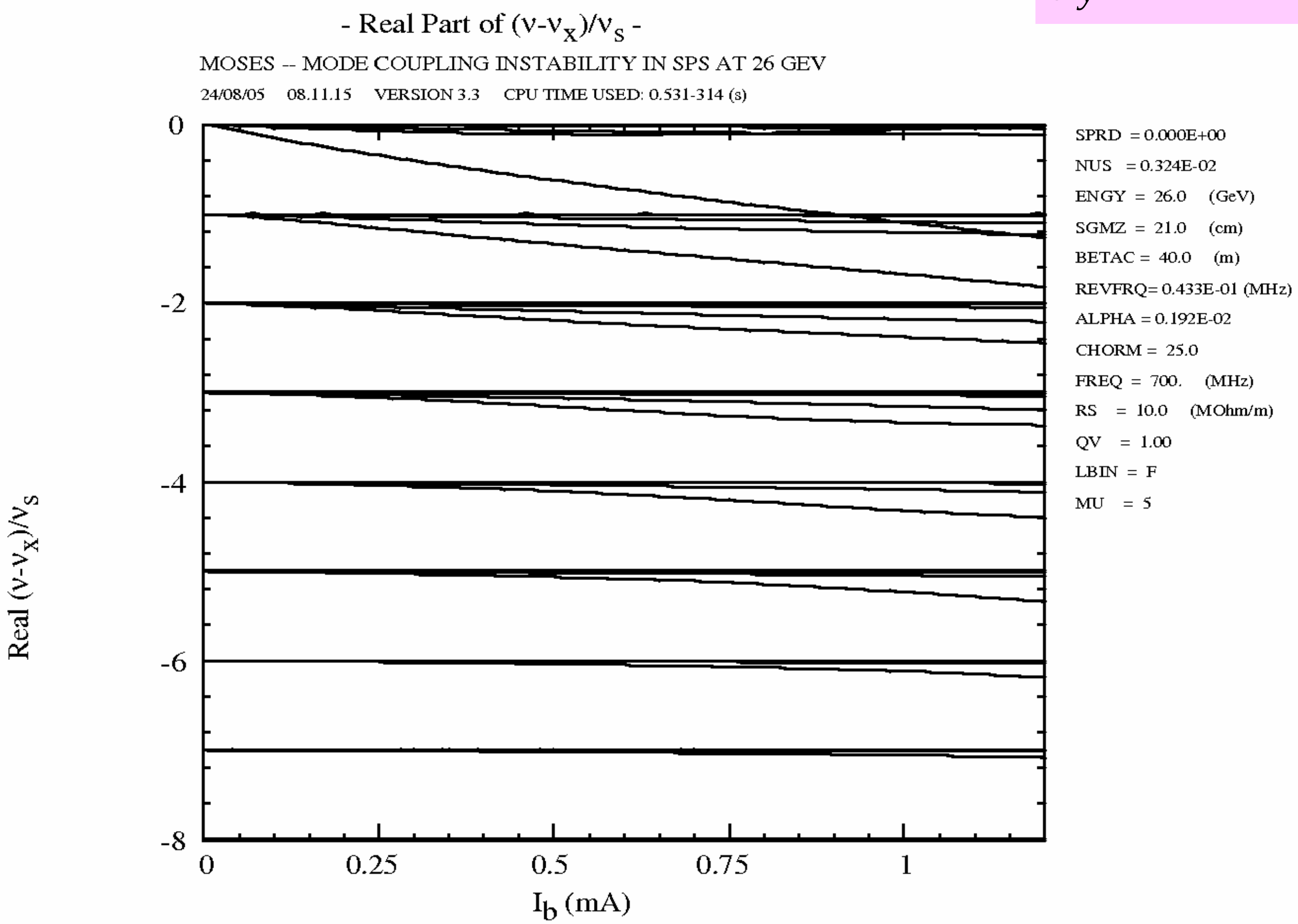
$\xi_y = 20 \,/\, Q_y \approx 0.77$

$\alpha \approx 0.1$



HEAD-TAIL & TMC INSTABILITY THEORY (16/27)

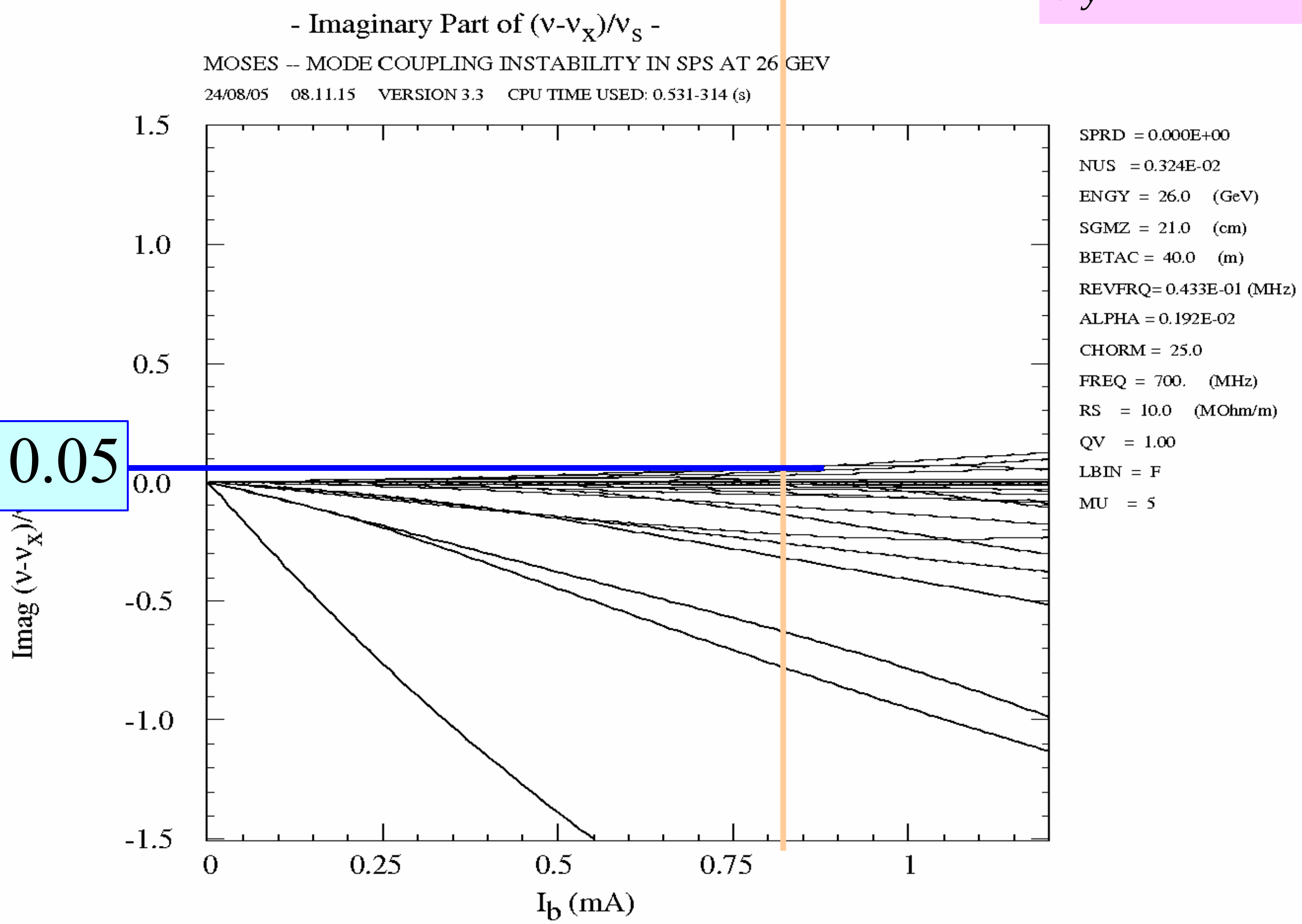
$$\xi_y = 25 \, / \, Q_y \approx 0.97$$



HEAD-TAIL & TMC INSTABILITY THEORY (17/27)

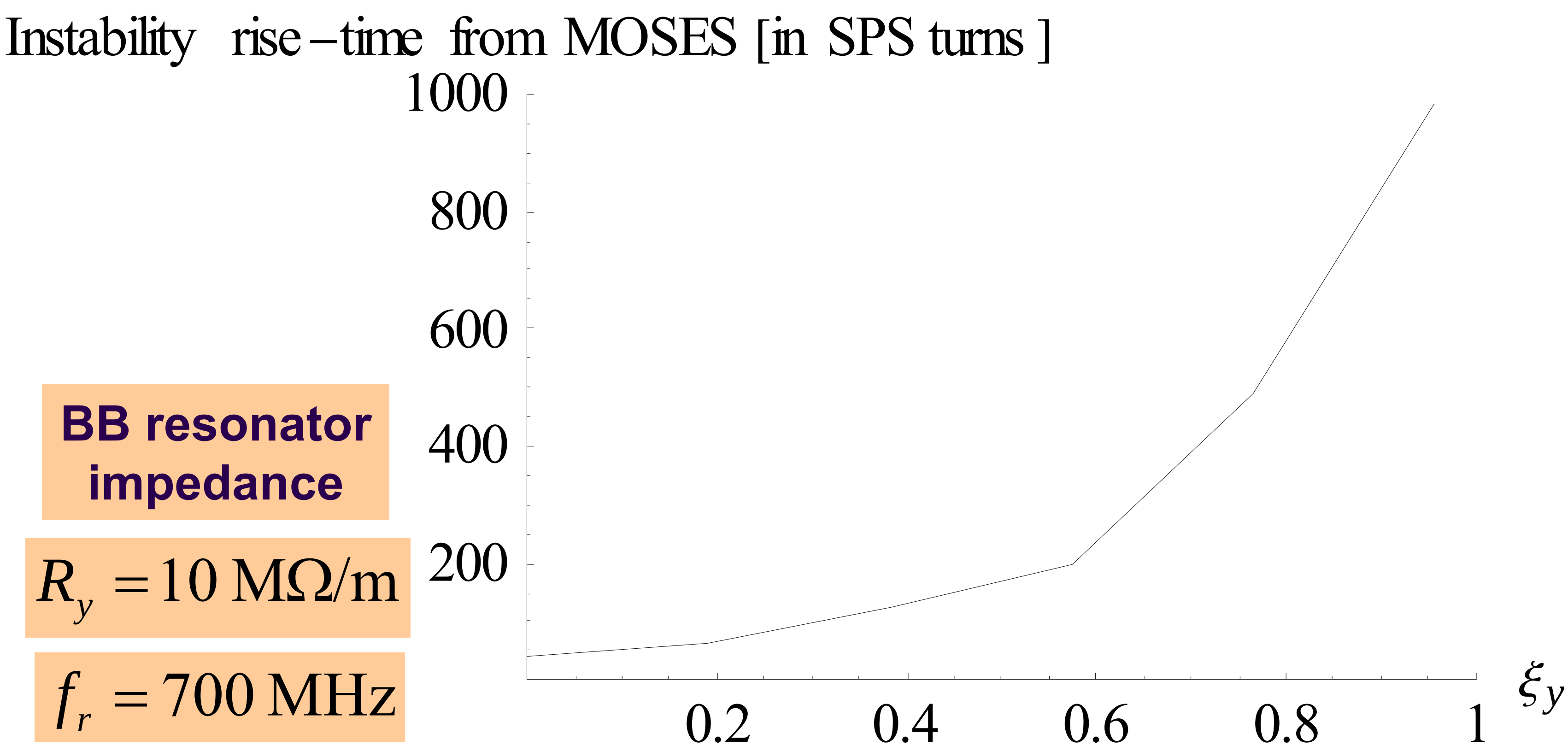
$I_b = 0.83\,\mathrm{mA}$

$\xi_y = 25 / Q_y \approx 0.97$



$\alpha \approx 0.05$

HEAD-TAIL & TMC INSTABILITY THEORY (18/27)



