# REVIEW OF SOME LANDAU DAMPING RESULTS

E. Métral

- **FZ's sentence at RLC meeting 07/07/06 "combine results from Hereward, Möhl-Schönauer, Chao, Métral-Ruggiero"**

- **Action of RLC meeting held on 07/07/06: "There was some discussion by EM and FZ on the differences or similarities between Hereward and Mohl-Schönauer, i.e., whether they disagree or whether Hereward did not study the same case. FZ mentioned that he could not find the 1969 paper of Hereward quoted by Kornilov ⇒ ACTION: Clarify differences in dispersion relations? (EM?)"**

# What Hereward, Möhl-Schönauer, Chao, Métral-Ruggiero did?

- Laslett-Neil-Sessler (1965)
- Hereward (1969)
- Möhl-Schönauer (1974) and Möhl (1995)
- Chao's book (1993)
- Berg-Ruggiero (1996)
- Métral (1997) $\Rightarrow$ Coupled Landau damping
- Métral-Ruggiero (2004)
- Métral-Verdier (2004) $\Rightarrow$ Computation for a beam collimated at an arbitrary number of sigmas

… D. Sagan, R.D. Kohaupt, A. Hofmann, K. Hübner - A.G. Ruggiero - V.G. Vaccaro …

# Chao's book (1993) (1/2)

- Dispersion relation derived and solved considering an externally given beam frequency spectrum and using the single-particle formalism $\Rightarrow$ Distribution function appears in the dispersion integral

- Used (to try) to explain the physical origin of Landau damping

# Chao's book (1993) (2/2)

- **It is said on page 219:**

In an accelerator, the spread in natural frequency of the beam comes from several sources. A dependence of the betatron frequency $\omega_\beta$ on the energy of the particle, together with an energy spread in the beam, leads to a spread in $\omega_\beta$. Nonlinearities in the focusing system cause a dependence of $\omega_\beta$ on the particle's betatron amplitude. A spread in betatron amplitudes then leads also to a spread in $\omega_\beta$.[5] In the longitudinal case the source of frequency spread depends on whether the beam is bunched or unbunched. For bunched beams, a spread in the synchrotron frequency $\omega_s$ can result from nonlinearity in the rf focusing voltage. For unbunched beams, dependence of the revolution frequency on the particle energy plays a similar role. In the following, for all cases studied, we will simply assume a frequency spread specified by an externally given beam frequency spectrum.

[5] In the following, we assume the spread in $\omega_\beta$ is independent of the amplitude of excitation due to the instability. This will not be true if the $\omega_\beta$ spread is caused by nonlinearities. This subtlety, however, will not be pursued. Interested reader may refer to H. G. Hereward, CERN Report MPS/DL 69-11 (1969), where it is shown that the equivalent of Eq. (5.7) reads

$$\langle x \rangle = -\frac{\pi}{2} A e^{-i\Omega t} \int_0^\infty dJ \, \frac{J\rho'(J)}{\omega(J) - \Omega}, \qquad (5.1)$$

# Hereward (1969)

- The case where the frequency spread comes from the longitudinal momentum spread of the beam is straightforward (for a coasting beam), because the longitudinal momentum is a constant, which just affects the coefficients in the equations of motion of the transverse oscillations, and hence their frequencies. It can be dealt with the same method as in Chao. The same result applies also if one considers a tune spread that is due to a non-linearity in the other plane. However, this result is no longer valid if the non-linearity is in the plane of coherent motion. In this case, the steady-state is more involved because the coherent motion is then a small addition to the large incoherent amplitudes that make the frequency spread, and it is inconsistent to assume that it can be treated as a linear superposition. One needs to consider "second order" non-linear terms and one finds then that the dispersion integral does not contain the distribution function but its derivative. This is easier to show starting from the Vlasov equation. The work of Hereward1969 was to derive the same dispersion relation as the one obtained with the Vlasov equation starting from the single-particle equation formalism

# Möhl-Schonaeur (1974) and Möhl (1995) (1/2)

- Use the single-particle equation formalism (considering also the "second" order nonlinear terms as proposed by Hereward) but add the nonlinear space charge effects

Incoherent force

$$\frac{d^2 x}{d\,t^2} + \Omega^2 \left( Q_{x00}^2 + 2\, Q_{x00}\, \Delta Q_{incoh}^x \right) x = -2\, \Omega^2\, Q_{x00} \left( \Delta Q_{coh}^x - \Delta Q_{incoh}^x \right) \overline{x}$$

External focusing force

Coherent force

# Möhl-Schonaeur (1974) and Möhl (1995) (2/2)

- **Definition of the transverse impedance**

$$Z_x \propto \Delta Q^x_{coh} - \Delta Q^x_{incoh}$$

- **Ex.1: Case of the space charge impedance (round beam in a round pipe)**

$\Delta Q^x_{incoh}$ $\Delta Q^x_{coh}$

$$Z_x^{SC} = - \frac{j \, L \, Z_0}{2 \, \pi \, \beta \, \gamma^2} \left[ \frac{1}{a^2} - \frac{1}{b^2} \right]$$

- **Ex.2: Case of a flat vacuum chamber**

**From the detuning (or quadrupolar $\Rightarrow$ incoh.) wake**

$$\Delta Q^x_{coh} = \Delta Q^x_{Z_x} + \Delta Q^x_{incoh}$$

**$\Rightarrow$ This explains why a zero horizontal coherent tune shift is measured in flat chambers (as e.g. in the SPS)**

# Métral-Ruggiero (2004)

- Work from Berg-Ruggiero (1996) who derived (using the Vlasov's formalism) and solved the dispersion relation using the 2-dimensional tune spread introduced by an octupole
  - Quasi-parabolic distribution used $\Rightarrow$ Valid for a beam extending up to ~ 3.2 $\sigma$
  - 2-dimensional tune spread already present in the work by Möhl-Schönauer but more precise computation here (solved analytically)
- Extension of this work introducing the nonlinear space-charge effects (same physics as the one from Möhl-Schönauer but expected to be more precise mathematically)
  - Quasi-parabolic distribution used
  - Space charge derived self-consistently
  - Solved analytically in an approximate case