BB resonator impedance

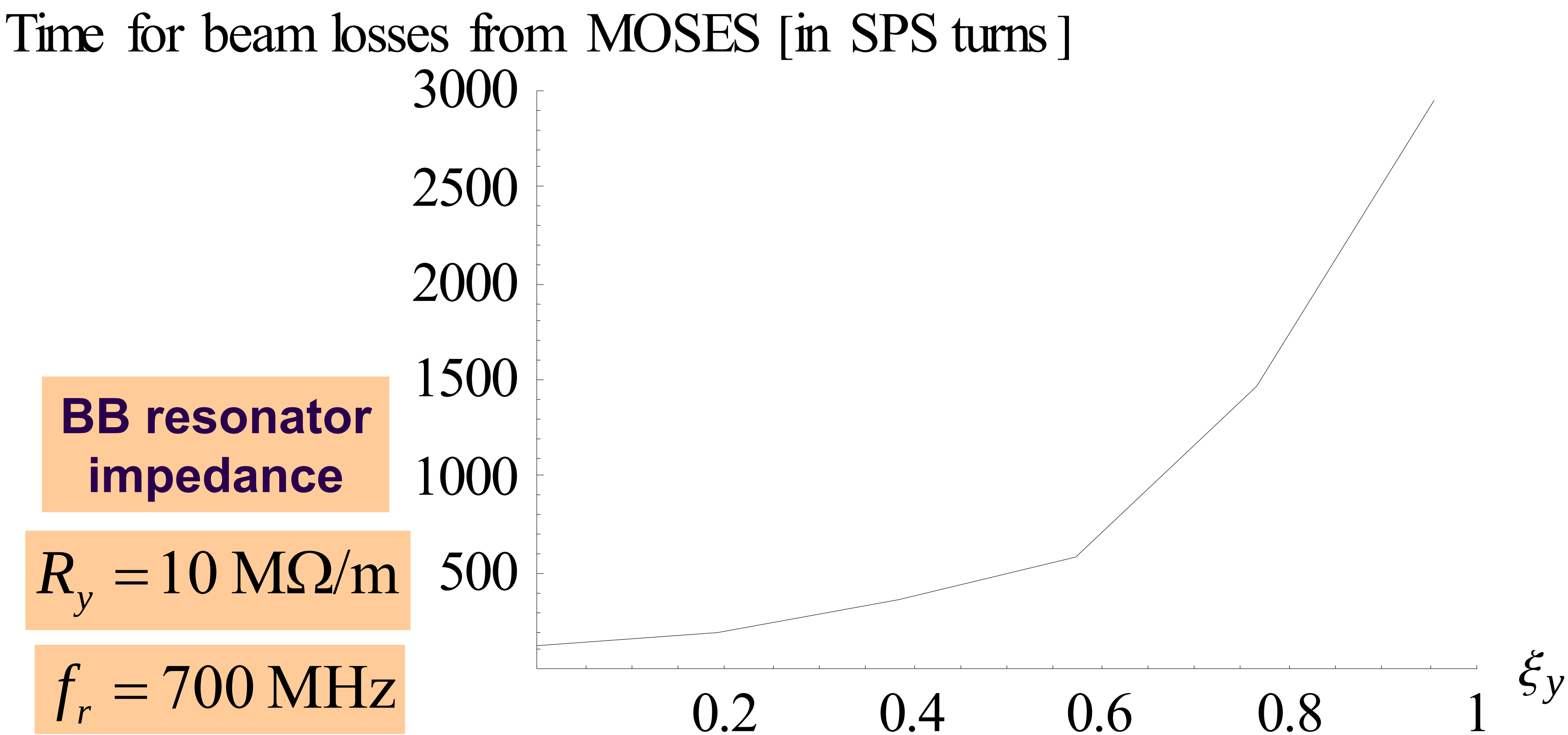
$R_y = 10\ \mathrm{M\Omega/m}$

$f_r = 700\ \mathrm{MHz}$

$Q = 1$

HEAD-TAIL & TMC INSTABILITY THEORY (19/27)

Assuming beam losses after 3 rise-times, i.e. after an amplification by a factor 20



BB resonator impedance

$R_y = 10 \text{ M}\Omega/\text{m}$

$f_r = 700 \text{ MHz}$

$Q = 1$

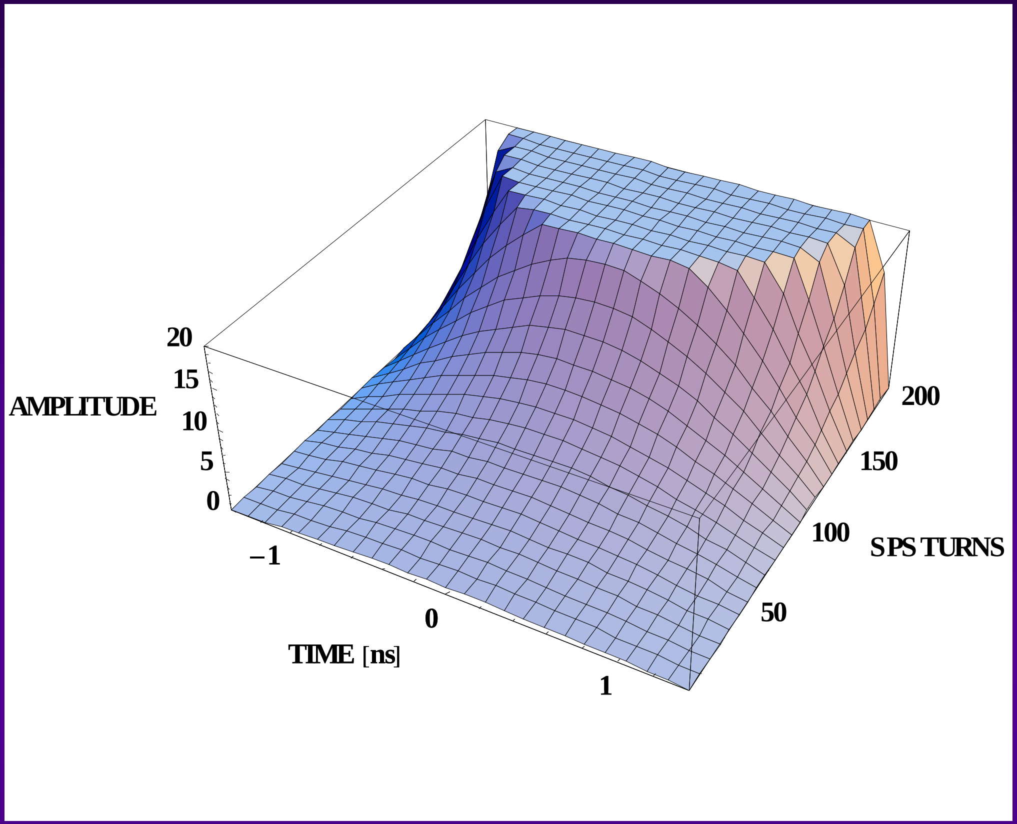
HEAD-TAIL & TMC INSTABILITY THEORY (20/27)

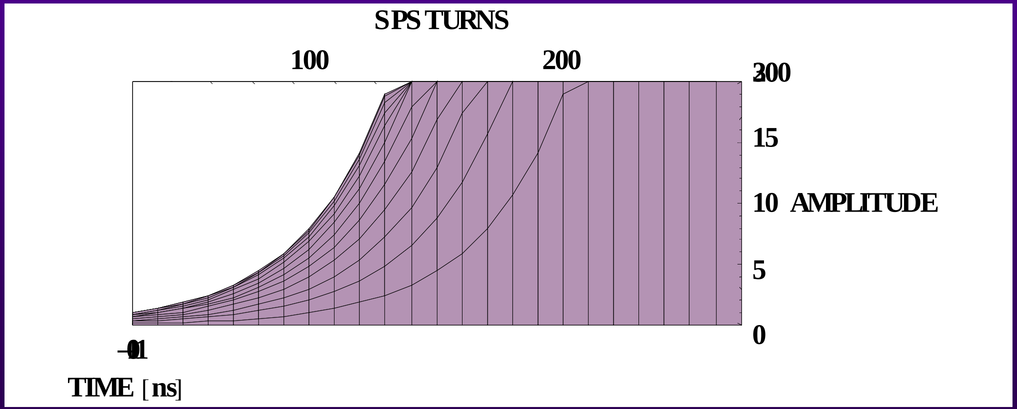
IF the different head-tail modes could be treated separately (i.e. below the TMC threshold), one would expect vertical signals as below (assuming the same rise-time from MOSES)

HEAD-TAIL & TMC INSTABILITY THEORY (21/27)

- **For mode**

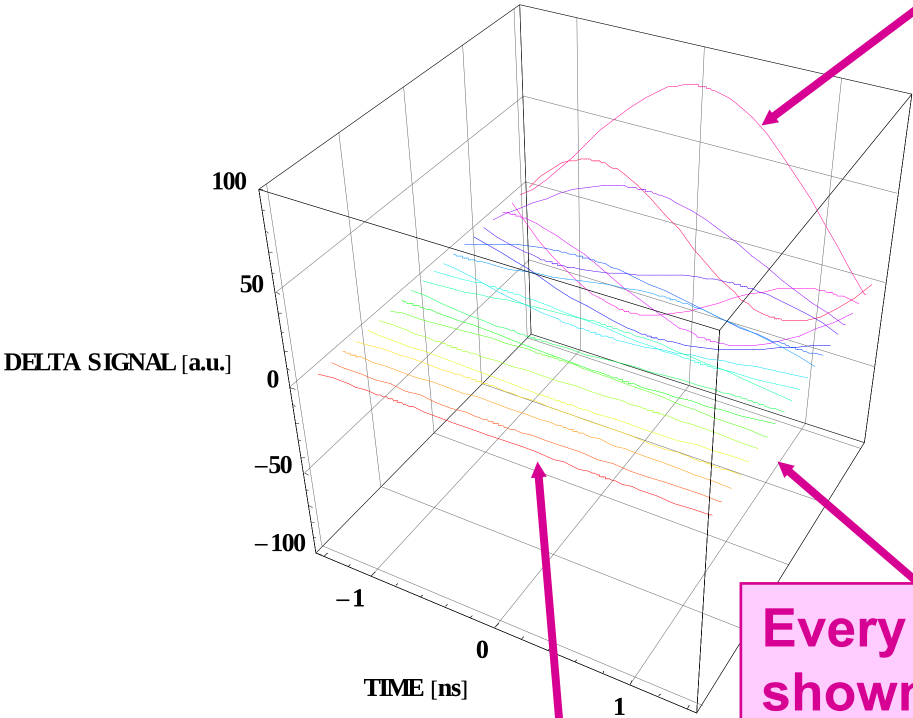
$$m = 0$$





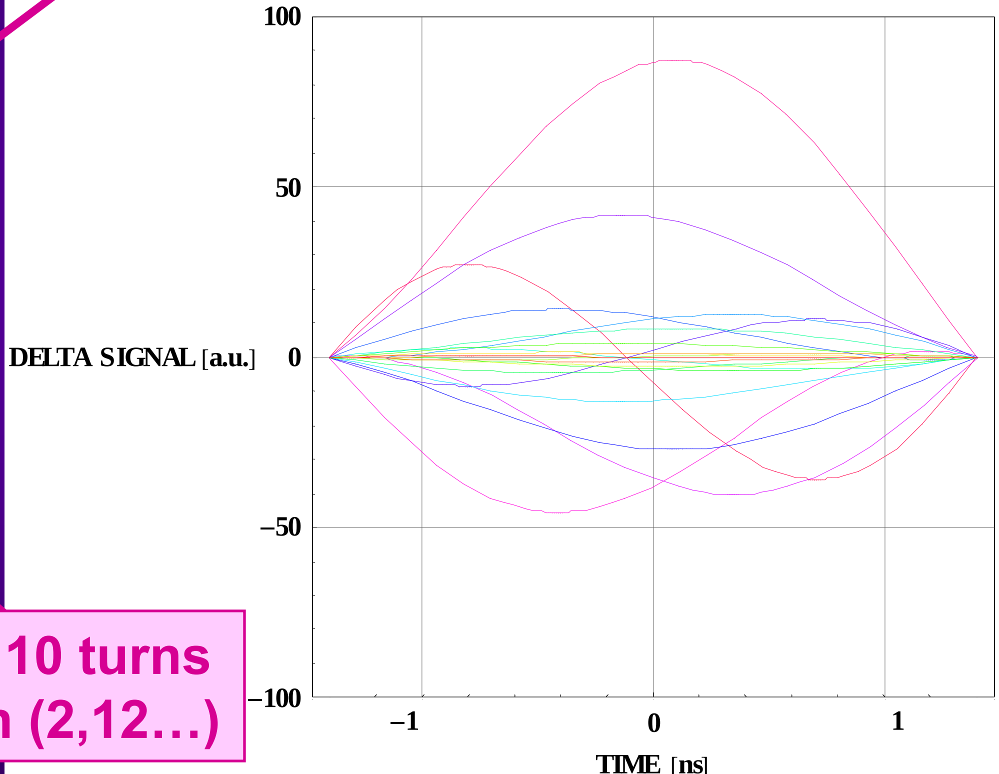
HEAD-TAIL & TMC INSTABILITY THEORY (22/27)

Last trace = turn 202



Every 10 turns shown (2,12…)

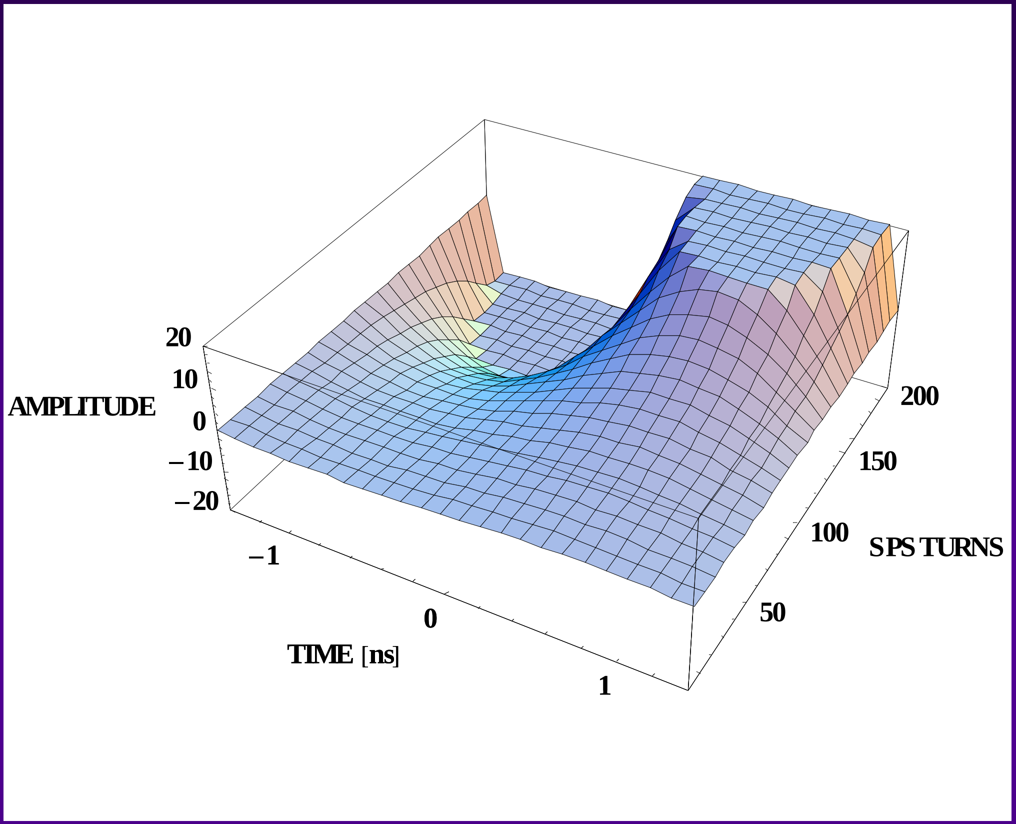
1st trace = turn 2

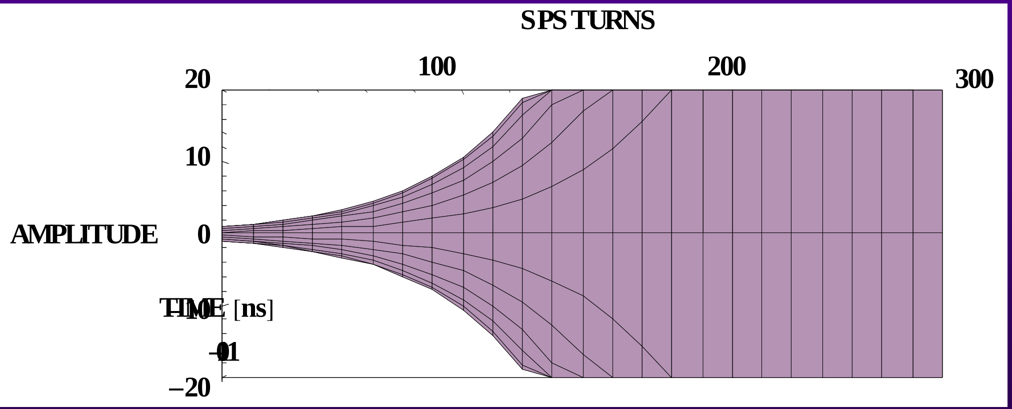


HEAD-TAIL & TMC INSTABILITY THEORY (23/27)

- **For mode**

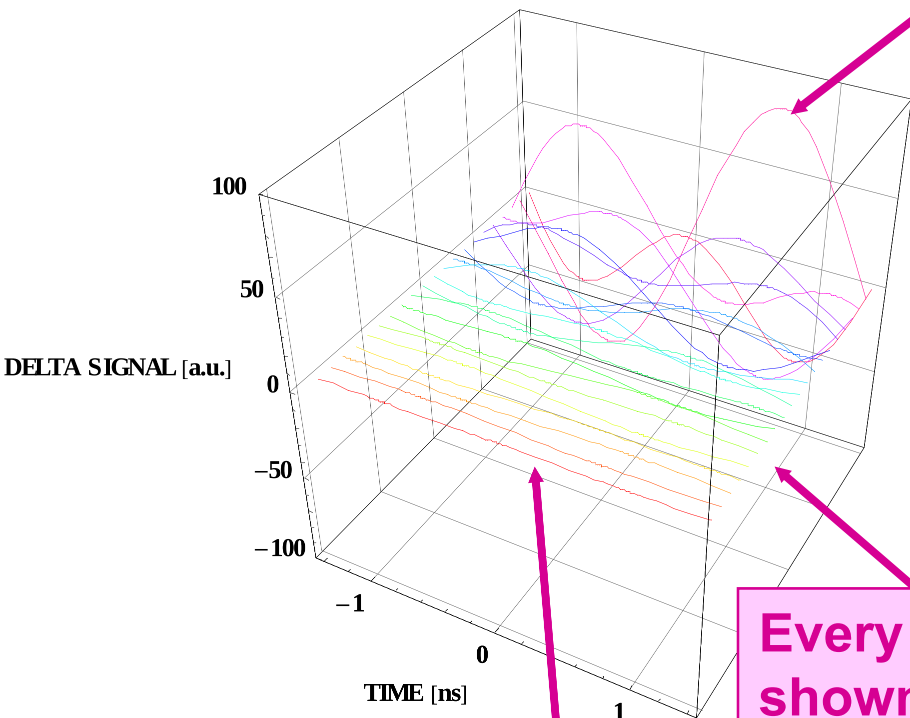
$$\left| m \right| = 1$$





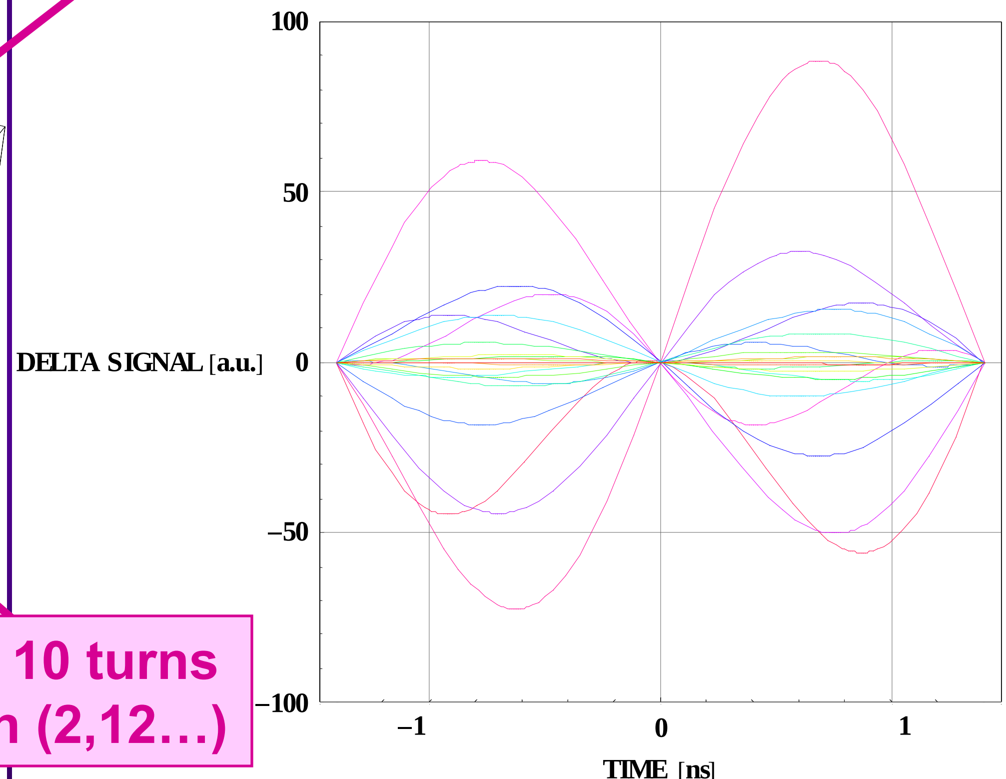
HEAD-TAIL & TMC INSTABILITY THEORY (24/27)

Last trace = turn 202



Every 10 turns shown (2,12…)

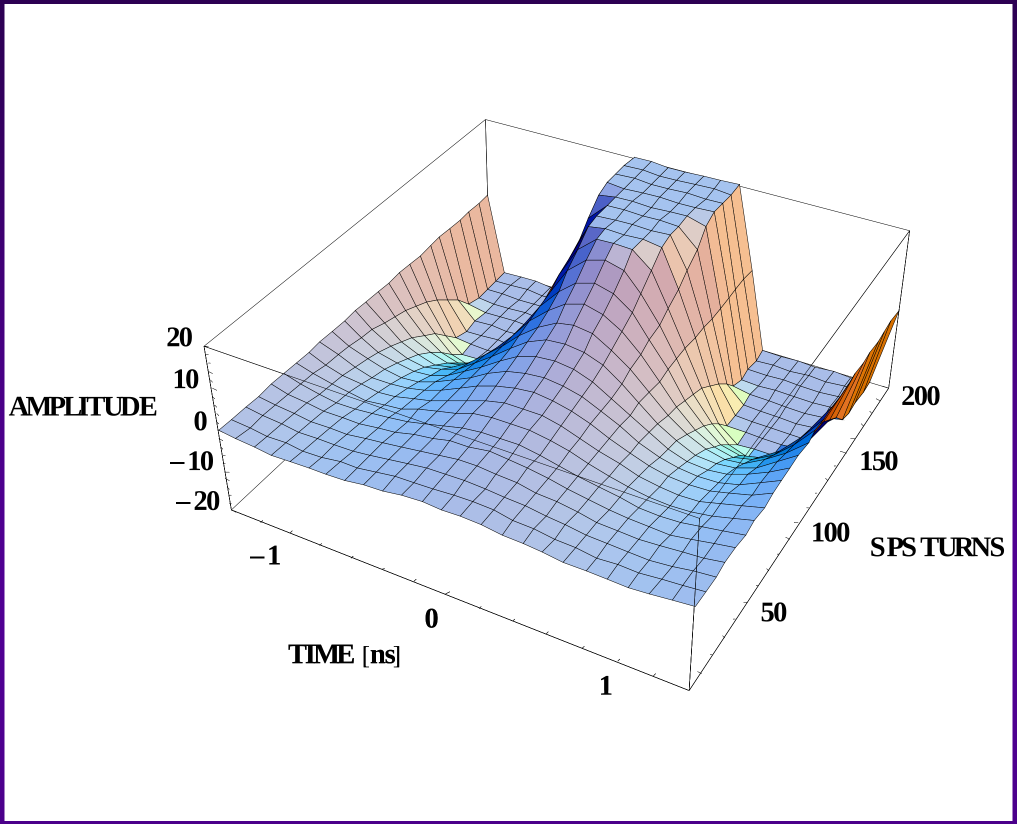
1st trace = turn 2

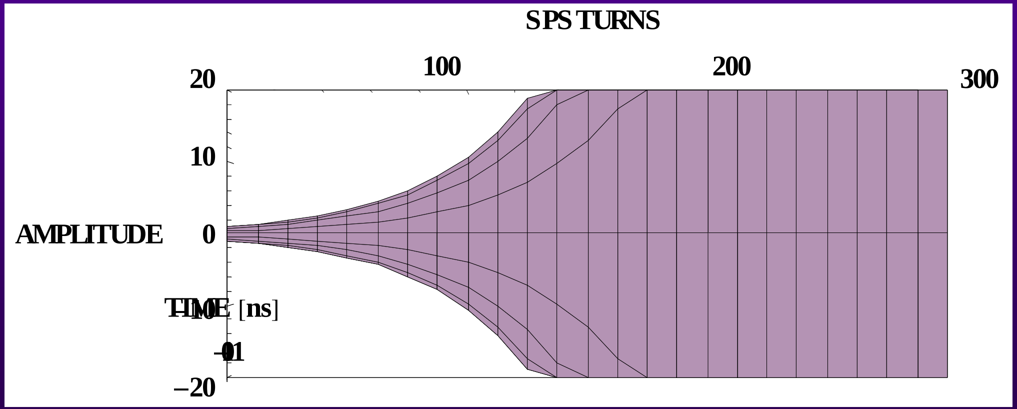


HEAD-TAIL & TMC INSTABILITY THEORY (25/27)

- **For mode**

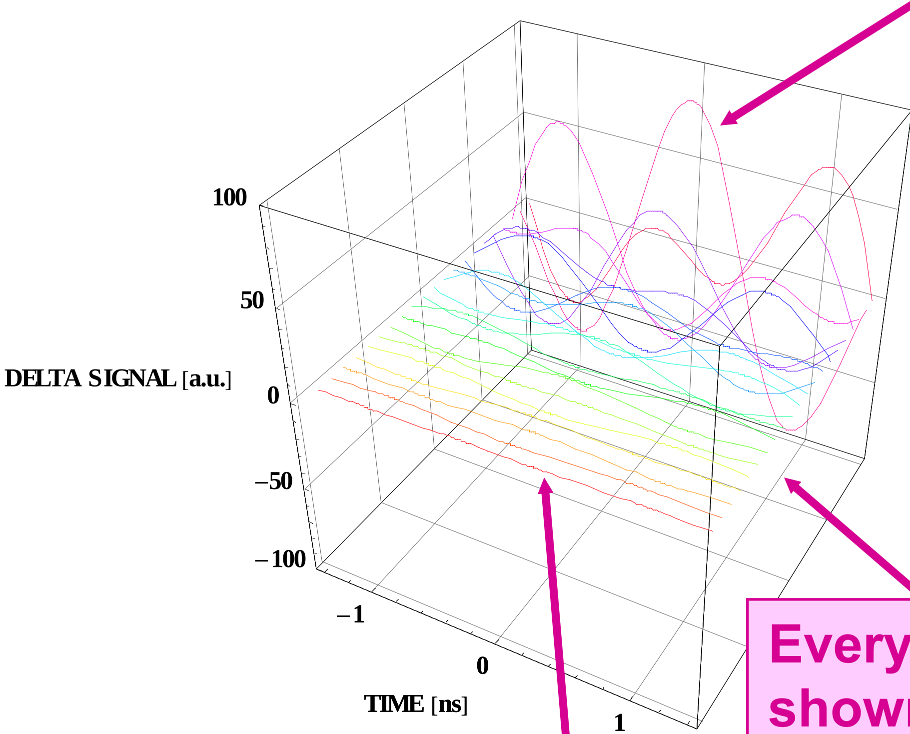
$$\boxed{|\,m\,| = 2}$$





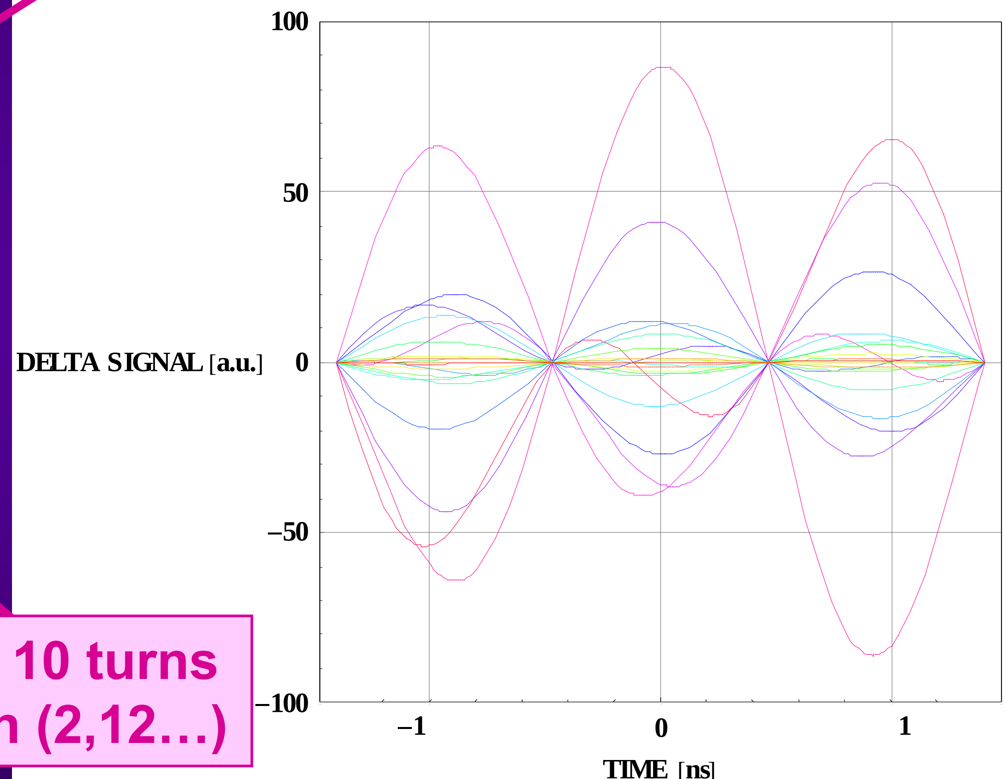
HEAD-TAIL & TMC INSTABILITY THEORY (26/27)

Last trace = turn 202

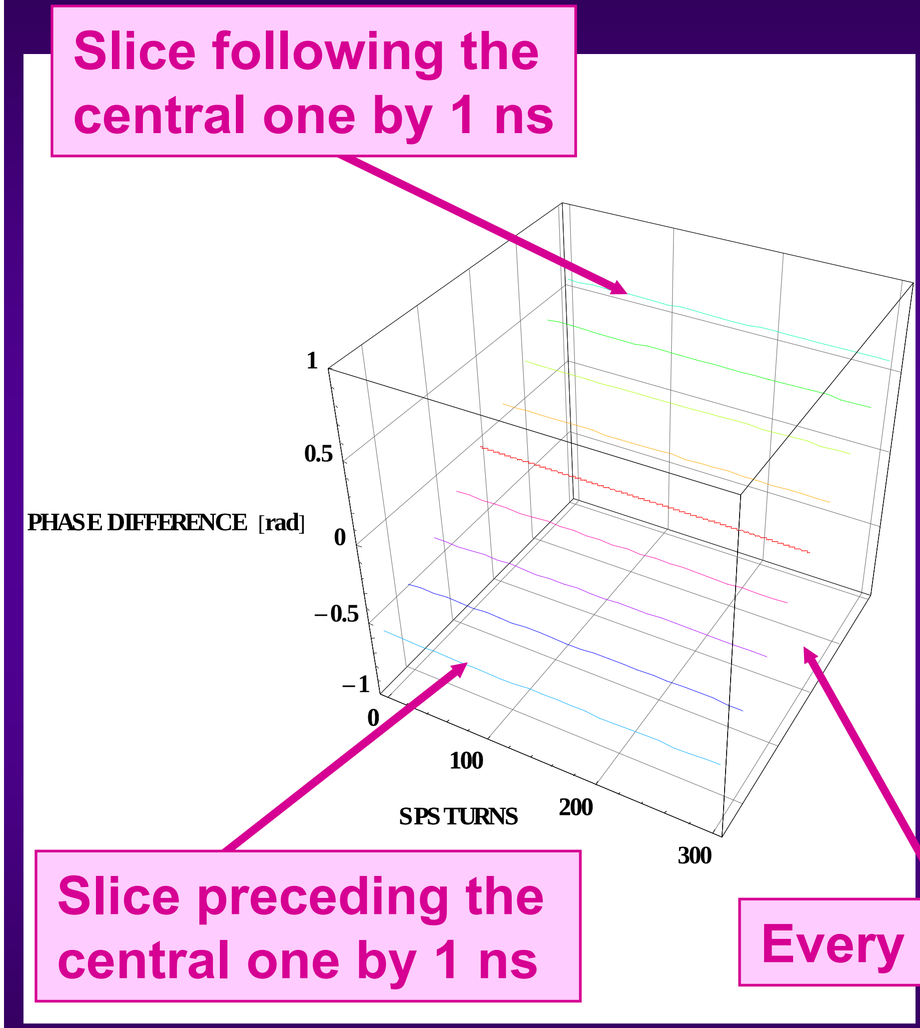


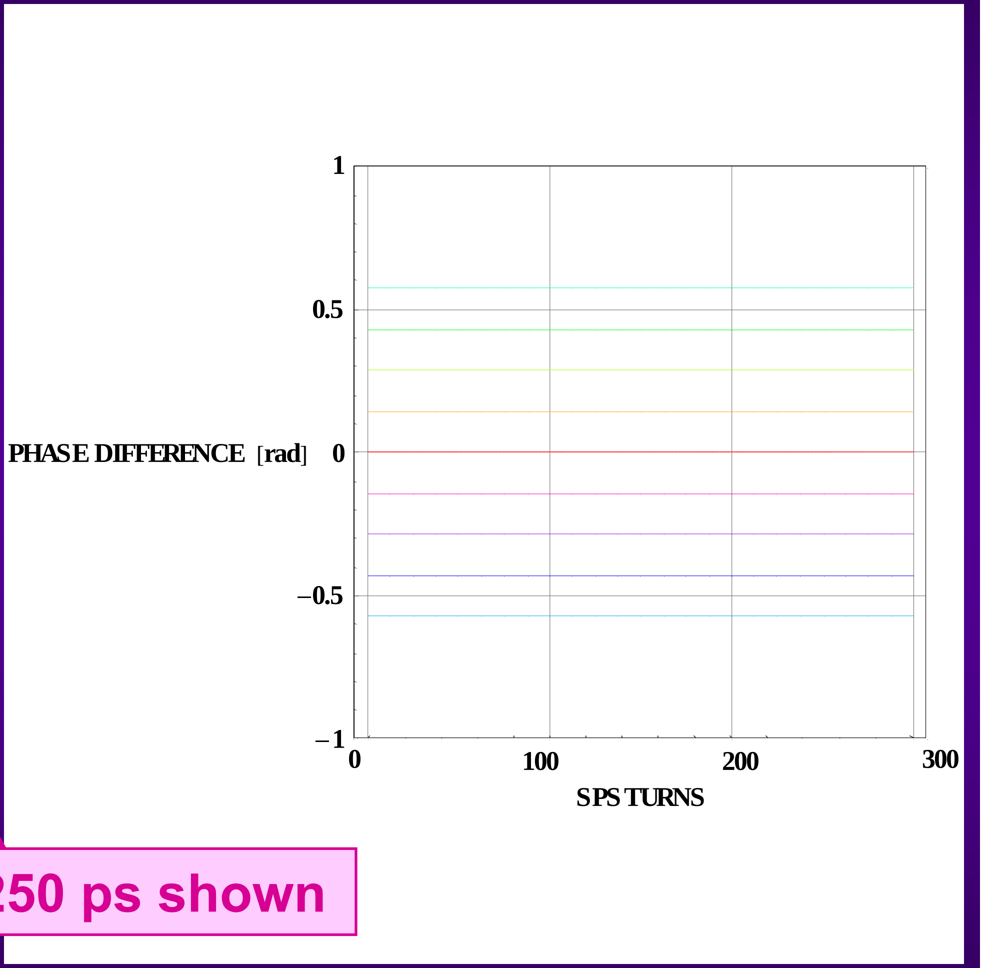
Every 10 turns shown (2,12…)

1st trace = turn 2



HEAD-TAIL & TMC INSTABILITY THEORY (27/27)





BBU INSTABILITY THEORY (1/9)

Yokoya DESY 86-084, Brandt&Gareyte (CERN SPS/88-17 (AMS))

$\xi_y = 0$

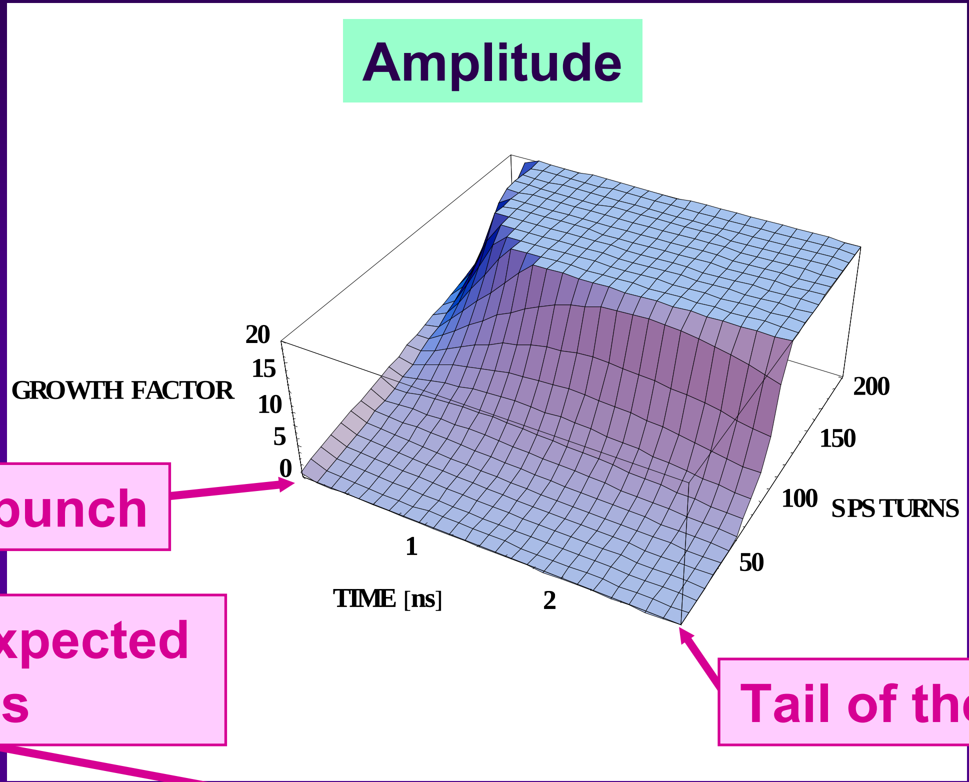
$T_s = \infty$

BB resonator impedance

$R_y = 10\ \mathrm{M\Omega/m}$

$f_r = 700\ \mathrm{MHz}$

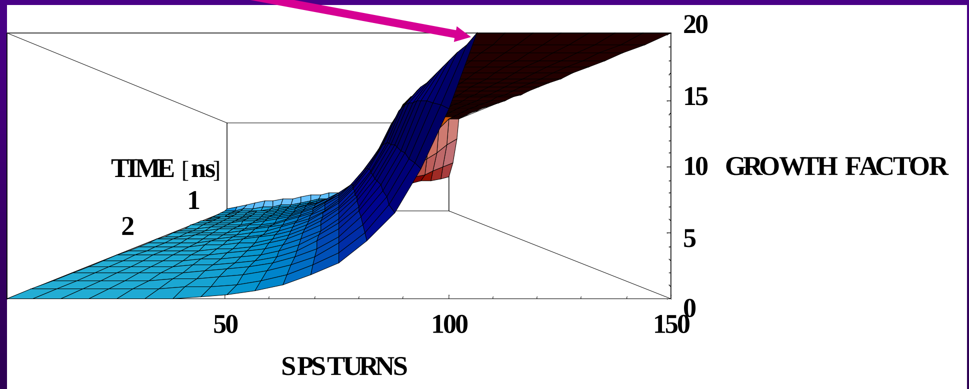
$Q = 1$



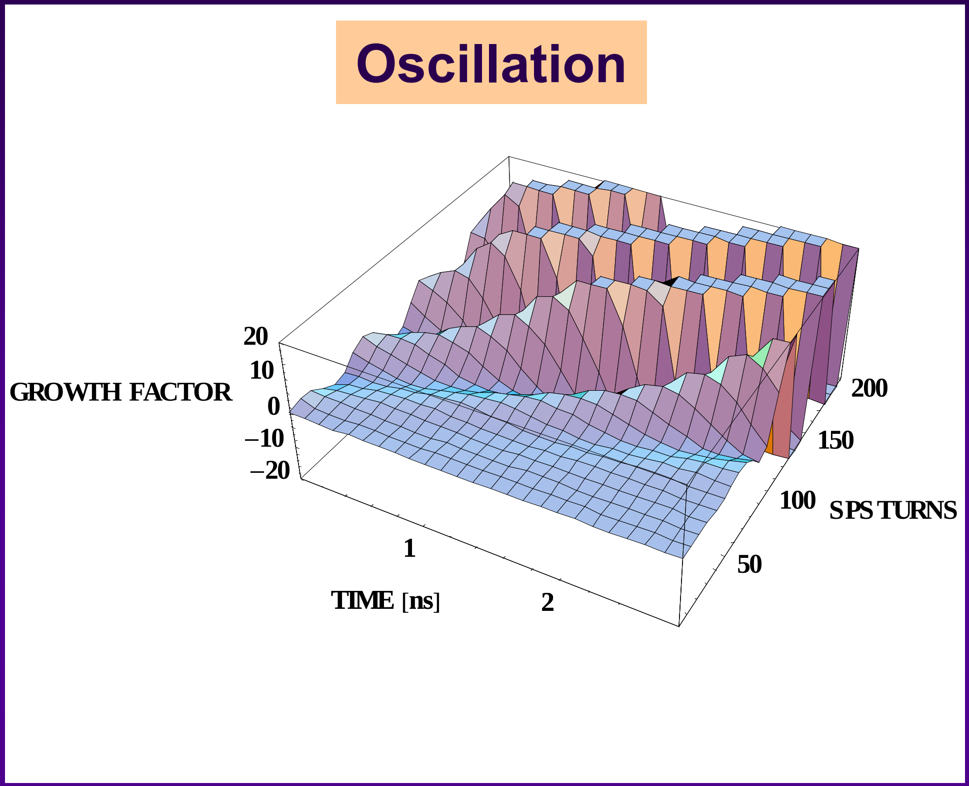
Head of the bunch

Tail of the bunch

Beam losses expected after ~ 100 turns



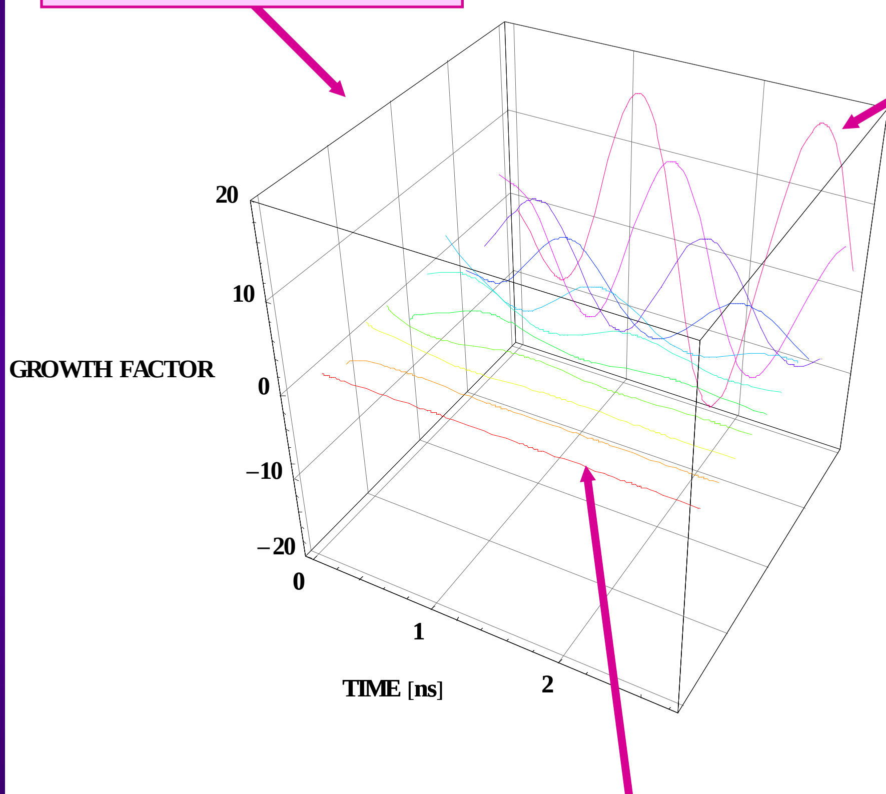
BBU INSTABILITY THEORY (2/9)

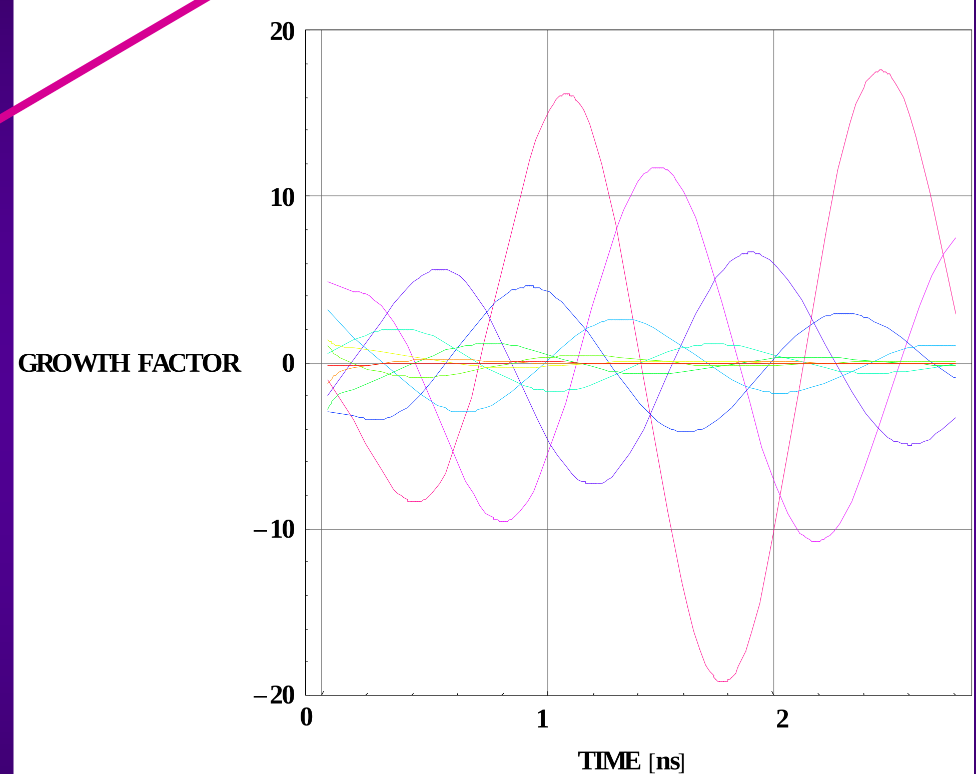


BBU INSTABILITY THEORY (3/9)

Every 10 turns shown (2,12…)

Last (11th) trace = turn 102

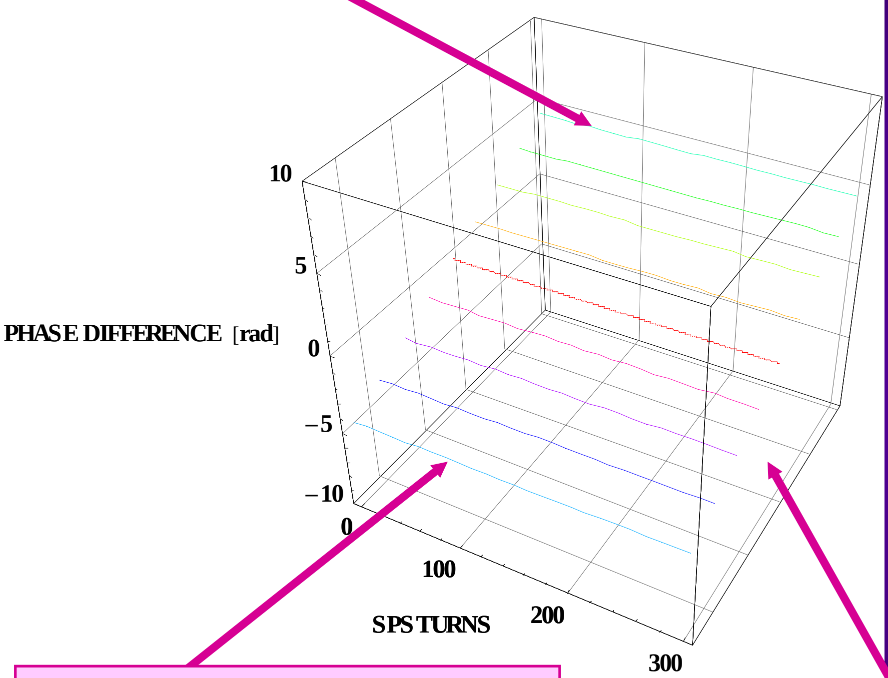




1st trace = turn 2

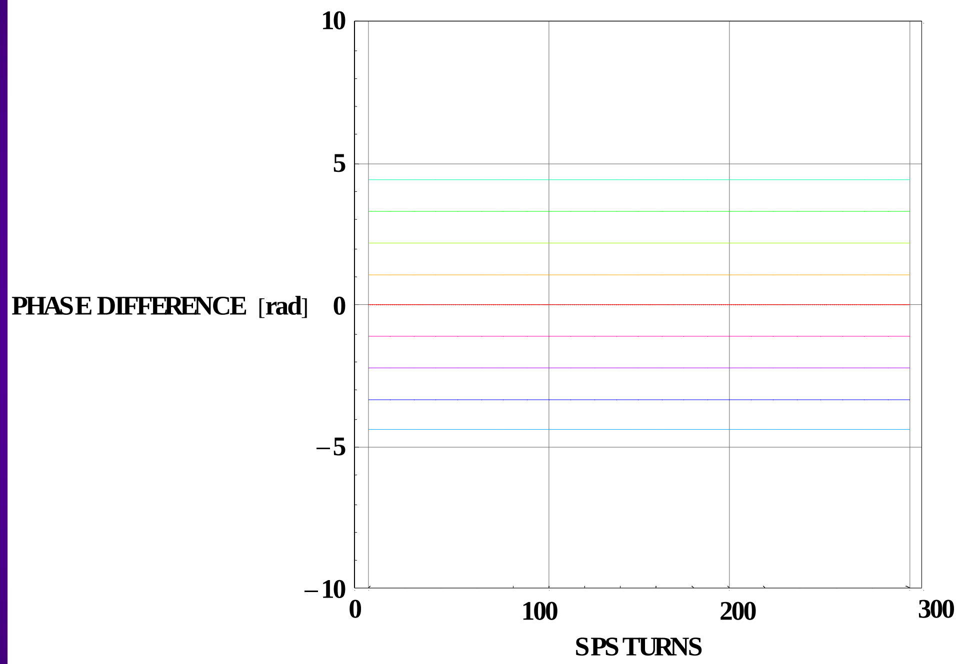
BBU INSTABILITY THEORY (4/9)

Slice following the central one by 1 ns

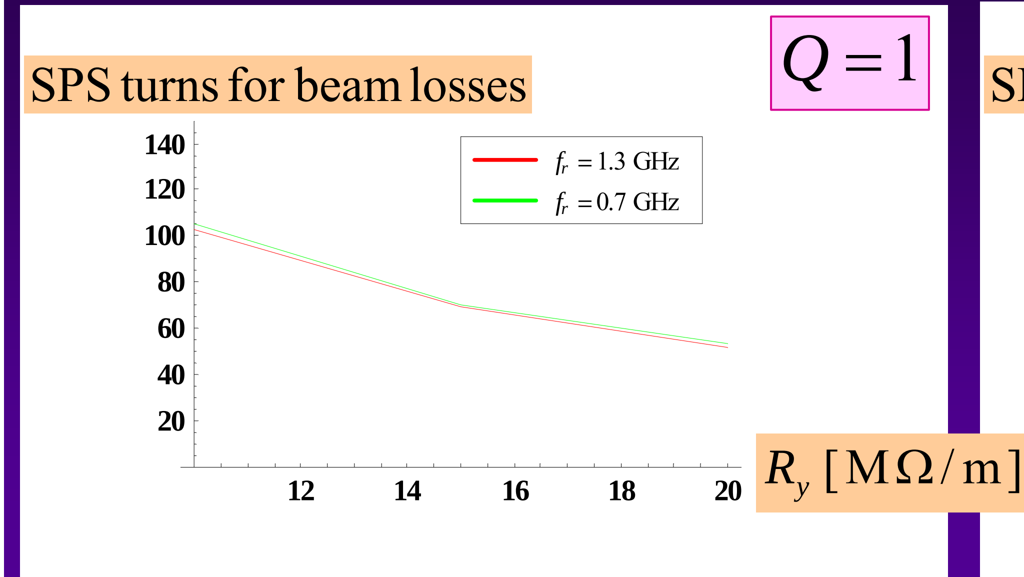


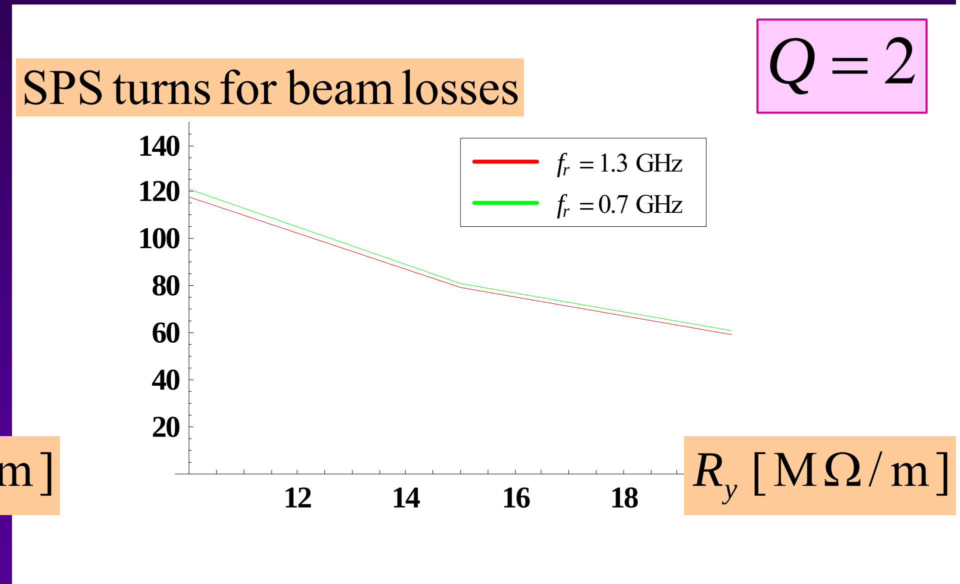
Slice preceding the central one by 1 ns

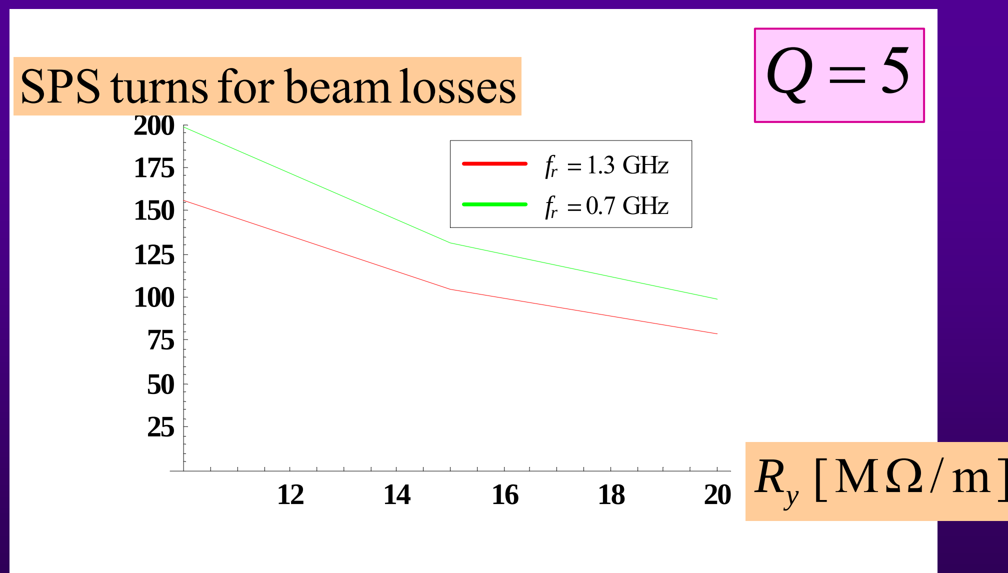
Every 250 ps shown



BBU INSTABILITY THEORY (5/9)



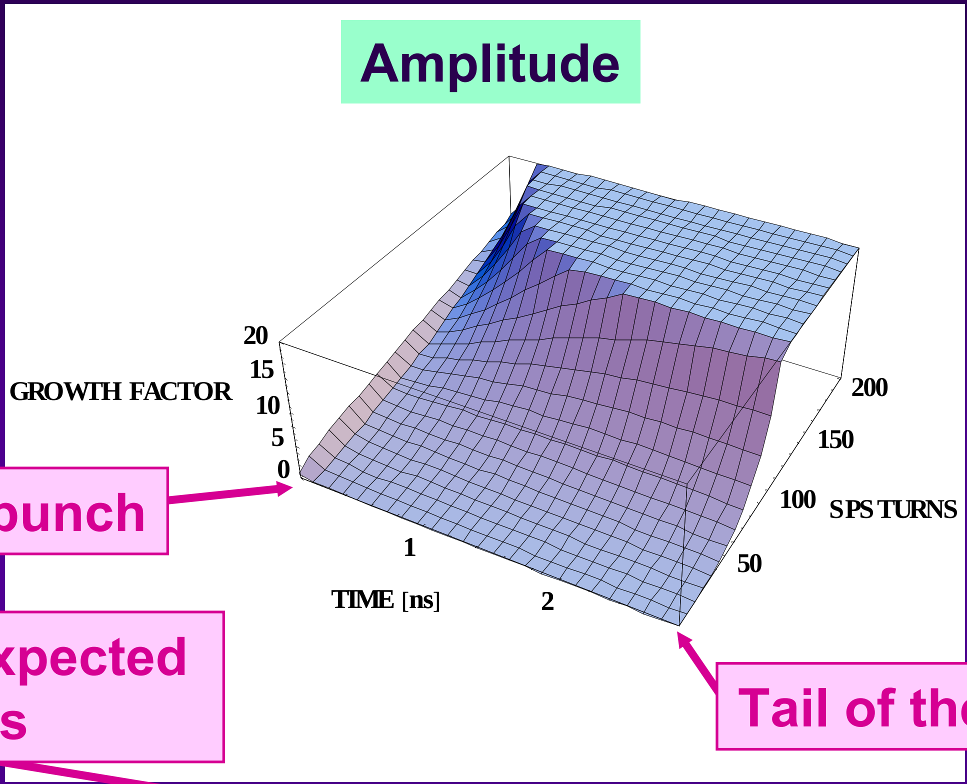




BBU INSTABILITY THEORY (6/9)

$\xi_y = 0$

$T_s = \infty$



BB resonator impedance

$R_y = 10 \text{ M}\Omega/\text{m}$

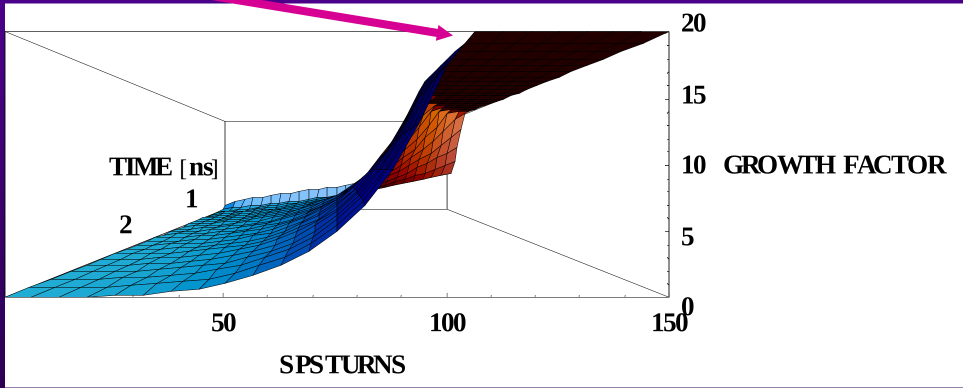
$f_r = 400 \text{ MHz}$

$Q = 1$

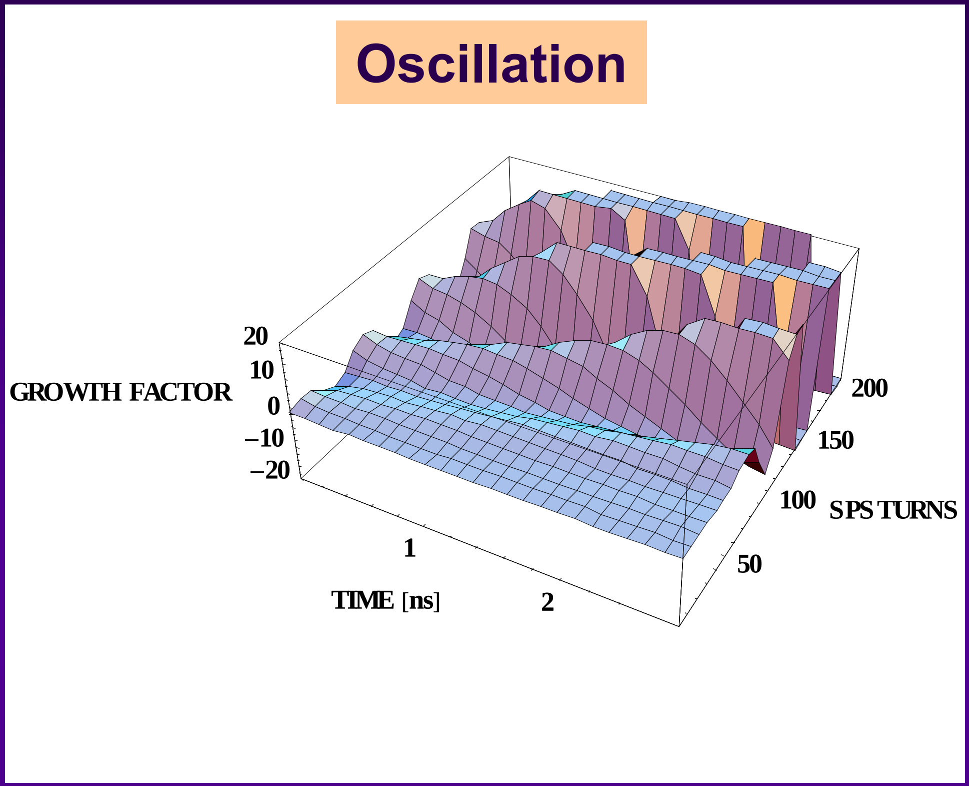
Head of the bunch

Tail of the bunch

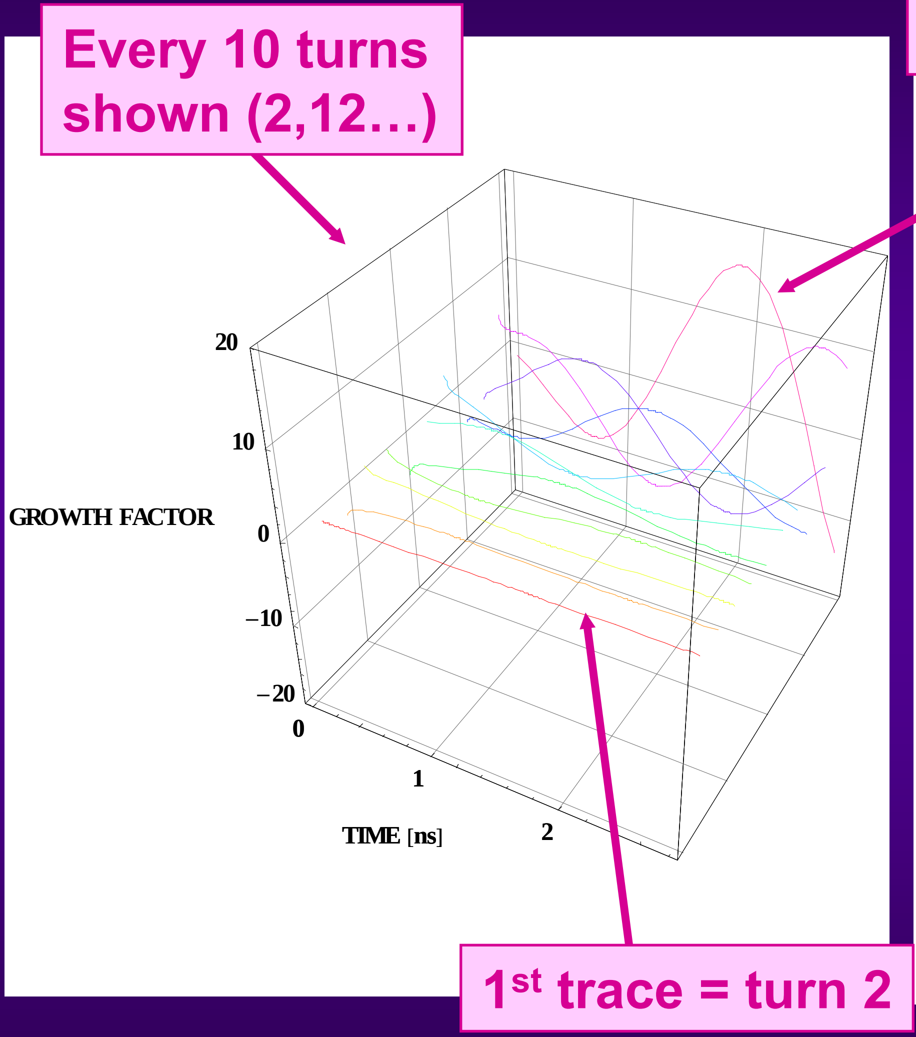
Beam losses expected after ~ 100 turns

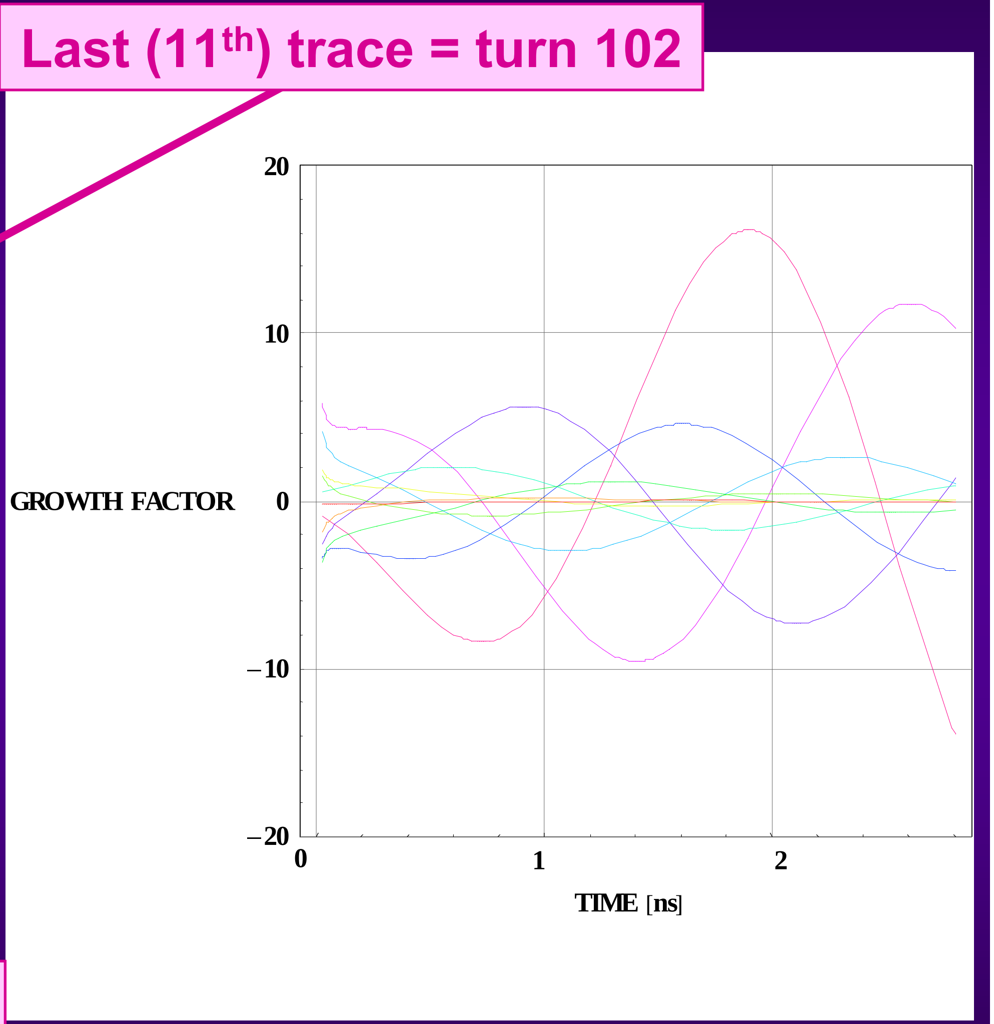


BBU INSTABILITY THEORY (7/9)



BBU INSTABILITY THEORY (8/9)

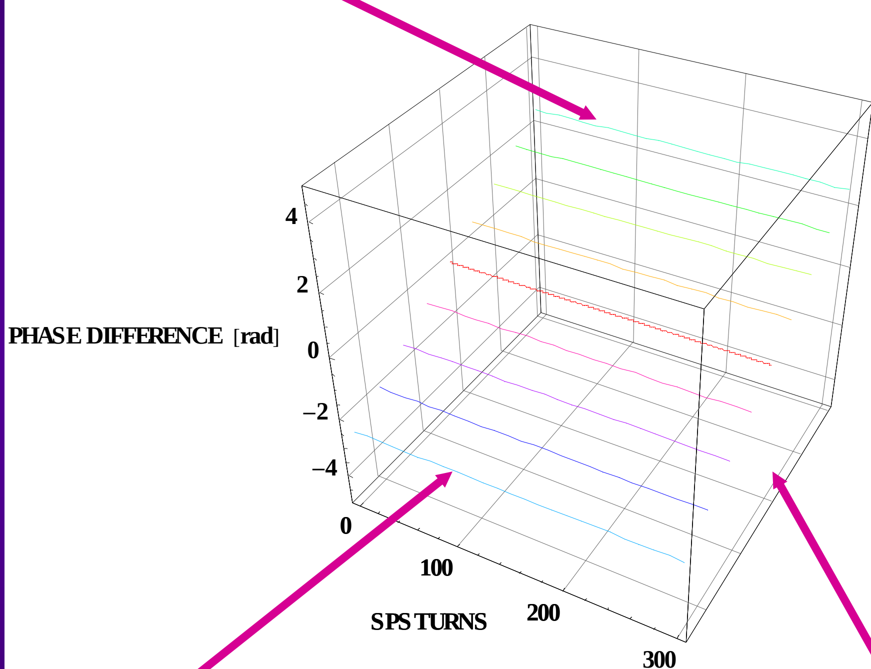




BBU INSTABILITY THEORY (9/9)

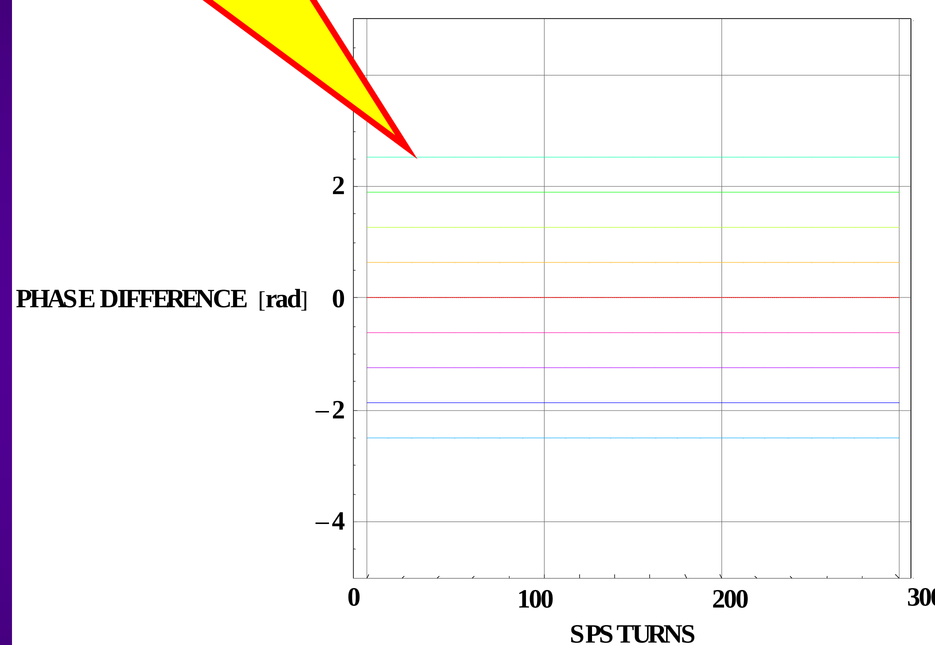
Very close to the measurements! (cf. p. 5)

Slice following the central one by 1 ns



Slice preceding the central one by 1 ns

Every 250 ps shown



CONCLUSION (1/5)

- The time evolution from injection to beam losses seems to be in good agreement with both TMC and BBU theory (for low chromaticity)

- Assuming a Broad-Band resonator impedance with

$$\boxed{R_y = 10\ \mathrm{M\Omega/m}} \quad \boxed{f_r = 700\ \mathrm{MHz}} \quad \boxed{Q = 1}$$

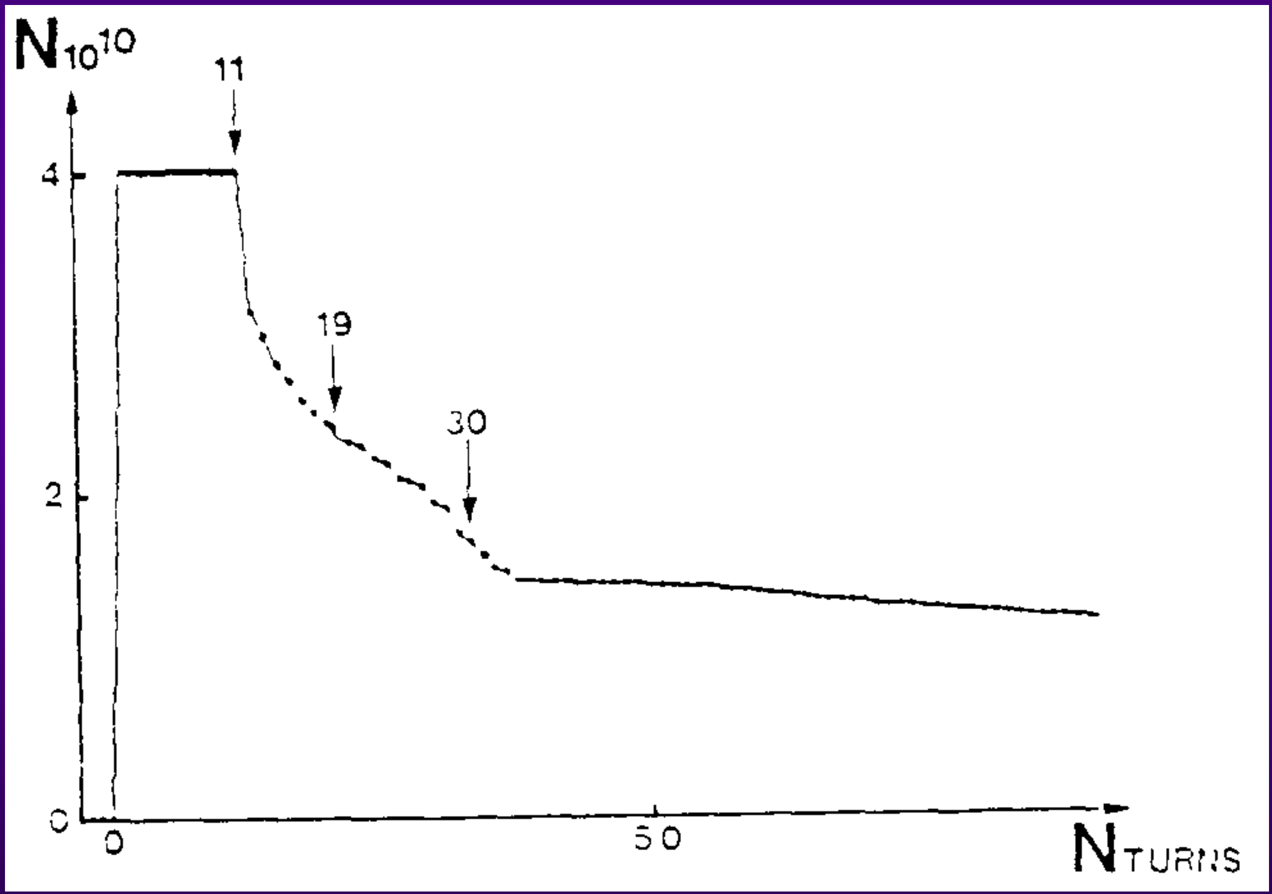
yields beam losses after

  - ~ 130 SPS turns from the TMC theory
  - ~ 105 SPS turns from the BBU theory

whereas during the MD beam losses occurred after ~ 100-150 turns

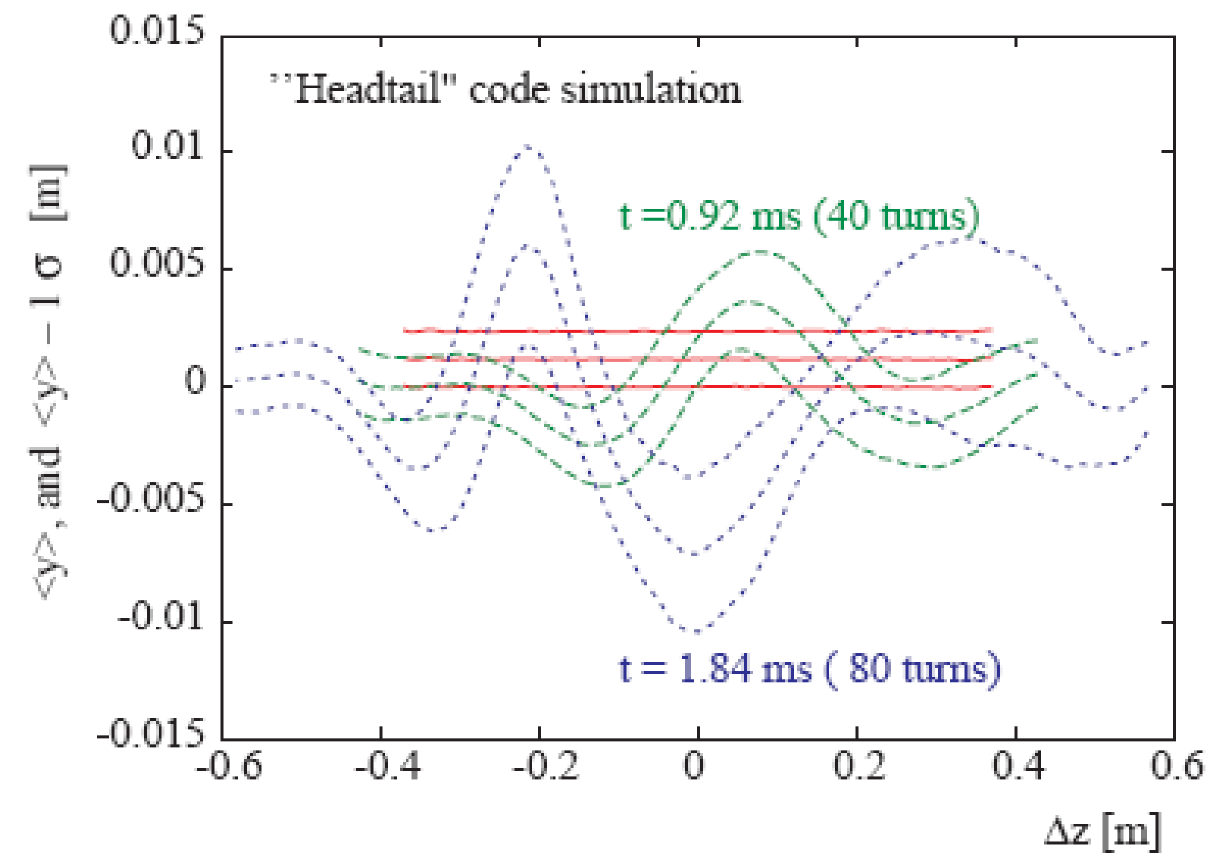
CONCLUSION (2/5)

- The BBU theory was already used in 1988 by Gareyte&Brandt to explain the fast (11 turns) instability of positron bunches observed at SPS injection



CONCLUSION (3/5)

- More detailed simulations are planned with the HEADTAIL code. One case already studied (with similar beam parameters) is shown below (cf. CERN-AB-2004-055). This picture is similar to the one obtained with the BBU formalism (careful check of parameters!...)



$$\sigma_\tau = 0.7 \text{ ns}$$
$$\Leftrightarrow \sigma_z = 21 \text{ cm}$$

CONCLUSION (4/5)

- The TMC instability formalism clearly shows the increase of the instability rise-time with chromaticity, as observed qualitatively. A quantitative comparison remains to be done

- The analysis of the betatron phase differences between different temporal slices with respect to the central slice (=centre of the bunch) revealed interesting features, as seen below

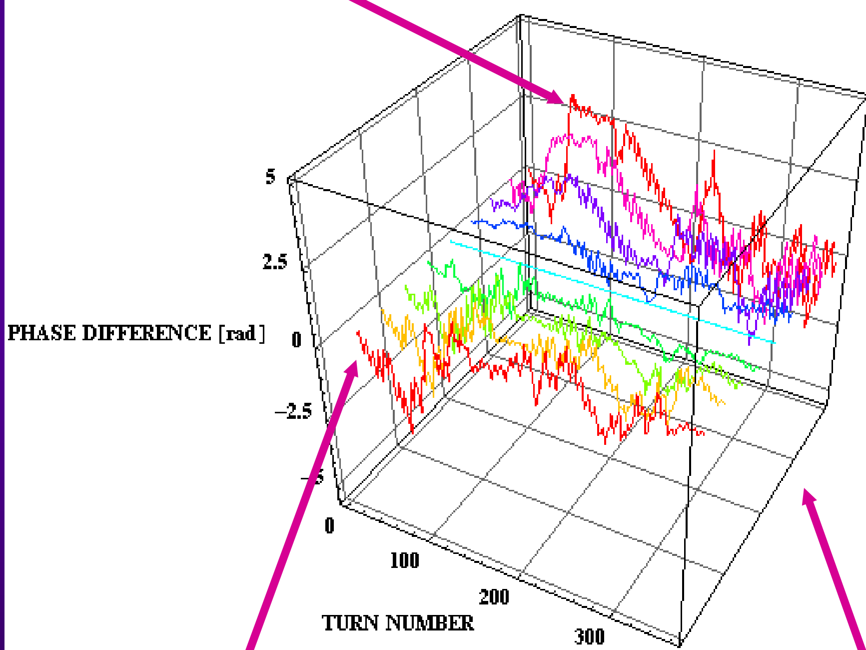
CONCLUSION (5/5)

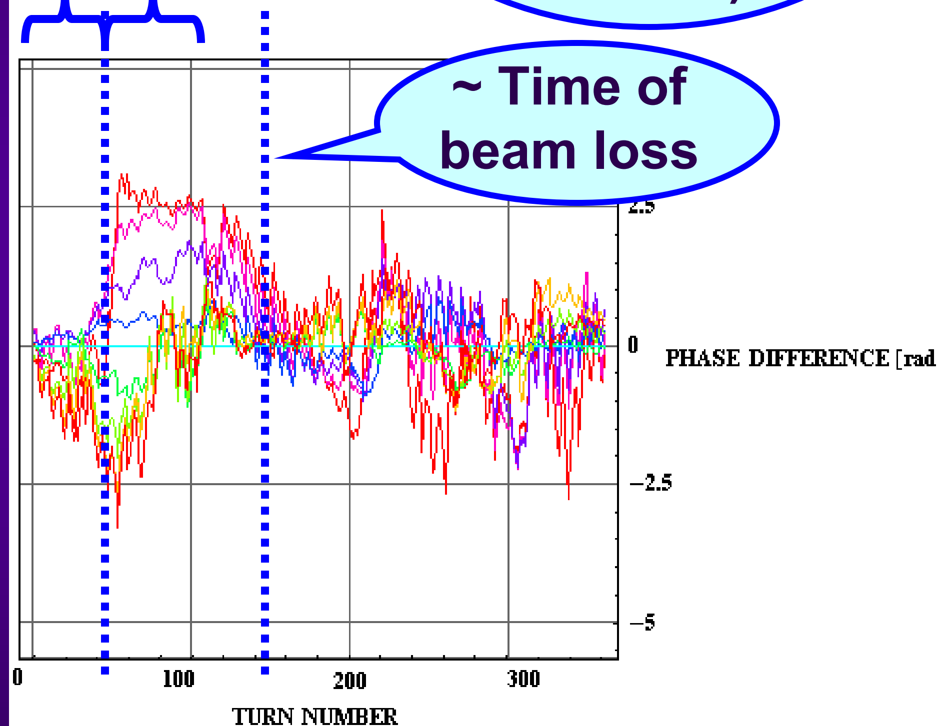
Measurements (see p. 5)

It takes a certain time to arrange the phases

The phases are arranged for a coherent instability (the centre of mass changes along the bunch)

Slice following the central one by 1 ns





~ Time of beam loss

Slice preceding the central one by 1 ns

Every 250 ps shown